Coupling between Different Oscillation Branches in a Waveguide-Mode Free-Electron Laser

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The coupling between different oscillation branches of a waveguide-mode free-electron laser has been studied. It was observed that the wave form of the upper branch of a waveguide-mode free-electron laser is modulated at the rate of the period of the mode-locked oscillation of the lower branch for a wide range of experiment parameters. This phenomenon is explained as the temporally periodic modulation of the gain of the upper-branch oscillation by the electromagnetic radiation of the lower-branch oscillation through the periodic bunching of the electron beam.

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Free-electron lasers using a cavity resonator of the Fabry-Pérot type have only one oscillation branch [1-3]. When a waveguide is used as the cavity resonator of the oscillator, the free-electron laser has two independent oscillation branches due to the nonlinearity of the dispersion curve of the electromagnetic radiation in the waveguide. These branches have different oscillation frequencies and are called the upper and lower branches of the free-electron-laser oscillation. In this paper, our observation of the coupling between different oscillation branches of a waveguide-mode free-electron laser is described, and the mechanism of the coupling is discussed.

The experimental setup is shown in Fig. 1. An electron beam from a plasma cathode [4] is accelerated electrostatically and introduced into a waveguide cavity resonator through a small hole at the corner of the waveguide oscillator. The acceleration voltage can be varied from 500 to 540 keV. The time constant of the voltage decay is about 20 μs, and thus the voltage change is about 1% for 200 ns. The wave form of the electron-beam current is not reproducible, and the average current is about 20 A. The energy spread of this electron beam ($\Delta E/E$) is expected to be of the order of 0.1%, judging from the characteristics of the cold relativistic-electron-beam source [4] which is the origin of the electron beam used in this experiment. The number of periods of the wiggler is 20. The adiabatic regions consists of 2.5 pitches, in which the magnetic-field strength of the wiggler (B_g) gradually increases. The pitch of the wiggler period (λ_w) is 5 cm. The waveguide resonator is composed of a straight waveguide tube with two corners attached on both ends. Two mirrors made of flat copper plates are attached on the ends of both corners. One of the mirrors has a coupling hole of 3 mm in diameter to extract output power. The dimensions of the waveguide and the length between mirrors along the axis of the waveguide are 2.29 cm × 1.02 cm and 122 cm, respectively. A longitudinal magnetic field (B_g) of 2.1 kG is applied in order to guide the electron beam along the axis of the waveguide. The output power of the free-electron-laser oscillation is introduced into a crystal detector (1N26) through a tapered waveguide and then through a waveguide operating as a high-pass filter for the output signal of the free-electron-laser oscillation. The cutoff frequency of this waveguide is 21.1 GHz. The length is 2 m, which is long enough for it to operate as a high-pass filter for a signal higher than the cutoff frequency. The signal is attenuated at a rate of 70 dB in order to obtain linear operation characteristics of the crystal detector. A pair of microwave horns were used for the electrical insulation between the cavity resonator and the signal detection system. The behavior of the electron beam in the wiggler and the small signal gain have been measured in previous papers [5,6].

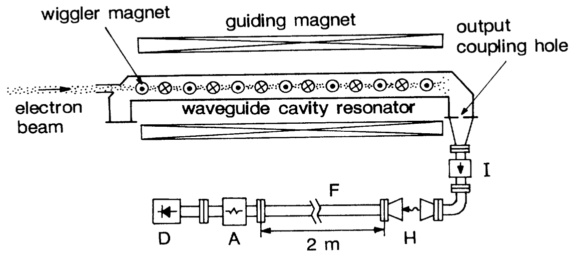

FIG. 1. Experimental setup of the waveguide-mode free-electron laser. *I*: isolator; *H*: microwave horns; *A*: attenuator; *D*: microwave detector (1N26 crystal diode); and *F*: high-pass filter.

The dispersion relation of the electromagnetic radiation in the waveguide is given by

$$(2\pi f)^2=(ck)^2+(2\pi f_c)^2\,, \qquad (1)$$

where f and k are the angular frequency of the electromagnetic radiation and the wave number of the waveguide, respectively. c is the light velocity. f_c is the cutoff frequency of the TE_{10} waveguide mode, which was calculated to be 6.55 GHz. The dispersion relation for the space-charge wave, which is produced by the oscillating

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electron beam in the wiggler, is given by

$$2\pi f = \beta c (k + k_w) , \qquad (2)$$

where f and k are the frequency and the wave number, respectively. β is defined by $\beta = v/c$, where v is the parallel velocity component of the electron beam. k_w is the wave number of the wiggler and defined by $k_w = 2\pi/\lambda_w$, where λ_w is the pitch length of the wiggler. Equations (1) and (2) are shown in Fig. 2. The experimental parameters of $E = 526$ keV, $B_w = 0.36$ kG, and $B_g = 2.1$ kG are used for Fig. 2.

Since the operation of the waveguide-mode free-electron laser is defined as the coupling between the space-charge wave and the waveguide mode, it is determined from the solutions to Eqs. (1) and (2). A pair of solutions are obtained at the crossing points in Fig. 2, which are defined by the oscillation frequencies of the upper branch (f_u) and the lower branch (f_l). For the experimental parameters chosen in this experiment, these are calculated to be $f_u = 34.4$ GHz and $f_l = 6.81$ GHz.

Figure 3 shows typical wave forms of the output power of the free-electron laser. Figure 3(a) shows a periodically modulated wave form, which appears occasionally and is considered to be the self-mode-locked oscillation of a free-electron laser [7-9]. Figure 3(b) shows a randomly modulated wave form, which is considered to be due to interference between a few longitudinal oscillation modes of the cavity resonator. The crystal diode can detect only the signal of the upper-branch oscillation, since f_l is lower than the cutoff frequency of the waveguide operating as a high-pass filter.

Figures 4 and 5 show the pulse-train period of the output wave form as a function of the wiggler magnetic field (B_w) and the energy of the electron beam E, respectively.

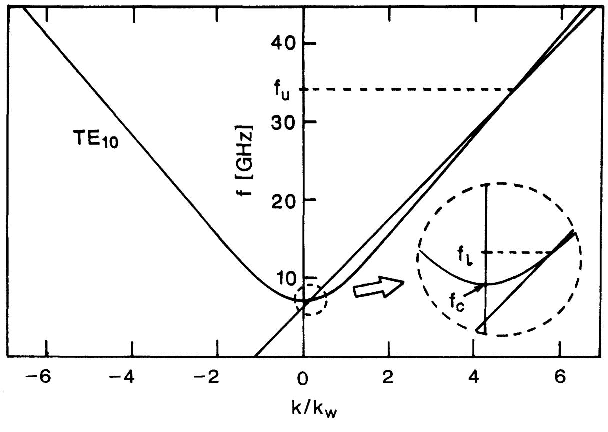
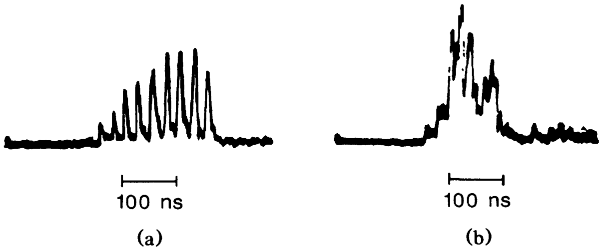


FIG. 2. Dispersion relations of the TE$_{10}$ mode in the waveguide cavity resonator and a space-charge wave. f_u and f_l are the frequencies of the upper-branch and lower-branch oscillations, respectively. f_c is the cutoff frequency of the waveguide cavity resonator. The horizontal scale is normalized by the wave number of the wiggler.



(a) (b)

FIG. 3. Typical wave forms of the output of the waveguide-mode free-electron laser. (a) A periodically modulated wave form, which appears occasionally and is considered to be a self-mode-locked oscillation. (b) A randomly modulated wave form, which is considered to appear due to the interference between a few longitudinal modes of the cavity resonator.

In the waveguide-mode oscillation, the pulse-train period of the mode-locked oscillation corresponds to the round-trip time of the electromagnetic radiation in the cavity with a group velocity (v_g) [7].

The solid lines in Figs. 4 and 5 are the theoretical calculations of the round-trip time of the cavity resonator using the group velocity of the lower branch. For wide ranges of B_w and E, the experimental results are in good agreement with the theoretical calculations. The theoretical calculation of the round-trip time for the upper branch is about one-third of that for the lower branch, and is not in good agreement with the experimental results. The discrepancies between the theoretical and the experimental values for smaller B_w ($B_w < 0.45$ kG) and higher E ($E > 530$ keV) are considered to be due to the fact that f_l approaches the cutoff frequency of the waveguide cavity resonator, and the loss of the cavity increases abruptly in these experimental parameter regions.

The interesting point of the experimental results is the

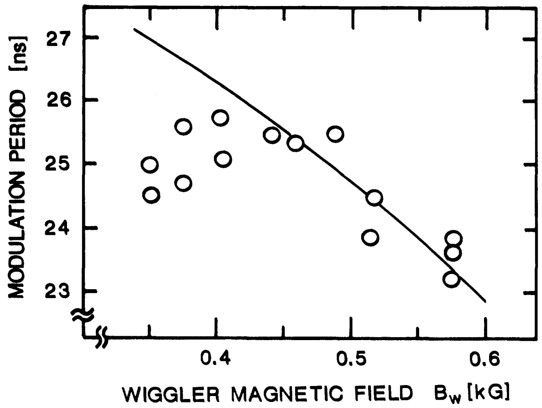


FIG. 4. Pulse-train period of the upper-branch oscillation as a function of the wiggler magnetic field (B_w). The energy of the electron beam (E) is 524 keV. The solid line is the theoretical calculation of the round-trip time in the cavity resonator using the group velocity of the lower branch.

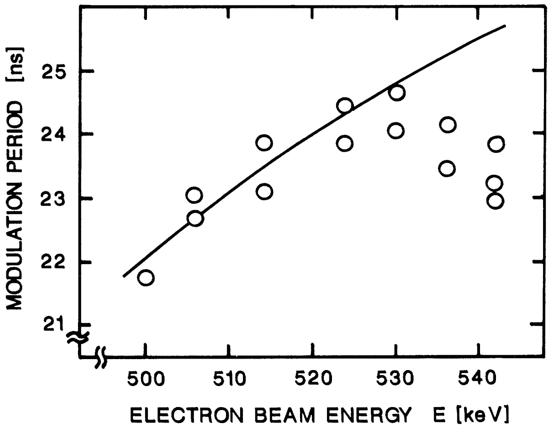


FIG. 5. Pulse-train period of the upper-branch oscillation as a function of the energy of the electron beam (E). The wiggler magnetic field (B_g) is 0.54 kG. The solid line is the theoretical calculation of the round-trip time in the cavity resonator using the group velocity of the lower branch.

fact that the wave form of the upper branch is modulated periodically with a period equal to the self-mode-locked oscillation of the lower branch. These experimental results show that there exists a coupling between the electromagnetic waves of the upper-branch and the lower-branch oscillations in the waveguide cavity resonator through the electron beam. A possible explanation for this phenomenon is now given.

Because of the high gain and high Q value of the cavity for the lower branch, oscillation of the lower branch can be easily initiated in the waveguide cavity resonator, but cannot be extracted out of the cavity due to the small coupling hole. When the self-mode-locked oscillation of the lower branch occurs, the interaction between the electron beam and the electromagnetic radiation of the lower branch is the strongest on the peak of the mode-locked wave form. Bunching of the electron beam occurs only in this region, where the interaction is strongest. In this bunched region of the electron beam, it is impossible for the electron beam to have gain in the upper branch. On the contrary, in the region between two adjacent peaks of the mode-locked oscillations, the interaction between the electromagnetic radiation of the lower branch and the electron beam is very weak. At the center of two adjacent peaks of the mode-locked oscillation, the strength of the electromagnetic field becomes completely zero and there is no more bunching of the electron beam by the lower branch. Therefore, the upper branch can have gain only in this region and the gain for the upper branch is modulated periodically by the period of the mode-locked oscillation of the lower branch. This temporal gain modulation of the upper branch, which is caused by the self-mode-locked oscillation on the lower branch, is considered to be the origin of the phenomenon observed in this experiment.

In conclusion, the coupling between different oscillation branches of a waveguide-mode free-electron laser has been studied. It was observed that the wave form of the upper-branch oscillation of a waveguide-mode free-electron laser is modulated by the period of the mode-locked wave form of the lower-branch oscillation over a wide range of operation parameters. This phenomenon is explained as the temporally periodic modulation of the gain of the upper branch by the electromagnetic radiation of the lower-branch oscillation through the periodic bunching of the electron beam.

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