## Demonstration of a New Free-Electron-Laser Harmonic Interaction

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The first experimental demonstration of a harmonic free-electron-laser amplifier utilizing a periodic position instability is described for a planar wiggler configuration. The interaction occurs at the even harmonics of the fundamental. <sup>A</sup> maximum gain of <sup>7</sup> dB was observed over a frequency band ranging from 14 to 15 GHz. The experimental results are compared with predictions from the three-dimensional simulation code wlGGLIN with excellent agreement. Improvements due to a tapered wiggler for this interaction are discussed.

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The free-electron laser (FEL) dates back over three decades [1,2], and has been intensively studied for over a decade. Recently, harmonic generation has become an important topic for either extending the frequency range of fixed-voltage facilities or reducing the beam voltage required at a given frequency. Reduced beam voltage would have a significant impact on potential applications. This paper describes the first measurement of evenharmonic amplification utilizing a periodic position instability [2,3].

For conventional planar-wiggler FELs the interaction occurs at the fundamental and the odd harmonics [4-11], due to the velocity harmonics present in the unperturbed undulations of the electrons. These harmonics are present even for ideal wigglers with perfect beam injection, and give rise to the periodic velocity instability of the FEL. The even-harmonic interaction considered here, however, requires no higher velocity harmonics. Rather, it depends on a synchronism in the electron position with respect to an antisymmetric radiation field. The interaction can occur with either a transverse or axial electric field. The transverse field must be odd in the direction of the wiggle motion, and the axial field must be even for the respective interactions to occur. For a secondharmonic interaction, the radiation goes through two cycles as the electron beam traverses one wiggler period  $\lambda_w$ .

For the transverse interaction, the on-axis electric field is zero, and the field peaks off axis. Considering only the central part of the beam, the essentials of the transverse interaction are shown in Fig. 1(a) where the electron motion is greatly exaggerated and the transverse profile of the field is included (in this case, the  $TE_{11}$  rectangular waveguide mode). As seen in the figure, the electron will always be in either a decelerating or a zero electric field. Although a particle displaced from the horizontal center of the beam will be in an accelerating field a portion of the time, the bulk of the beam will be in a decelerating field most of the time, leading to a net amplification. The axial interaction is shown in Fig. 1(b), again for the central part of an on-axis beam. The transverse profile in this case represents the axial field of the  $TM_{11}$  mode. Here, even the central particle sees both an accelerating and a decelerating field. The electron is in a decelerating field on axis where the field is at its maximum and the axial velocity at a minimum, and in an accelerating field off axis where the field is reduced and the axial velocity is maximum. However, the transverse variation of the electric field is greater than the transverse variation of the axial velocity. This results in a stronger interaction on axis which, again, leads to net amplification.

Although the axial and transverse interactions have been considered separately in the preceding paragraph, it is difficult to completely separate the two interactions. In fact, computer simulations indicate that the overall performance at the second harmonic is improved when the two interactions are combined. Simulation also shows that the second-harmonic periodic position interaction

## Transverse Interaction



FIG. 1. Physical representation of the periodic position interaction.

can have a stronger growth rate than the fundamental interaction, and is a significantly stronger interaction than the third-harmonic FEL interaction for the current range of experimental parameters. The remainder of this paper includes a description of the experiment, as well as a comparison with simulation results using the threedimensional nonlinear FEL amplifier code WIGGLIN [10,12,13].

The experimental configuration is described in Ref. [9]. The drive frequency was between 12 and 18 GHz. The experiment used a cylindrical electron beam tunable in voltage from about 30 to 250 kV, with a 100-kV nominal operating voltage for the second-harmonic interaction. The beam voltage was measured as the output voltage of the modulator using a capacitive voltage divider. The current was measured at the gun with a current transformer, and at two downstream locations with resistive current monitors. One measurement was taken before the input coupler region and the other just after the interaction region. The wiggler consisted of a permanentmagnet-assisted electromagnet with a period of 3 cm and an amplitude variable over 670-1300 G. This corresponds to a large perturbation of the electron motion at these low voltages, and the ratio of transverse to axial velocity is in the range of 0.23-0.43. The pole pieces extended partially down the sides of the waveguide to provide wiggle-plane focusing, and resulted in a very flat profile near the center of the waveguide with the field rising sharply near the wall.

The experiment operated as an amplifier in an oversized waveguide  $(3.485 \times 1.58$  cm) with the input signal injected using a novel coupler capable of launching the  $TE_{01}$  and  $TE/TM_{11}$  modes. These are the lowest-order modes with the odd transverse symmetry necessary for the periodic position interaction. Simulations of this coupler suggest there was a 9 to 1 split between the  $TE_{11}$ and  $TM_{11}$  modes with very little power in other modes. The output radiation was analyzed via mode-selective output couplers, and the microwave power was measured with calibrated detectors at each of the output coupler ports. By comparing the signals from the output couplers and by utilizing the uncoupled dispersion curves, the interaction was positively identified as a second-harmonic interaction with the 1,1 modes. The input coupler was also switched to launch the  $TE_{01}$  mode (the lowest-order mode for the FEL interaction) to verify that no interaction occurred at these parameters.

Gain due to the second-harmonic periodic position interaction was measured at beam voltages of 78-106 kV and currents of 6-10 A (measured downstream from the interaction region). This contrasts with voltages in the range of 200-250 kV required for the fundamental interaction at the same frequency. Operation at frequencies of 12.5-16.<sup>5</sup> GHz was achieved by both voltage and wiggler-field tuning. The maximum observed gain was approximately 7 dB. The measured gain spectrum will be

presented later in comparison with the theoretical analysis. The interaction could not be saturated at this value of gain with the available drive power, but the maximum unsaturated efficiency obtained was 1.1%.

An oscillation also occurred at beam voltages of 115- 130 kV (depending on the wiggler strength) which had a significant effect on the transported beam current, reducing it by as much as 12% and indicating a strong interaction. The measured frequency was 10.4 GHz, corresponding to the cutoff frequency of the 1,1 modes. An uncoupled dispersion analysis indicated the oscillation was a backward-wave second-harmonic periodic position instability. This was supported by the observation of a higher power exiting the input coupler than was measured at the output couplers. The measured power exiting from the input coupler was 41.5 kW, corresponding to an efficiency of over 3%. The actual power inside the device was uncertain due to the unknown response of the input and output couplers at 10.4 GHz for the  $TE_{11}$  and  $TM_{11}$  modes. Although the fraction of the total power that was actually coupled out from the input coupler is unknown, the apparent strength of this oscillation indicates the potential of the periodic position interaction.

The experiment was not optimized for the secondharmonic periodic position interaction. The primary limitations were electron-beam generation and injection. The electron gun was designed for a different experiment, and new focusing and transport systems were devised to match the beam to the wiggler. A good match was difficult to achieve as the beam was transported from a solenoidal field into the planar wiggler, and a significant portion of the beam was lost in the transition. The problems in the transition region also resulted in a larger than desired beam diameter. Because of the nature of the interaction, an increasing portion of the beam becomes essentially noninteracting as the beam diameter increases, thus limiting the gain. In addition, a large diameter also gives rise to a large wiggler-induced velocity spread which limits the operating efficiency.

The experimental observations were compared with simulations using wIGGLIN, which includes the simultaneous integration of a slow-time-scale formulation of Maxwell's equations as well as the complete Lorentzforce equations for an ensemble of electrons. No average of the orbit equations is performed. As such, WIGGLIN implicitly includes both the well-known odd harmonic interaction in a planar wiggler and the periodic position interaction. No further fundamental modification is required to model the experiment. In this formulation, the electrons are assumed to be initially monoenergetic but with a pitch-angle spread that describes an axial energy spread [12,13].

The wiggler model describes an inhomogeneity in the wiggle direction (i.e., the x axis). The measured field was quite uniform about the symmetry axis, and rose sharply toward the edges of the interaction region. As such, we employ the following wiggler model [14]:

$$
B_{w,x}(\mathbf{x}) = \left[ \left( \sin k_w z - \frac{\cos k_w z}{k_w} \frac{d}{dz} \right) B_w(z) \right] \left[ \sinh k_w y - \frac{Y(k_w y)}{2k_w^2} \frac{d^2}{dx^2} \right] \frac{1}{k_w} \frac{d}{dx} X(x) , \tag{1}
$$

$$
B_{w,y}(\mathbf{x}) = \left[ \left( \sin k_w z - \frac{\cos k_w z}{k_w} \frac{d}{dz} \right) B_w(z) \right] \left[ \cosh k_w y - \frac{k_w y \sinh k_w y}{2k_w^2} \frac{d^2}{dx^2} \right] X(x) , \tag{2}
$$

$$
B_{w,z}(\mathbf{x}) = B_w(z) \cos k_w z \left[ \sinh k_w y - \frac{Y(k_w y)}{2k_w^2} \left[ 1 + \frac{1}{k_w^2} \frac{d^2}{dx^2} \right] \frac{d^2}{dx^2} \right] X(x) , \tag{3}
$$

where  $B_w(z)$  describes the axial variation,  $k_w \equiv 2\pi/\lambda_w$ ,  $X(x)$  denotes the variation in the wiggle plane, and  $Y(k_{w}y) \equiv k_{w}y \cosh k_{w}y - \sinh k_{w}y$ . This field is not selfconsistent in that it is divergence-free but not curl-free. However, the approximation is good as long as  $B_w(z)$  and  $X(x)$  vary slowly compared with  $\lambda_w$ .

We choose  $B_w(z)$  to describe both the adiabatic injection of the beam into the wiggler over  $N_w$  wiggler periods and the downstream taper of the wiggler for efficiency enhancement. Hence,

$$
B_{w}(z) = \begin{cases} B_{w} \sin^{2}(k_{w}z/4N_{w}), & 0 \le z \le N_{w}\lambda_{w}, \\ B_{w}, & N_{w}\lambda_{w} < z \le z_{0}, \\ B_{w}[1 + k_{w}\varepsilon_{w}(z - z_{0})], & z_{0} < z, \end{cases} \tag{4}
$$

where  $B_w$  is the wiggler magnitude in the uniform region, and  $\varepsilon_w$  denotes the normalized taper. The variation in x is described for the general case by a polynomial

$$
X(x) = 1 + \frac{1}{2} (x/a_x)^{2m}, \qquad (5)
$$

where  $\alpha_x$  denotes the scale length for variation of the field, and *m* is an integer. As  $\alpha_x \rightarrow \infty$  this reduces to a wiggler with flat pole faces. A comparison of the actual field with  $X(x)$  as used in the code (a quartic with  $m=2$ and  $\alpha_x = 1.4938$ ) is shown in Fig. 2, and it is clear that the approximation gives a reasonable fit to the data.

The specific parameters used for comparison are a voltage and current of 99.4 kV and 6.6 A with a beam radius



FIG. 2. Comparison of the measured transverse wiggler variation and the quartic representation used in WIGGLIN.

of 0.4 cm. The wiggler was characterized by  $B_w = 1.295$ kG,  $\lambda_w = 3$  cm, an input taper of  $N_w = 3$ , and a total length of  $34\lambda_w$ . Both the TE<sub>11</sub> and TM<sub>11</sub> modes are included (at a ratio of 9 to 1) with a total input power of 300 W. Figure 3 contains a comparison of the observations with results from WIGGLIN over the unstable band for  $\Delta\gamma_z/\gamma_0$  = 0.0%, 0.025%, and 0.05%. The experimental points over the frequency band fall, for the most part, between the curves representing energy spreads of 0.025% and 0.05%. This is in good agreement with the estimated energy spread based upon trajectory calculations of the gun geometry. Observe that the power has not saturated in any of these cases. At 14.4 6Hz, the saturated gain is about 10 dB over  $40\lambda_w$  for  $\Delta\gamma_z/\gamma_0=0$ , which falls to about 8 dB over a distance of approximately  $50\lambda_w$  for  $\Delta \gamma_z / \gamma_0 = 0.025\%$ .

The effect of a tapered wiggler is shown in Fig. 4 for the case of  $\Delta \gamma_z = 0$  and  $\varepsilon_w = -0.00083$ . The efficiency enhancement is sensitive to the start-taper position, which must be close to the point at which the beam becomes trapped in the ponderomotive potential formed by the beating of the wiggler and radiation fields. For this example, the optimal start-taper point is  $z_0 \approx 30.2$  cm. Only the total signal and the  $TE_{11}$  mode are shown in the figure, and the large oscillations in the total power are caused by the  $TM_{11}$  mode. It is evident that the saturat-



FIG. 3. Comparison of the observed output spectrum and the calculations with WIGGLIN.

TE<sub>11</sub> and TM<sub>11</sub> Modes ( $a = 3.485$  cm;  $b = 1.58$  cm)



FIG. 4. Simulation of the power vs axial distance for a tapered-wiggler interaction.

ed efficiency can be increased relative to that of the uniform-wiggler case for the present parameters by almost threefold through the use of a tapered wiggler.

In summary, the first experimental demonstration of a harmonic periodic position amplifier has been achieved. The interaction occurs at the even harmonics of the fundamental FEL interaction frequency in planar-wigglerrectangular-waveguide geometry where modes exist with an odd symmetry about the wiggler symmetry plane. The experiment permitted positive identification of the interacting modes, and the experiment was seen to be in close agreement with predictions from the wIGGLIN simulation code. Improvements in the interaction efficiency by means of a tapered-wiggler interaction have been demonstrated in simulation; however, these results are far from optimized, and we expect that substantial improvements in performance are possible. It is important to note that the harmonic periodic position interaction is operative for other wiggler-waveguide geometries and for optical-resonator modes with a similar odd symmetry.

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