Inner Structure of a Charged Black Hole: An Exact Mass-Inflation Solution

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Recently, Poisson and Israel have shown how when an electrically charged black hole is perturbed its inner horizon becomes a singularity of infinite spacetime curvature—the *mass-inflation singularity*. In this paper we construct an exact mass-inflation solution of the Einstein-Maxwell equations, and use it to analyze the mass-inflation singularity. We find that this singularity is weak enough that its tidal gravitational forces do not necessarily destroy physical objects which attempt to cross it. The possible continuation of the spacetime through this weak singularity is discussed.

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The issue of the final state of gravitational collapse is a long-standing, open question in general relativity (GR). It is widely accepted today that everything falling into a black hole, including the collapsing matter that formed the black hole, will eventually crash into a strong spacelike singularity of almost zero volume, in which spacetime ceases to be classical and quantum gravity dominates the physics. Nevertheless, there is an alternative possibility of gravitational bounce: Infalling objects may avoid the singularity and emerge out of a "white hole" into another asymptotically flat universe [1]. The Reissner-Nordstrom (RN) geometry, which is the unique solution of the Einstein-Maxwell equations for static, spherically symmetric, electrically charged black holes, is an archetype for this scenario. In the extended RN geometry, the central singularity is timelike and all free-falling (electrically neutral) objects avoid it, and eventually are ejected into another external universe. Test particles in the extended Kerr geometry show a similar behavior (however, we shall restrict our attention here to RN-based models). When a strictly spherical charged object collapses, it leaves behind it a RN exterior. If the object is made of charged dust, we can solve analytically for the interior geometry as well [2]. For some range of initial conditions, the interior dust evolves in a completely regular manner, producing a "tunnel" to another universe [3].

The main objection to this idea is that the internal parts of both the RN and Kerr geometries are unstable. That is, the energy-momentum associated with various massless test fields diverges at a certain null hypersurface inside the black hole, called the *Cauchy horizon* (CH) or the inner horizon [4]. This instability is crucial to the question discussed here, because in the RN geometry any object that falls into the black hole must cross the CH. It is widely believed that if one were to consider, self-consistently, the back reaction of the field's diverging energy-momentum on the geometry, the regular CH would become a curvature singularity. One would like to know the features of this singularity. In particular, is it sufficiently strong and violent to be regarded as a physical boundary of classical spacetime?

Recently, Poisson and Israel (PI) invented a simple model to explore the possible back-reaction effect of the diverging perturbations on the CH [5,6]. To simplify the analysis, they modeled the infinitely blueshifted radiation by an ingoing spherically symmetric stream of massless particles. With such an ingoing stream, the RN geometry is converted into the charged Vaidya solution (CVS) [7]. In this solution, the CH is in fact a curvature singularity. However, as was shown by PI, this singularity seems to be rather weak. This is expressed by the fact that for a suitable choice of the coordinates the metric functions approach a regular limit on the CH, and $\det(g)$ remains nonzero [6]. In addition, none of the scalars constructed by contraction of the Riemann tensor or its products is divergent there.

However, PI have shown that this situation is drastically changed if one considers, in addition, beneath the hole's event horizon, a flux of *outgoing* massless particles. Such particles model a piece of the ingoing field that has been backscattered by the hole's curvature and thereby has become outgoing. (Such backscattering will always be present.) We do not know the explicit solution for the case of two cross flows. Nevertheless, PI showed that the mass function m (a generalization of the Schwarzschild mass and the Vaidya mass function to generic spherical geometries; see Ref. [6]) blows up at the CH. As this divergence is exponential with retarded time (which is infinite at the CH), PI call this phenomenon *mass inflation.* This divergence of m guarantees that the scalar $R^{\mu\nu\rho\lambda}R_{\mu\nu\rho\lambda}$ is infinite at the CH, and therefore that the mass-inflation singularity is somewhat stronger than the CH singularity of the original CVS.

The most important physical consequences of the singularity are tied to the question of whether it is strong enough to destroy objects which hit it. The formalism developed by PI does not answer this question, as it does not give explicit expressions for the metric functions. It is the main goal of this paper to construct an exact mass-inflation solution and to use it to analyze the structure and strength of the mass-inflation singularity.

As was shown by PI, the structure of the mass-inflation singularity is virtually independent of the details of the outgoing flux. We shall therefore consider an extremely short pulse of outgoing flux, beyond which the geometry is described by another CVS. Such a short pulse can be modeled, mathematically, as a null layer of energy with vanishing thickness. We shall construct an exact mass-inflation solution by matching two patches of CVS through such an outgoing null "thin layer" (see Fig. 1). Then, using this explicit solution, we shall show that the mass-inflation singularity is, in fact, so weak a singularity that extended objects hitting it are not necessarily destroyed. Hence, if the mass-inflation singularity is a generic feature of gravitational collapse, as suggested by PI, the intriguing possibility of objects crossing the Cauchy horizon is not ruled out by the instability of the CH.

The charged Vaidya solution is given by the line element

$$ds^2 = -f(r,v)dv^2 + 2dr\,dv + r^2 d\Omega^2, \tag{1}$$

where $f \equiv 1 - 2m(v)/r + e^2/r^2$, the constant e is the black hole's electric charge, and v is an ingoing null coordinate. The arbitrary mass function $m(v)$ determines the flux of ingoing radiation, which is proportional to dm/dv.

Let us now consider the matching of two patches of CVS (denoted region 1 and region 2 in Fig. 1) along an outgoing thin null layer S. We shall use the subindex 1 (2) for quantities defined in patch 1 (2) (unindexed quantities will refer to both patches). We require that the metric tensor be continuous at S; hence, the coordinate r is continuous. Since the null fluid is assumed to be electrically neutral and pressureless, the thin layer S which models it has a vanishing electric charge and a vanishing surface tension (i.e., vanishing nonradial components of surface stress-energy). This implies that (i) the constant e is the same in both regions 1 and 2, and (ii) the affine parameter λ along S is the same on its two sides [8]. We shall take λ to increase with time, and set $\lambda = 0$ at the CH.

Since the overall geometry cannot be described by the CVS, we shall also use double null coordinates:

$$ds^2 = -2e^{2\sigma}dU\,dV + r^2 d\Omega^2. \tag{2}$$

Here V is an ingoing radial null coordinate (namely, ingoing photons move on V=const orbits), and U is the outgoing radial null coordinate. We choose the coordinate V to coincide with λ along S. The coordinate U is as yet unspecified, except that it increases with time and we set $U=0$ at S. We define the function $R(\lambda)$ to be the value of r along S, namely, $R(\lambda) \equiv r(V=\lambda, U=0)$. We shall now show how this function determines the geometry on both sides (up to an integration constant).

From Eq. (1) it is clear that along any line U=const we have

$$dr/dv = \tfrac{1}{2}f = \tfrac{1}{2}(1 + e^2/r^2) - m/r. \tag{3}$$

In particular, we obtain for $U=0$

$$R'/v' = \tfrac{1}{2}(1 + e^2/R^2) - m/R, \tag{4}$$

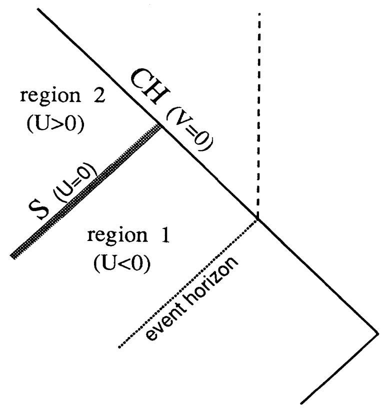

FIG. 1. Penrose diagram of the spacetime formed by the matching of two CVS patches along the thin layer S.

where the prime denotes a derivative with respect to λ. In addition, the geodesic equation for v'' reads for $U=0$

$$v'' = v'^2(e^2/R^3 - m/R^2). \tag{5}$$

Combining Eqs. (4) and (5) yields a closed equation for $z(\lambda) \equiv R/v'$:

$$z' = \tfrac{1}{2}(1 - e^2/R^2). \tag{6}$$

From Eqs. (4) and (6), we obtain the three matching equations in an explicit form:

$$m(\lambda) = (R/2)(1 + e^2/R^2) - zR', \tag{7}$$

$$v(\lambda) = \int^{\lambda}(R/z)d\lambda, \tag{8}$$

$$z(\lambda) = Z + \tfrac{1}{2}\int_0^{\lambda}(1 - e^2/R^2)d\lambda. \tag{9}$$

Here Z is an integration constant. We omitted the integration constant in Eq. (8) because adding a constant to v does not make any physical difference. Equations (7)-(9) completely determine the CVS on both sides of S, once $R(\lambda)$ and the integration constants Z_1 and Z_2 are given. The (λ-dependent) mass of the thin layer, $\Delta m(\lambda) \equiv m_2(\lambda) - m_1(\lambda)$, is obtained directly from Eqs. (7) and (9):

$$\Delta m(\lambda) = (Z_1 - Z_2)R'(\lambda). \tag{10}$$

In order to apply this formalism to the problem of interest, we must determine the relevant function $R(\lambda)$ and the two integration constants Z_1 and Z_2. $R(\lambda)$ and Z_1 are to be determined from the well-known features of the CVS in region 1, which is, in fact, a slightly perturbed RN solution. In particular, the CH ($\lambda=0$), which corresponds to $v_1 \to \infty$, is located at $r=r_0$, where $r_0 \equiv m_0 - (m_0^2 - e^2)^{1/2}$, and m_0 is the final mass of the black hole after it has absorbed all the ingoing radiation. It is convenient to express the mass function in region 1 as $m_1(v_1) = m_0 - \delta m(v_1)$, where $\delta m(v_1)$ represents the mass contribution associated with the radiative tail which dominates the late-time behavior of realistic perturbations. The relevant asymptotic form of $\delta m(v_1)$ is given by

$$\delta m(v_1) \propto v_1^{1-p}, \tag{11}$$

where the constant p depends on the perturbation under discussion, and is ≥ 12 [6].

Since $R = r_0$ at $\lambda = 0$, we can approximate Eq. (9) near $\lambda = 0$ by

$$z(\lambda) \cong Z + \tfrac{1}{2}(1 - e^2/r_0^2)\lambda = Z - k_0 r_0 \lambda, \tag{12}$$

where $k_0 \equiv (2r_0)^{-1}(e^2/r_0^2 - 1) > 0$. Hereafter, the $\cong$ sign means "equals, asymptotically, as $\lambda \to 0$." The divergence of v_1 at $\lambda = 0$ demands $v_1'(\lambda = 0) = \infty$, which, in view of Eqs. (8) and (12), implies $Z_1 = 0$ and therefore yields $v_1(\lambda) \cong -(1/k_0)\ln|\lambda|$.

The relevant function $R(\lambda)$ may be determined from the mass function $m_1(v_1)$ by applying Eq. (3) to $U = 0$. By linearizing this equation in δm and $\delta R \equiv R - r_0$, we obtain $\delta R(v_1) \cong (k_0 r_0)^{-1}\delta m$, which, in terms of λ, becomes

$$\delta R(\lambda) \propto (-\ln|\lambda|)^{1-p}. \tag{13}$$

To obtain a positive-energy thin layer at S we choose $Z_2 > 0$. Equations (10) and (13) now yield the mass-inflation formula, $\Delta m(\lambda) \propto |\lambda|^{-1}(-\ln|\lambda|)^{-p}$, or, correspondingly,

$$\Delta m(v_1) \propto v_1^{-p} \exp(k_0 v_1). \tag{14}$$

This conforms with the result fo PI [cf. Eq. (4.12) in Ref. [6]].

We will be mainly interested in the mass function $m_2(v_2)$, as it completely determines the geometry in the mass-inflation region ($V \approx 0$, $U > 0$). From Eqs. (8) and (12) we obtain $v_2(\lambda) \cong (r_0/Z_2)\lambda$, and therefore

$$m_2(v_2) \cong \Delta m(v_2) \propto |v_2|^{-1}(-\ln|v_2|)^{-p}. \tag{15}$$

It is convenient to use the (U,V) coordinates to describe the geometry in the mass-inflation region. To compute $r(U,V)$ we use Eq. (3), which reads

$$\partial r/\partial V \propto \partial r/\partial v_2 \cong -m_2/r \propto -(r|V|)^{-1}(-\ln|V|)^{-p},$$

and can be immediately integrated. To calculate $\sigma(U,V)$, we recall that the transformation from (r,v) to (U,V) coordinates yields

$$e^{2\sigma} = -(\partial r/\partial U)dv_2/dV = -(\partial r/\partial U)v_2'.$$

In the following we shall restrict our attention to the close vicinity of $U = 0$. For a suitable choice of the coordinate U one obtains, to the leading terms in U and $(\ln|V|)^{-1}$,

$$r(U,V) \cong r_0 - U + (-\ln|V|)^{1-p} \times \text{const}, \tag{16}$$

$$\sigma(U,V) \cong \sigma_0 + U(-\ln|V|)^{1-p} \times \text{const}, \tag{17}$$

where σ_0 is a constant. Clearly, both σ and r are finite at $V = 0$. This follows from the fact that, although the mass function in Eq. (15) diverges, its integral over v_2 is finite. Recall that the term $-U$ in Eq. (16) represents the contraction of the CH with time, due to the focusing effect of the outgoing flux. This contraction continues until the CH shrinks to $r = 0$. [Equations (16) and (17) are valid only for small U, where $r \approx r_0$; the extension of this analysis to later stages is straightforward, but we shall not present it here because of space limitations.]

We are now in a position to discuss the physical properties of the mass-inflation singularity. Unlike the original CVS singularity at the CH, some of its curvature scalars blow up (e.g., $R^{\mu\nu\rho\lambda}R_{\mu\nu\rho\lambda} \propto m^2$). More important physically than this blowup, however, is the issue of whether the tidal forces associated with this curvature singularity are strong enough to destroy any realistic object which attempts to cross it [9]. Because of the divergence of the tidal force, we can neglect all internal stresses, and model any extended object falling into the singularity as a collection of pointlike particles that move on timelike geodesics. If such an "object" suffers an infinite distortion (either a compression or a stretch) due to the diverging tidal forces, we shall regard the singularity as a strong one (this criterion is almost identical to that of Tipler [9]). If the distortion is finite in all three directions, we shall regard the singularity as weak. The strength of the singularity is crucial to the possibility of classically extending the spacetime beyond the singularity. For a strong singularity such an extension is meaningless, because an observer will never be able to cross the singularity: He will be completely destroyed during its approach to the singularity. On the other hand, in the case of a weak singularity, the possibility exists of crossing the singularity without being destroyed; hence, one cannot exclude the possibility of a classical extension.

From Eqs. (16) and (17) one can show that the rate of growth of the curvature (and the tidal forces), as expressed by the tetrad components associated with a free-falling observer, is proportional to $\tau^{-2}|\ln|\tau||^{-p}$, where τ is the observer's proper time. [This is also the rate of growth of curvature in the singular CH of the original CVS in patch 1; cf. Eq. (B8) in Ref. [6].] The tidal forces are proportional to the second time derivatives of the distances between various points of the object. By integrating this expression twice, one finds that as the mass-inflation singularity is approached ($\tau = 0$), the distortion remains finite. This is closely related to the fact that both metric functions r and σ are finite at $V = 0$ [Eqs. (16) and (17)], and that $\det(g)$ is nonvanishing. The criteria for strength of singularities given by Tipler [9] and by Ellis and Schmidt [10] give this same result: The mass-inflation singularity is a weak one; hence, classical continuation beyond it is not excluded.

If classical spacetime does extend beyond the CH, how can we determine the right extension? Of course, one of the candidates is the analytic RN continuation, in which the black hole is a tunnel to other universes. But many other extensions are possible as well. Classical general relativity (GR) cannot uniquely determine the extension: Even in the case of pure RN (where there are no perturbations and the CH is completely regular) classical GR cannot give a definite prediction about the geometry beyond the CH. (Such a prediction requires additional initial data.)

It has long been recognized that at some stage of the evolution of spacetime during gravitational collapse, classical GR must be replaced by a more fundamental theory of spacetime—presumably quantum gravity (QG). This more fundamental theory is expected to prevent the formation of the spacetime singularities, which, according to classical GR, must exist inside black holes. It is probably QG that will determine the right continuation beyond the CH. We expect QG to tell us (i) whether spacetime extends classically beyond the CH; (ii) if it does, what the right classical continuation is; and (iii) if it does not, what kind of existence there is beyond the CH. Unfortunately, despite great effort in recent decades, we do not have yet a complete formulation of QG. However, some basic elements of that theory are well understood, and perhaps this will enable one to gain some insight into the question of the right continuation.

Even if spacetime extends classically beyond the CH, the CH itself and its immediate vicinity must be described by QG. The possibility of a quantum state which forms a bridge between two classical states might look somewhat strange, but a simple analog in a one-dimensional Schrödinger scattering problem shows that it is not unlikely. Consider a well-localized wave packet (a semiclassical particle, analogous to the semiclassical geometry) which moves freely until it hits a narrow potential barrier. Of course, inside the barrier itself (which is analogous to the CH) the classical description is meaningless. Still, if the barrier is sufficiently narrow, the wave packet will tunnel through it, with a negligible scattering. The state after the scattering is again semiclassical.

It should be pointed out that any classical extension beyond the mass-inflation singularity will require an infinite ingoing flux of negative energy along the CH. This is a consequence of the diverging derivatives of the metric functions there [11]. One should not regard this as an impossible obstacle, because, as discussed above, the CH itself must be described by QG. Already in the framework of quantum field theory in classical curved spacetime, which is often regarded as the first step toward QG, one finds fluxes of negative energy. Interestingly, analyses of the renormalized stress-energy of the quantum field in RN spacetime (based on a simplified, two-dimensional model) show *an infinite ingoing flux of negative energy along the CH* [12]. Therefore, there is no reason to assume that QG will forbid such a flux of negative energy. One may compare the situation here to the semiclassical particle discussed above. When this particle is crossing the barrier, its kinetic energy is negative, which is classically forbidden.

Although it is not at all clear whether the spacetime extends classically beyond the CH, it is rather disturbing that such a possibility exists. It is often believed (or hoped) that classical GR protects itself against such ambiguous situations in which an observer may reach (alive) the limit of predictability. This is the essence of the strong cosmic censorship conjecture. The model discussed here suggests that perhaps this is not the case. Just how generic the mass-inflation singularity is remains an open question. PI [6] suggest that the mass-inflation phenomenon may be rather generic, and not limited to spherically symmetric perturbations. This suggestion is strongly supported by a recent analysis of the evolution of nonspherical scalar field perturbations on the spherical mass-inflation background [13].

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