Edge States in the Fractional Quantum Hall Effect

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We have studied *nonlocal* electrical transport over macroscopic distances in the regime of the fractional quantum Hall effect (FQHE). Experiments clearly demonstrate dissipationless edge-state conduction associated with the FQHE. Surprisingly, our data imply that there is no edge-state conduction near filling factor $\nu = \frac{1}{2}$ and on the low-ν side of the $\nu = 1$ QHE state, while edge-state conduction is observed near $\nu = \frac{3}{2}$ and on the high-ν side of the $\nu = 1$ QHE state. We also observe that the electron-hole symmetry is broken for the edge states in a confined geometry.

The integer quantum Hall effect [1] (IQHE) can be understood in terms of transport by edge channels corresponding to an integer number of fully occupied Landau levels [2–4]. In this picture, near an integral Landau-level filling $\nu = i$, when the chemical potential lies in the gap of the localized bulk states, all current is carried by the dissipationless edge channels and the Hall resistance is quantized to h/ie^2. Dissipative transport (between $\nu = i$ and $\nu = i + 1$) occurs because current is carried *both* by extended bulk states of the partially occupied topmost Landau level *and* by the extended edge states. Much experimental research on edge-state transport has focused on two-dimensional electron systems (2DES) confined within narrow and small gated devices, where the effect of edge-state conduction is enhanced. Recent measurements [4] of nonlocal four-terminal magnetoresistance (FTMR) by McEuen *et al.*, however, dramatically demonstrate that edge-channel transport may be truly dissipationless over macroscopic distances of ~1 mm in 2DES samples exhibiting the IQHE.

In the IQHE, interpretation in terms of edge channels is straightforward since the edge channels are formed in one-to-one correspondence to the bulk Landau levels defined in the single-electron density of states [2–4]. The fractional quantum Hall effect [5] (FQHE) occurs at certain simple rational values of ν and is fundamentally a many-body phenomenon. For $\nu = 1/m$, where m is an odd integer, the FQHE states are described very well by the Laughlin incompressible states [6]. The rest of the FQHE states occur at $\nu = p/q$, where q is odd. These states, usually called "hierarchy states," have been obtained in the Haldane-Halperin theory [7] and, more recently, in the Jain theory [8]. These theories [6–8] describe only the bulk FQHE states, however.

Several theories of the edge states in the FQHE regime have been proposed recently [9–11]. Chang and Cunningham [12] and Kouwenhoven *et al.* [13] experimentally studied adiabatic edge-state transport in gated samples; their results have been discussed in terms of "edge channels" defined in the single-electron density of states. Chang and Cunningham performed measurements of a gate-induced barrier resistance on the $\nu = \frac{1}{3}$ and $\frac{2}{3}$

FQHE plateaus. Kouwenhoven *et al.* employed adjustable barriers as current and voltage probes and measured the Hall resistance. They interpreted their results in terms of selective population and detection of edge channels at $\nu = \frac{2}{3}$ and concluded that each fractional edge channel contributes a conductance of $\frac{1}{3} e^2 / h$.

In this paper we report observation of nonlocal magnetoresistance in ungated 2DES samples in the FQHE regime at arbitrary ν. Our experiments unambiguously (model independently) demonstrate dissipationless edge-state conduction associated with the FQHE. We find that the edge-state conduction persists over macroscopic distances of several mm in the 2DES samples tested. Surprisingly, our data imply that there is no edge-state conduction near $\nu = \frac{1}{2}$ and on the low-ν side of the $\nu = 1$ IQHE state, while there is edge-state conduction near $\nu = \frac{3}{2}$ and on the high-ν side of the $\nu = 1$ IQHE state. We develop a picture of edge-state structure consistent with these observations.

Standard, simply connected "Hall-bar" patterns (see inset in Fig. 1) were defined by wet etching of low-disorder GaAs/AlGaAs heterojunctions with density $n = (7–12) \times 10^{10}$ cm^{-2} and mobility $\mu = (6–20) \times 10^5$ cm^2/V s. A brief illumination by a red-light-emitting diode was used to prepare a 2DES. FTMRs were measured using the standard low-frequency lock-in technique, with measurement currents between 0.4 and 2 nA. Standard longitudinal R_{xx} was measured with current passed between probes 1 and 4 and voltage measured between probes 2 and 3 ($R_{14,23}$) or 5 and 6 ($R_{14,56}$). Hall resistance R_{xy} was measured across the Hall bar between probes 2 and 6 or 3 and 5. Using the same Hall-bar patterned samples, nonlocal FTMRs $R_{26,35}$ and $R_{35,26}$ were also measured. For one such sample with a Hall-bar pattern with 2.2 squares separating current and voltage probe pairs, Fig. 1 shows the nonlocal FTMR as well as R_{xx} and R_{xy}. The sample also displays FQHE at $\nu = \frac{1}{5}$ at higher B, not shown in Fig. 1. In Fig. 2, the nonlocal FTMR trace is blown up for $\nu \geq 1$. In Fig. 3(a) we show nonlocal FTMR data taken at several different temperatures. R_{xx} at 350 mK is shown in Fig. 3(b) for comparison.
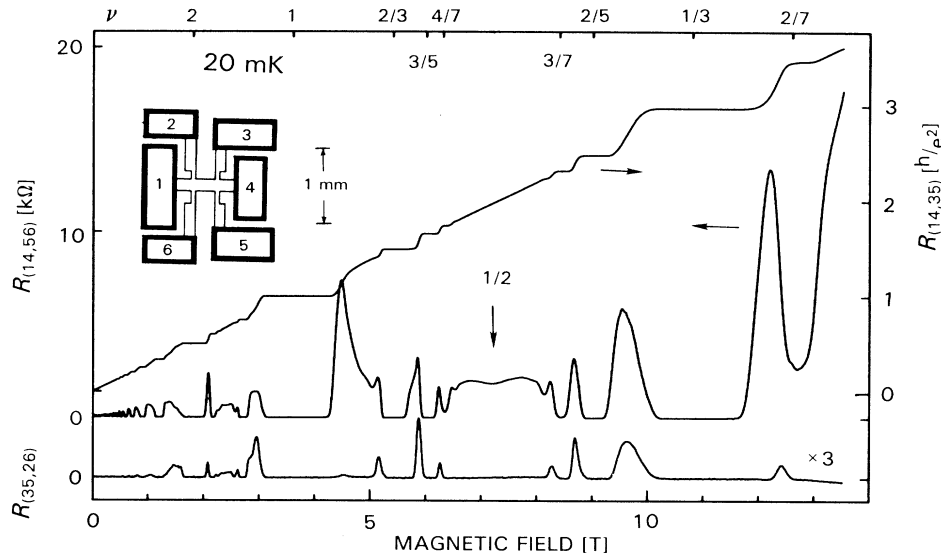
FIG. 1. Longitudinal $(R_{14,56})$, Hall $(R_{14,35})$, and "nonlocal" $(R_{35,26})$ four-terminal magnetoresistances (FTMR) obtained in the sample geometry shown in the inset. While nonzero longitudinal resistance implies dissipative transport, nonzero nonlocal FTMR implies current transport by *both* dissipative bulk and dissipationless edge states at that filling factor v. Note the qualitatively different shapes of the longitudinal and nonlocal FTMRs. $R_{35,26}$ is magnified by a factor of 3 compared to $R_{14,56}$. Inset: Outline of the sample ("Hall bar"); thick-line rectangles are the contacts.

Examination of Figs. 1–3 immediately reveals drastic differences between R_{xx} and the nonlocal FTMR. In the nonlocal FTMR data of Fig. 1 regions of negligibly small resistance correspond to prominent peaks in R_{xx}. For classical, dissipative, homogeneous 2DES transport in this geometry, $R_{35,26}$ (nonlocal FTMR) would be the same as $R_{14,56}$ (R_{xx}), except for being reduced by a *B-inde-pendent* geometric factor of $\sim 10^{-3}$ ($R_{35,26}$ resistance at $B=0$ is $1.3 \times 10^{-3} R_{xx}$). At high B, however, peaks in nonlocal FTMR are roughly 3×10^{-1} of those in R_{xx}, far from being attenuated by 10^{-3}. The nonlocal FTMR also differs qualitatively from R_{xx} in positions and widths (in B) of peaks, even in the presence or absence of some peaks. R_{xx} and the nonlocal FTMR demonstrate mark-

edly different dependences on temperature, as is shown in Fig. 3, also inconsistent with dissipative bulk-only transport. At 350 mK, the magnitudes of peaks in R_{xx} generally decrease little, while the peaks in the nonlocal FTMR all diminish considerably with increasing temperature, so that only three are discernible above 300 mK.

The above data and observations demonstrate adiabatic transport in the FQHE regime. The occurrence of nonzero R_{xx} does imply existence of dissipative current paths (in the bulk) generating potential differences within the 2DES sample. However, nonzero nonlocal FTMR of comparable magnitude implies that at certain v potential differences extend unattenuated (dissipationless transport) for macroscopic distances ~ 1 mm away from dissipative bulk current paths. Thus, our data demonstrate that dissipationless edge states exist in the FQHE, similar to the IQHE regime. It should be noted that this nonlocal transport behavior, which requires suppression of scattering between the edge states and the bulk, is not a prerequisite for QHE. Thus, relatively precise Hall resistance quantization and the corresponding minima in R_{xx} are observed at higher temperatures, at lower magnetic fields, and at higher applied currents, where nonlocal behavior is suppressed (Figs. 1 and 3).

The IQHE is usually treated within the second quantization (single-electron density of states), while the FQHE is understood in terms of wave functions describing the relevant many-electron states. As was pointed out by Jain [8], the IQHE can be equally well understood in the language of many-electron wave functions. In this language we can consider the IQHE states $|i\rangle$ corre-
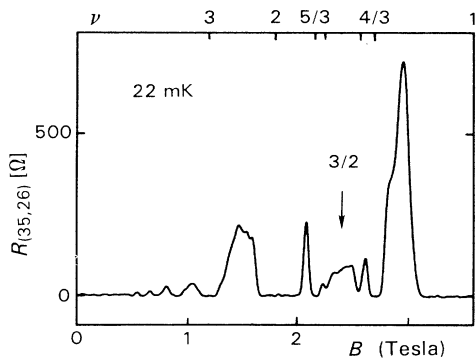

FIG. 2. Nonlocal FTMR expanded for $v \geq 1$. Note the nonzero nonlocal FTMR near $v = \frac{3}{2}$.
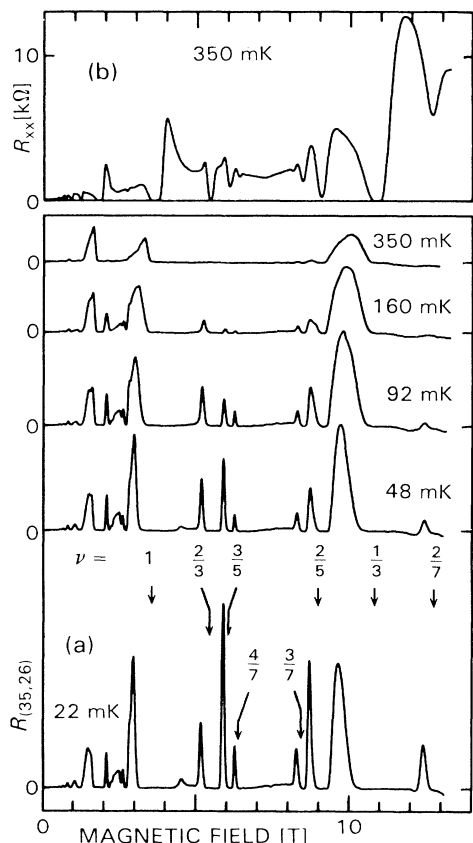
FIG. 3. (a) Nonlocal FTMR at higher temperatures; all traces have same resistance scale. Note the markedly different temperature dependences of the peaks at 5 and 6 T and also of the peaks at 9.5 and 12.5 T (discussed in the text). The origin of the weak peak at 4.5 T is not known at present; its disappearance at 65 mK seems to rule out the $v=1$ IQHE state. (b) Longitudinal ($R_{14,56}$) FTMR at 350 mK.

sponding to an integer number of fully occupied Landau levels (LL); then quasielectrons are simply the occupied states of the next higher LL and quasiholes are the empty states in the topmost filled LL. States $|v\rangle$ at v such that $i < v < i+1$ can be viewed, equivalently, as the $|i\rangle$ state plus $v-i$ quasielectrons or as the $|i+1\rangle$ state plus $i+1-v$ quasiholes.

In the wave-function language, in the IQHE regime, the sample can be considered as an $|i\rangle$ state extending from edge to edge plus the interior of the sample ("bulk") containing the quasiparticles needed to produce the given v. The $|i\rangle$ state at the periphery of the sample constitutes the edge channel [14]. This picture applies equally well to the FQHE. We first consider the case of exact filling, where $v=p/q$ in the interior of the sample. In this situation the edge channel is the same $|p/q\rangle$ FQHE state as the bulk state, or, in other words, the bulk state extends throughout the sample. If a potential difference $\delta\mu$ is created between a pair of contacts to such

sample, a (net) current of $(p/q)(e^2/h)\delta\mu$ is carried along one side (determined by the direction of B) of the sample between this pair of contacts. The carriers are electrons, according to Ref. [8], for any $|p/q\rangle$.

As v is varied from exact filling, quasiparticles are created in the interior of the sample thus forming the bulk state; the edge states remain unaffected until the deviation from the exact filling is substantial. The quasiparticles are localized by the disorder potential so that no current is transported through the bulk and R_{xy} is still quantized [6]. As the deviation from the exact filling becomes larger, the bulk state becomes delocalized and a part of the current is carried by the bulk and the remainder by the underlying edge channel.

The precise nature of the bulk state in this regime is not known at present [15], but our data imply that the edge channel is derived from the *nearest* observed exact-filling QHE state at *lower* v. For example, consider the nonlocal FTMR peaks on either side of $v=\frac{2}{3}$ in Fig. 3. The peak at $B=5$ T ($v>\frac{2}{3}$) survives to a higher temperature than the peak between $v=\frac{2}{3}$ and $\frac{3}{5}$. This fact strongly suggests that the peak at $B=5$ T is supported by the $|\frac{2}{3}\rangle$ state while the peak between $v=\frac{2}{3}$ and $\frac{3}{5}$ is supported by the weaker $|\frac{3}{5}\rangle$ state. Likewise, the nonlocal FTMR peak at 12.5 T is supported by the $|\frac{2}{7}\rangle$ state; it disappears by 160 mK, while the peak at 9.5 T, supported by the $|\frac{1}{3}\rangle$ state, is prominent even at 350 mK. These observations imply that the electron-hole symmetry is broken in a confined geometry. Thus, in our picture, the edge current is always carried by electrons, while the bulk current may be transported by fractionally charged quasiparticles [6,8] when the underlying state is a FQHE state. There is substantial difference in strength of the FTMR peaks $|\frac{1}{3}\rangle \rightarrow |\frac{2}{5}\rangle$ and $|\frac{1}{3}\rangle \rightarrow |\frac{2}{7}\rangle$, which seems to be consistent with Jain's theory [8,15].

This picture implies a relatively large gradient of the confining potential defining the edge of the sample. This situation seems always to be realized in heterostructure samples [16]. If the gradient of the confining potential could be made very small, less than $\sim0.1en/\varepsilon$, the sample can be considered to be nonuniform, with lower-n regions on the periphery of the sample forming multiple edge channels [9–11].

Within the above qualitative description of QHE transport, more conclusions arise regarding edge states in specific regimes of FQHE. Nonzero R_{xx} and zero nonlocal FTMR over an interval of B near $v=\frac{1}{2}$ implies absence of a distinct dissipationless edge channel in this regime. Indeed, within the edge-state picture discussed above there is no underlying incompressible FQHE state near $v=\frac{1}{2}$ since $v=\frac{1}{2}$ lies *between* sequences of the FQHE states. Thus, there is an edge channel of the underlying $|\frac{3}{7}\rangle$ state in the range of $\frac{3}{7} \le v < \frac{4}{9}$, and there is an edge channel of the underlying $|\frac{4}{7}\rangle$ state in the range of $\frac{4}{7} \le v < \frac{3}{5}$, but there is no underlying FQHE state for $\frac{4}{9} \le v \le \frac{4}{7}$ around $v=\frac{1}{2}$.

Nonzero nonlocal FTMR at $v = \frac{3}{2}$ (see Fig. 2) arises from the edge channel of the IQHE state $|v=1\rangle$. It is easy to see that while the $|\frac{7}{5}\rangle$ state can support an edge channel for $\frac{7}{5} \leq v < \frac{10}{7}$, at some v in this interval the FQHE correlations of $v = \frac{7}{5}$ disappear while the IQHE correlations of $|1\rangle$ emerge and persist in the range of v near $\frac{3}{2}$. This is in contrast to situation near $v = \frac{1}{2}$, where no other QHE state may emerge to support an edge channel. The absence of nonlocal FTMR in the range of $\frac{3}{4} \leq v \leq 1$, on the low-$v$ side of the $|1\rangle$ state, likewise implies absence of a distinct dissipationless edge channel in this regime and is explained in a similar manner.

In conclusion, we presented experimental observation of nonlocal four-terminal magnetoresistance in the FQHE regime. These experiments demonstrate existence of edge channels, distinct from the bulk and dissipationless over macroscopic distances of ~ 1 mm. Our experiments demonstrate that in the FQHE, unlike in the IQHE, there is no one-to-one correspondence between the peaks in R_{xx} and the nonlocal resistance. This qualitative difference gives insight into the fundamental nature of the bulk FQHE states.

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