Superfluid Density from Heat-Pulse Propagation near the λ Line in ^{4}He-Aerogel Systems

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We determined the superfluid density ρ_s from the speed of heat pulses, using a hydrodynamic model that contains coupling between the aerogel and the helium. To investigate the universality of the exponent of ρ_s, measurements of ρ_s were taken from saturated vapor pressure to 29 bars. Fitting ρ_s with a pure power law in reduced temperature leads to a pressure-dependent exponent. Inclusion of a confluent singular term in the fits reveals a pressure-independent exponent $\zeta=0.755\pm0.003$ which differs from $\zeta=0.672$ for bulk helium.

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Recent experiments on the superfluid transition of ^{4}He contained in open-structure glasses such as aerogels and xerogels have shown that the nature of the transition differs considerably from that of bulk helium [1,2]. While the transition is sharp at reduced temperatures $t=1-T/T_c$ as small as 5×10^{-5} and the superfluid fraction ρ_s/ρ can be approximated by a power law, the exponents derived from a fit by

$$\rho_s/\rho=kt^{\bar{\zeta}} \tag{1}$$

are 0.81 for aerogel and 0.89 for xerogel [2], to be compared with 0.67 for bulk helium [3]. The existence of sharp transitions with exponents different from bulk helium suggests that these helium-glass systems belong to different universality classes. Measurements of the specific heat [1] C_p for the helium-aerogel system at saturated vapor pressure (SVP) reveal a peak at T_c, with a magnitude comparable to what one would expect from two-scale-factor universality [4,5] and the value of the ρ_s amplitude. However, close to T_c, C_p is nearly a linear function of t, suggesting a specific-heat exponent α close to -1, in apparent violation of the hyperscaling relation $3\zeta=2-\alpha$.

In order to shed more light on this interesting problem, we made measurements of ρ_s over a pressure *range* from saturated vapor pressure to the bulk freezing pressure near 29 bars, and in aerogels [6] with densities ranging from 0.14 to 0.25 g/cm^3. As is the case for bulk helium [3], we found that effective exponents $\bar{\zeta}$ obtained by fitting data with Eq. (1) are pressure dependent and that only the inclusion of singular corrections to the leading power law in the fitting function yields a pressure independent, and thus presumably universal, exponent. However, the exponent *value*, although somewhat lower than the previous estimates at SVP, is close to 0.76 and still significantly above the bulk value. Thus our results considerably strengthen the evidence for, but of course do not prove the existence of, a new universality class. Through hyperscaling a ρ_s exponent of 0.76 yields a C_p exponent of -0.27. This value is even more difficult to reconcile with the C_p measurements [1] than the value of -0.43 which follows from a pure power-law exponent of 0.81 [1,2].

Bulk helium supports two propagating modes, called first and second sound. Second sound is a propagating temperature wave, and its velocity gives the superfluid fraction ρ_s/ρ (ρ is the density of bulk liquid helium). Confined in a *rigid* medium with sufficiently small pores, helium does not support the second-sound mode anymore, because the normal fluid is viscously locked to the substrate and is immobile. The superfluid remains free to move and supports a pressure wave, fourth sound, the speed of which then gives ρ_s/ρ. It was recognized by McKenna, Slawecki, and Maynard that this is changed for helium in *highly compliant* media, like silica aerogels [7]. In that case the normal fluid, though locked to the substrate, drags the substrate along. As a result, the substrate adds both its mass and a restoring force to the normal-fluid motion. A hydrodynamic model that takes this into account [7] shows that there are once more two propagating modes, a slow critical mode, and a fast one, which resembles first sound. The velocities of the two propagating modes, c_s and c_f, are given by the roots of the secular equation [7]

$$(c^2-c_1^2)(c^2-c_2^2)+(\rho_a/\rho_n)(c^2-c_a^2)(c^2-c_4^2)=0\,, \tag{2}$$

where $c=(c_s,c_f)$, $c_1^2=(\partial P/\partial\rho)_T$, $c_2^2=(\rho_s/\rho_n)c_0^2$ with $c_0^2=TS^2/C_p$, and $c_4^2=(\rho_s/\rho)c_1^2+(\rho_n/\rho)c_2^2$ (in bulk helium, c_1, c_2, and c_4 correspond to the velocities of first, second, and fourth sound, respectively). The velocity of sound in empty aerogel is c_a, and ρ_n/ρ and ρ_a are respectively the normal-fluid fraction and the aerogel density. The sound speed c_s of the slow mode can again be used to determine ρ_s/ρ. From Eq. (2) one obtains

$$\frac{\rho_s}{\rho}=\frac{c_s^2}{c_1^2}\left[\frac{y+c_s^2/c_1^2}{y+1}+\frac{c_0^2}{c_1^2}\right]^{-1}, \tag{3}$$

with $y=(\rho_a/\rho)(c_a^2-c_s^2)/(c_1^2-c_s^2)$.

Equation (3) depends upon the thermodynamic parameters of helium confined in a particular aerogel which are not known directly. We do not expect these parameters to differ greatly from their bulk values [8]. Further, ρ and c_1 are only weakly dependent upon temperature. They were taken from Ref. [9] except that ρ was replaced by $\mathcal{P}\rho$, where $\mathcal{P}$ is the porosity of the aerogel. The term containing c_0 is a small correction of order 1%. The ve-

TABLE I. The porosity $\mathcal{P}$ and the measured values of the length l, the density ρ_a, and the sound speed c_a.

Aerogel	l (cm)	ρ_a (g/cm^3)	$\mathcal{P}$ (%)	c_a (m/s)
A	0.579	0.140	93.6	255
B	1.105	0.188	91.4	288
C	0.841	0.247	88.8	462

locities c_a were measured in a separate experiment at 4.2 K.

The apparatus was similar to one used for second-sound pulse-shape measurements in bulk helium [10]. A 32 mm×32 mm slab of aerogel was mounted inside an aluminum frame with epoxy [11] which prevented a direct bulk-superfluid path around the sample. The aerogel was then sandwiched between two glasses plates, one with a heater and the other with a bolometer evaporated on it as in Ref. [10]. Table I gives the relevant properties of the aerogels. The cell was sealed in a cooper enclosure and could be filled and pressurized with liquid helium through a capillary. It was immersed in a pumped liquid-helium bath whose temperature was controlled to about 1 μK using a germanium thermometer. The pressure in the cell was held constant within about 0.4 mbar by bleeding in or out appropriate amounts of helium as the temperature was changed. Square heat pulses approximately 2 μsec long were launched at the heater. Well below T_c, the power dissipated at the heater was approximately 50 mW/cm^2. For $t \lesssim 1\times10^{-3}$, it had to be reduced to avoid effects dependent upon the amplitude of the pulses. The pulses at the bolometer were digitized with a time resolution of 1 μsec.

Figure 1 shows typical signals received at the bolometer. The pulses were distorted and broadened considerably, but for a wide range of heater pulse widths, their shape and in particular their first arrival times did not change significantly. The received pulses often contained structure corresponding to the transit times of the fast mode. Successive echoes of the fast mode typically decayed within one transit time of the slow mode, but were regenerated upon reflection of the slow pulse from the cavity ends. We do not yet have a detailed understanding of sound propagation in these systems. The hydrodynamic model [7] does not include such effects as dispersion and viscous damping. Important for the present work is that the onset of the received pulse was always well defined by a sharp rise in the bolometer signal. This first arrival time was used to determine c_s.

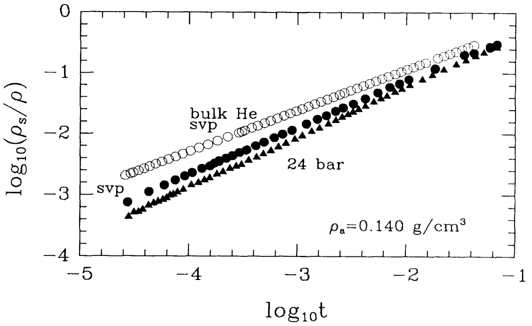

FIG. 2. Double-logarithmic plot of ρ_s/ρ vs t for ρ_a=0.140 g/cm^3 at SVP and 24 bars, and for bulk helium at SVP.

Figure 2 shows results for ρ_s/ρ at SVP and at 24 bars as a function of t on logarithmic scales. For comparison, ρ_s/ρ for bulk helium [3] at SVP is shown as well. As noted by others [1,2], the slope of the data, which corresponds to the exponent $\bar{\zeta}$, is significantly larger for helium in aerogel than for bulk helium. Fitting ρ_s, for a given pressure and for $10^{-4} < t < 10^{-2}$, by the power law Eq. (1) results in a pressure-dependent exponent $\bar{\zeta}$. Figure 3 shows the values of $\bar{\zeta}$ as open symbols for all three samples. The parameters T_c, k, and $\bar{\zeta}$ were least-squares adjusted. The statistical errors are very small, and there clearly is a significant increase of $\bar{\zeta}$ with pressure. However, particularly at the higher pressures the data show systematic deviations from the fits. This is already no-

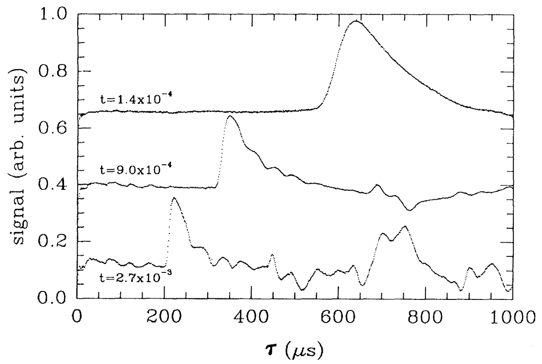

FIG. 1. Typical pulse shapes for ρ_a=0.140 g/cm^3. The numbers are the values of reduced temperature t.

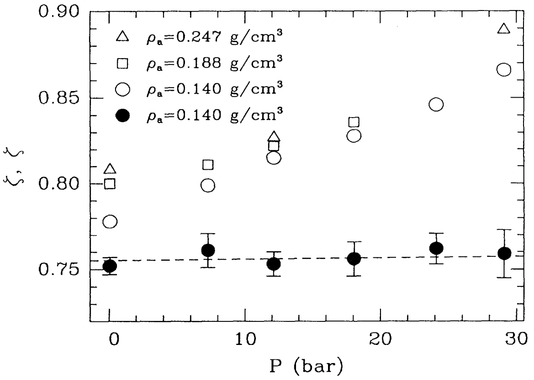

FIG. 3. The exponents $\bar{\zeta}$ and ζ, as a function of pressure. The open symbols correspond to $\bar{\zeta}$ and the closed symbols to ζ.

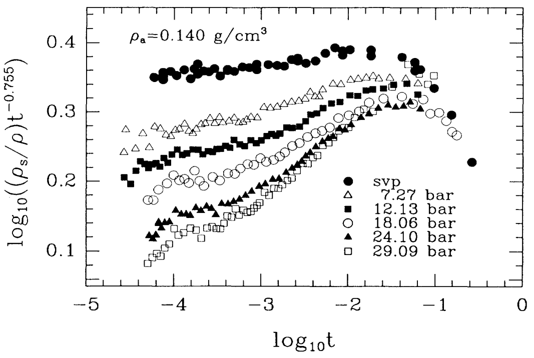

FIG. 4. High-resolution plot of the superfluid fraction ρ_s/ρ for $\rho_a=0.140$ g/cm^3, with the leading singularity taken out, along isobars from SVP to 29 bars.

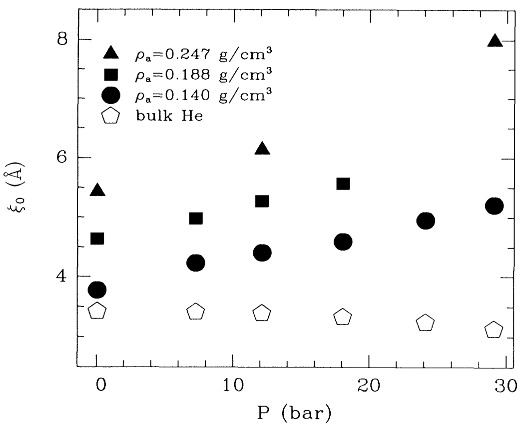

FIG. 5. Amplitude ξ_0 of the correlation length as a function of pressure. The solid symbols correspond to helium in aerogels and the open symbols to bulk helium [13].

ticeable by a close inspection of Fig. 2, which reveals that the results at 24 bars do not really fall on a straight line even for $t<0.01$. Our result at SVP in the sample with $\rho_a=0.188$ g/cm^3 ($\bar{\zeta}=0.80$, $k=2.88$) is in good agreement with previous results obtained by a torsional-oscillator method on a sample of the same batch [12]. This agreement supports the validity of the acoustic model [7] used by us.

To obtain a genuine critical exponent ζ, ρ_s/ρ for the sample with $\rho_a=0.140$ g/cm^3 was fitted by [3,13]

$$\rho_s/\rho=k_0(1+k_1t)(1+at^y)t^\zeta, \tag{4}$$

which includes the confluent singular term at^y and the regular background term k_1t. The nonuniversal amplitudes k_0, k_1, and a, as well as T_c, are expected to be pressure dependent. Together with ζ, they were least-squares adjusted separately at each pressure. The confluent-singularity exponent y was fixed at 0.5. Values close to 0.5 are found theoretically [14] for all physically relevant n-vector models, and experimentally [3,13] for bulk helium. Inclusion of the regular term permitted extension of the range of the fit [13] to $t\simeq0.1$. The values of ζ are shown in Fig. 3 as solid circles. The statistical errors, shown as vertical bars, are considerably larger than those for $\bar{\zeta}$ because of the enhanced correlation between the larger number of parameters. Within these errors, and consistent with the expected universality exponents, the inclusion of the confluent singular term gives a pressure-independent value

$$\zeta=0.755\pm0.003. \tag{5}$$

Most of the data for the denser aerogels did not extend close enough to T_c to allow fits by Eq. (4). Only for $\rho_a=0.188$ g/cm^3 at SVP and 7 bars was it possible to do a full fit, which yielded $\zeta=0.757\pm0.007$ and $\zeta=0.755\pm0.007$, respectively, in excellent agreement with Eq. (5). The other data for the denser aerogels, although not adequate for a separate fit by Eq. (4), were also consistent with the result Eq. (5).

In Fig. 4, $(\rho_s/\rho)t^{-0.755}$ is plotted as a function of t on a log-log scale for the sample with $\rho_a=0.140$ g/cm^3 at various pressures. The leading singularity is thus taken out. The curvature of the plots for $t\lesssim0.01$ shows the importance of including a confluent singular term. The y intercepts for small t give the leading amplitude k_0. The downturn at large t shows the effect of the regular term k_1t. Figure 4 is remarkably similar to the equivalent figure for bulk helium [3], but for $(\rho_s/\rho)t^{-0.67}$ vs t. This shows that the general sizes of confluent and regular terms are not very different for the two systems, and that the difference in the leading exponent is the most important one. Figure 4 also illustrates the remarkable sharpness of the transition. At $t=3\times10^{-5}$, there is as yet no evidence of any "rounding."

The correlation-length amplitude ξ_0 can be determined from the measured values of T_c and from k_0 by using the relation [5]

$$\xi_0=m_4^2k_BT_c/\hbar^2\rho k_0, \tag{6}$$

where m_4 is the mass of a helium atom, k_B is Boltzmann's constant, and $\hbar$ is Planck's constant. The values of k_0 were obtained from fitting ρ_s/ρ by Eq. (4) with ζ fixed at 0.755. The results are plotted versus pressure in Fig. 5. One sees that ξ_0 increases both with pressure and aerogel density. Also shown is ξ_0 for bulk helium [13], which has the opposite pressure dependence.

Figure 6 shows the relative depression $t_c=1-T_c/T_\lambda$ of the transition temperature as a function of pressure [15]. The solid lines correspond to the reduced temperature at which the bulk-helium correlation length has the constant value corresponding to t_c at SVP. Clearly, t_c in aerogels does not correspond to a temperature at which the correlation length in bulk helium is equal to some length characteristic for the aerogel.

Determination of ρ_s from c_s using a hydrodynamic model has a significant advantage over torsional-oscillator [1] measurements. It does not require any background

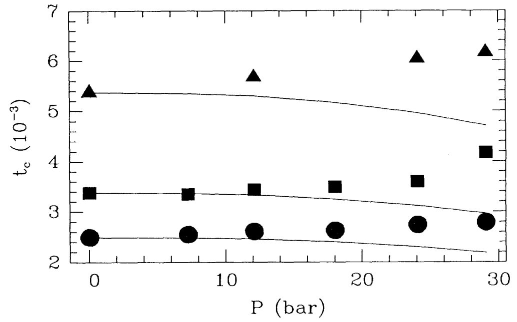

FIG. 6. Depression of the transition temperature as a function of pressure. The solid lines correspond to a bulk-helium correlation length kept constant at its value at T_c at SVP.

subtraction. This makes it possible to measure, in principle, arbitrarily small values of ρ_s. However, the accuracy of the measurement is limited by the uncertainty in the thermodynamic parameters which enter into the model. The effective ρ_a and ρ are modified by the absorption of an inert layer of helium on the aerogel.

Between T_c and T_λ, we observed a propagating slow mode, suggesting the existence of a finite ρ_s. The signals were not clean enough to determine a sound speed. It was not possible to distinguish whether this mode was a genuine propagating mode in the helium-aerogel system or a signal due to bulk helium in percolating large voids. Close to T_c it became impossible to distinguish between the critical and the background modes. This effect was compounded by small signals due to the requirement that the temperature excursions in the excitation be small compared to the distance to T_c.

Machta [16,17] has suggested two different possible explanations of the critical behavior of helium in aerogels. In one picture [16] the system is treated as a fractal percolating network. An exponent greater than the value for bulk helium is obtained, together with the existence of two phase transitions, one at $T_c < T_\lambda$ and another at T_λ. Our pressure-independent result Eq. (5) is consistent with this interpretation. However, the scaling relation $3\zeta = 2 - \alpha$ with Eq. (5) predicts a specific-heat exponent $\alpha = -0.265$. This leads to strong disagreement with the measurements by Wong *et al.* [1]. We also note that the aerogels used in our experiments are base catalyzed and are not expected to show fractal structure over a very wide range of lengths scales. They are unlikely to have the often quoted fractal dimension $D = 2.4$ [18]. Alternatively [17], the effects of the aerogel are treated as correlated disorder. Integration of the Weinrib-Halperin recursion relations [19] leads to *effective* exponents greater than the bulk value. The higher exponent values are attributed to the existence of very slow transients originating from a small specific-heat exponent α. Similar slow transients, but for the case of a logarithmic C_p, arise in a model discussed by Narayan and Fisher [20]. A detailed comparison with these models must await the calculation of the pressure dependence of the effective exponents and of the depression of the transition temperature. Our pressure-independent experimental result Eq. (5) requires that the amplitudes of the transients are also approximately pressure independent.

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