Anomalous Magnetic Moment of the τ Lepton

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We evaluate the magnetic moment $\mu = ge\hbar/2m_\tau c$ of the τ lepton. This is interesting from a purely theoretical point of view, although an experimental measurement, though difficult, may be possible in the future. Our result, including strong- and weak-interaction contributions, which are large, is $a_\tau = (g-2)/2 = 11\,773(3)\times 10^{-7}$.

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The anomalous magnetic moments of the electron [1] and the muon [2,3] provide very precise tests of quantum electrodynamics (QED). In the case of the electron we have an almost pure QED system [a_e(hadronic) $=1.6\times 10^{-12}$]. However, in the case of the muon, the hadronic contribution is large and the weak-interaction contribution will likely be measurable in the new $g-2$ muon experiment, which is under way at Brookhaven National Laboratory. This will provide a good test of the standard model (SM), assuming that a_μ(hadronic) can be determined more precisely.

The τ lepton [4] was discovered in 1975. All evidence indicates that it is a SM lepton [5]. A very important property of the τ, which has yet to be calculated, is its magnetic moment,

$$\mu = ge\hbar/2m_\tau c\,, \tag{1}$$

given by its anomaly

$$a = (g-2)/2\,. \tag{2}$$

Although g has not yet been measured, it may be possible in the future, in spite of the obvious difficulty involved in such an experiment. This could be done by making use of the radiation amplitude zero [6] which occurs at the high-energy end of the lepton distribution in radiative τ decays. This zero amplitude will be noticeable only if $g\approx 2$. Of course, very good energy resolution would be necessary. A measurement of the anomalous magnetic moment of the τ lepton using this method has been suggested by Perl [7]. This could be done at a proposed Tau-Charm Factory, where approximately 10^8 τ pairs would be produced, via the reaction $e^+e^-\to\tau^+\tau^-$ at a c.m. energy of 3.67 GeV. An alternative method would be to make use of channeling [8] in a bent crystal, which has been suggested for measuring baryon magnetic moments, where the baryon has a short lifetime, $\tau\approx 10^{-13}$ s. [The lifetime of the τ lepton is $\tau = 0.303(8)\times 10^{-12}$ s.] The strong electric field is seen by the fast-moving particle as a large megatesla magnetic field and the spin will precess significantly before it decays. This can be measured from the angular distribution of its decay lepton. From the precession angle one can obtain g. Such an experiment [9], to test the method, is under way at Fermilab (E761). The decay $\Sigma^+\to p\gamma$ is being used to measure the magnetic moment of the Σ^+, μ_{Σ^+}. The crystal used is Si. Preliminary results indicate that the method works so that an accurate value for μ_{Σ^+} will likely be obtained [9]. This is a fixed-target experiment. For the τ one could use the decay $B^+\to\tau^+\nu$, which would produce polarized τ leptons. The spin would then precess in a bent crystal and could be measured from the angular distribution of the $\mu(e)$ in the leptonic decay of the τ, $\tau\to\mu(e)\nu\bar{\nu}$. Such an experiment might be possible at the Tevatron at Fermilab or at the Superconducting Super Collider, if fixed-target experiments will be possible there. We hope that this option will be seriously considered. However, independent of the experimental situation we believe a prediction for the magnetic moment of the τ lepton is important from a purely theoretical point of view.

We now outline our calculation. We follow the standard procedure used in the case of the muon by first calculating the mass-dependent contribution to $a_\tau - a_e$, and then in the end using the known value for a_e to obtain a_τ:

$$a_\tau = (a_\tau - a_e) + a_e\,. \tag{3}$$

In our case the τ receives contributions from both the electron and the muon:

$$a_\tau - a_e = I\left(\frac{m_\tau}{m_e}\right) + I\left(\frac{m_\tau}{m_\mu}\right) + I\left(\frac{m_\tau}{m_e},\frac{m_\tau}{m_\mu}\right), \tag{4}$$

where the third term, which depends on all three leptons, occurs only in sixth and higher orders. The mass-dependent contributions for a_e are negligible.

In fourth order the contributions from Figs. 1(a) and 1(b) are given by [3] [see Eq. (9) in Ref. [3]]

$$I^{(4)}(m_\tau/m_e) = 2.025\,61(6)(\alpha/\pi)^2 = 1.092\,91(4)\times 10^{-5} \tag{5}$$

and

$$I^{(4)}(m_\tau/m_\mu) = 0.362\,14(\alpha/\pi)^2 = 0.195\,39(4)\times 10^{-5}\,, \tag{6}$$

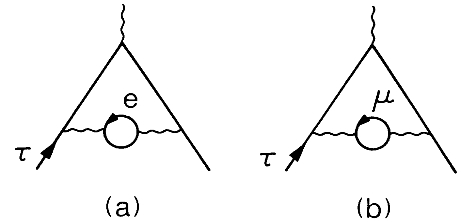

FIG. 1. Feynman diagrams contributing to the fourth-order contribution to $a_\tau - a_e$.

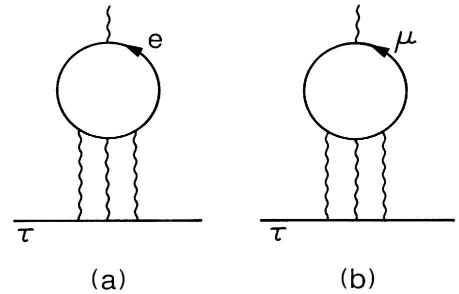

FIG. 2. Light-by-light scattering diagrams contributing to the sixth-order contribution to $a_\tau - a_e$.

respectively.

Thus the total contribution in fourth order is given by

$$(a_\tau - a_e)^{(4)} = 1.288\,30(6)\times 10^{-5}. \quad (7)$$

In sixth order the dominant contributions come from the light-by-light scattering diagrams illustrated in Fig. 2. The contribution of Fig. 2(a) is given by

$$I^{(6)}\left(\frac{m_\tau}{m_e},\gamma\gamma\right) = \left(\frac{2\pi^2}{3}\ln\frac{m_\tau}{m_e} + B\right)\left(\frac{\alpha}{\pi}\right)^3, \quad (8)$$

where B can be obtained from two independent calculations, which agree to the accuracy needed here:

$$B = \begin{cases} -14.13, & \text{M.A.S. [10]}, \\ -14.13, & \text{T.K. [11]}. \end{cases} \quad (9)$$

Thus we obtain

$$I^{(6)}(m_\tau/m_e,\gamma\gamma) = 39.6(\alpha/\pi)^3 = 4.96\times 10^{-7}. \quad (10)$$

Similarly, the contribution of Fig. 2(b) is

$$I^{(6)}(m_\tau/m_\mu,\gamma\gamma) = 4.47(\alpha/\pi)^3 = 5.60\times 10^{-8}. \quad (11)$$

Thus we obtain the total contribution

$$(a_\tau^{(6)} - a_e^{(6)})(\gamma\gamma) = 5.52\times 10^{-7}. \quad (12)$$

The sixth-order vacuum polarization (VP) contribution is small and is given by [see Eqs. (24)-(28) of Ref. [3]]

$$I^{(6)}(m_\tau/m_e,\mathrm{VP}) = 7.2670(\alpha/\pi)^3 = 9.11\times 10^{-8}. \quad (13)$$

Similarly we obtain

$$I^{(6)}(m_\tau/m_\mu,\mathrm{VP}) = -0.1222(\alpha/\pi)^3 = -1.53\times 10^{-9} \quad (14)$$

and

$$I^{(6)}\left(\frac{m_\tau}{m_e},\frac{m_\tau}{m_\mu}\right) = 1.679\left(\frac{\alpha}{\pi}\right)^3 = 2.1\times 10^{-8}. \quad (15)$$

Thus the total VP contribution is

$$(a_\tau - a_e)^{(6)}(\mathrm{VP}) = 1.11\times 10^{-7} \quad (16)$$

and the total sixth-order contribution is given by

$$(a_\tau - a_e)^{(6)} = 6.63\times 10^{-7}. \quad (17)$$

We will use

$$(a_\tau - a_e)^{(6)} = 6.6(1)\times 10^{-7}. \quad (18)$$

This gives us, for the QED contribution,

$$a_\tau^{\mathrm{QED}} - a_e^{\mathrm{QED}} = 1.354(1)\times 10^{-5}, \quad (19)$$

and using the very accurate result [1]

$$a_e = 1\,159\,652\,140(28)\times 10^{-12}, \quad (20)$$

we obtain

$$a_\tau^{\mathrm{QED}} = 117.319(1)\times 10^{-5}. \quad (21)$$

Because of the large mass scale m_τ the hadronic contribution a_τ(hadronic) and the weak-interaction contribution a_τ(weak) are significant. As we shall see the accuracy of our result will be limited by the error estimate for a_τ(hadronic).

The best way to calculate a_τ(hadronic) is to make use of the experimental cross section $\sigma_H(e^+e^-\to \text{hadrons})$ from threshold to high energies. We follow the methods of Refs. [12] and [13] and use the equation (Ref. [14])

$$a_\tau(\text{hadronic}) = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty}\sigma_H(s)k_\tau(s)ds, \quad (22)$$

where $k(s)$ is known explicitly and can be found in Ref. [13].

The dominant contribution comes from the ρ resonance. We use a Breit-Wigner formalism to obtain this contribution:

$$a_\tau(\rho) = 1.88\times 10^{-6}. \quad (23)$$

Adding the contributions from the ω, ϕ, and ψ and the continuum we obtain our result

$$a_\tau^{\mathrm{vac}}(\text{hadronic}) = 3.6(3)(1)\times 10^{-6}. \quad (24)$$

The first error comes from taking the absolute upper and lower bounds of the contribution to a_τ^{vac} corresponding to each continuum contribution to a_μ^{vac} in Ref. [13], Table III, while the second error comes from the error in the experimental cross section σ_H. Adding our estimate for the higher-order hadronic contribution

$$a_\tau(\mathrm{HH}) = -1.2(2)\times 10^{-7}, \quad (25)$$

we obtain our final result

$$a_\tau(\mathrm{hadronic}) = 3.5(3)(1)\times 10^{-6}. \quad (26)$$

As a check, the hadronic contribution is estimated by inserting quarks in place of the electron in the diagram of Fig. 1(a). We use a_μ(hadronic) to determine the masses of the quarks. The result is rather insensitive to m_c and m_s. We then insert these quark masses in Fig. 1(a) obtaining our result for a_τ(hadronic). We obtain

$$a_\tau(\mathrm{hadronic}) = 2.6\times 10^{-6}. \quad (27)$$

This result, which is not accurate due to unknown systematic errors, is about 25% below our result in Eq. (26). We will use the result in Eq. (26) which should be a reliable value for a_τ(hadronic) because the quoted error was estimated by using absolute upper and lower bounds.

The weak-interaction contribution is easily obtained by scaling the corresponding contribution for the muon

$$a_\tau(\mathrm{weak}) = (m_\tau/m_\mu)^2 a_\mu(\mathrm{weak}) = 5.560(2)\times 10^{-7}. \quad (28)$$

Finally, adding the results of Eqs. (21), (26), and (28) we obtain our result

$$a_\tau = 11\,773(3)\times 10^{-7} \quad (29)$$

or

$$g_\tau = 2.002\,355\,6(6). \quad (30)$$

It can be seen that the error in a_τ(hadronic) is comparable to the sixth-order contribution and the weak contribution and dominates the error in a_τ. We note that a_τ(hadronic) is about 50 times larger than a_μ(hadronic). A measurement accurate enough to see a_τ(hadronic) would be very interesting. Of course, first a measurement to verify that $g \approx 2$ is necessary. If one would measure a_τ directly as in the case of the muon and the electron, then a measurement with an accuracy of only 3 parts per 10^3 would allow one to see the hadronic contribution. We urge experimentalists to seriously consider making a measurement of g, first to verify that $g=2$, and then a_τ with a precision of a few parts per 10^3 to see the hadronic contribution.

These can be compared with the corresponding results for the electron and the muon,

$$a_e = 1\,159\,652\,140(28)\times 10^{-12}, \quad (31)$$

$$g_e = 2.002\,319\,304\,280(56), \quad (32)$$

and

$$a_\mu = 116\,591\,902(77)\times 10^{-11}, \quad (33)$$

$$g_\mu = 2.002\,331\,838\,04(154). \quad (34)$$

In conclusion, we have obtained the magnetic moment of the τ lepton. The hadronic- and weak-interaction contributions are large. The latter can be used as a test of the SM. Finally, we would like to urge experimentalists to consider attempting a measurement of the magnetic moment of the τ lepton.

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