Gravity Invasion Percolation in Two Dimensions: Experiment and Simulation

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We present experiments and computer simulations of slow drainage in a two-dimensional porous medium. The effect of gravity is systematically varied by tilting the system from the horizontal position. The width σ of the front between the fluids is found to scale with the dimensionless Bond number Bo (ratio between gravitational and capillary forces) as $\sigma \sim \mathrm{Bo}^{-0.57}$, as predicted by theory. The external perimeter of the invaded structure is shown to be fractal with the fractal dimension $D_e \simeq 1.34$ for length scales smaller than the front width.

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Slow fluid-fluid displacement in a porous medium under the influence of gravitational forces is important in oil production, in hydrology, in chemical engineering, and in the physics of disordered media. Front structures observed in two-phase flow exhibit patterns that range from compact to disordered and ramified [1,2]. Commonly studied processes are fast viscous fingering [3-5] dominated by viscous forces (modeled by the diffusion-limited aggregation algorithm [6]) and slow invasion percolation (IP) [7,8], where capillary forces dominate, which is simulated by the IP algorithm [9,10]. The interfaces between the fluids form fractal fronts [11,12] having no intrinsic length scale.

Most systems of practical importance are three-dimensional (3D) and include fluids of different densities. Therefore, it is important to study the effect of gravity on the front structure. Gravity causes hydrostatic pressure gradients in the fluids, and introduces a length scale that leads to crossover phenomena. If the less-dense fluid is on top of the heavier fluid, gravity effects stabilize the front. In quantitative terms the competition between gravity and capillary forces is described by the dimensionless Bond number: $\mathrm{Bo} = ga^2\Delta\rho/\gamma$, where g is the acceleration of gravity, a is a typical pore size, $\Delta\rho$ is the fluid density difference, and γ is the fluid interface tension.

Little experimental information on gravity effects is available and experiments with a systematic variation of Bo are needed. Clément and co-workers [13] performed 3D IP experiments. Nonwetting Woods metal was slowly injected from below into a column of crushed glass. Horizontal cuts of the solid material were analyzed to determine the spatial correlations of the metal and the fractal character of the front was studied. However, they used only one value of Bo.

Here we present two-dimensional (2D) experiments where, for the first time, Bo was varied systematically by tilting the plane of the experimental model from the horizontal plane. The resulting invasion front geometry was studied quantitatively and compared to theory and 2D computer simulations. Figure 1 shows pictures of experiments and results from simulations at two different Bo. We found (Fig. 2) that the front width scales as

$$\sigma \sim \mathrm{Bo}^{-0.57}. \qquad (1)$$

The exponent 0.57, consistent with the theoretical prediction of $\frac{4}{7}$, will be discussed below. We also found that the fronts between the two fluids had a fractal dimension $D_{\mathrm{exp}} = 1.34$, consistent with the fractal dimension of external perimeters [14,15] found in our simulations: $D_{\mathrm{sim}} = 1.39$; see Fig. 3. These values are consistent with heuristic arguments [14,15] which predict a fractal dimension of $\frac{4}{3}$ for the external perimeter of percolation clusters. Fronts, both experimental and simulated, cross over to a linear behavior at length scales larger than the front width.

In IP at breakthrough, the invading fluid forms a single

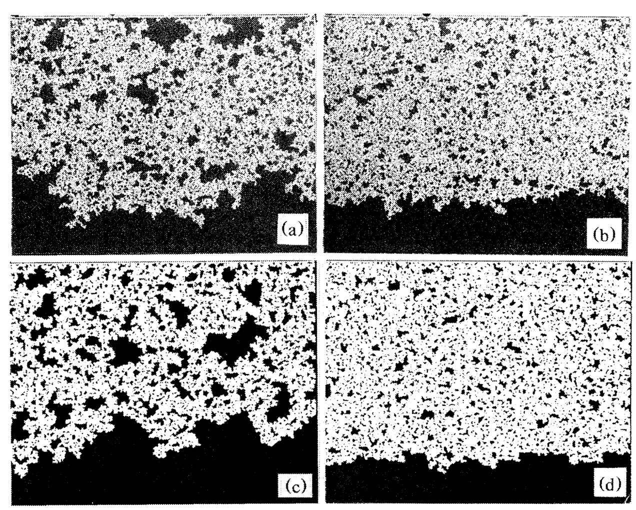

FIG. 1. (a) Slow displacement of glycerol-water mixture by air in a two-dimensional porous medium at angle $\theta = 3°$ to the horizontal (Bo=0.005) and (b) $\theta = 11°$ (Bo=0.018). (c) Numerical simulations of invasion percolation in a gradient, on a square lattice of size 400×400 at Bond number Bo=0.001 and (d) Bo=0.01.

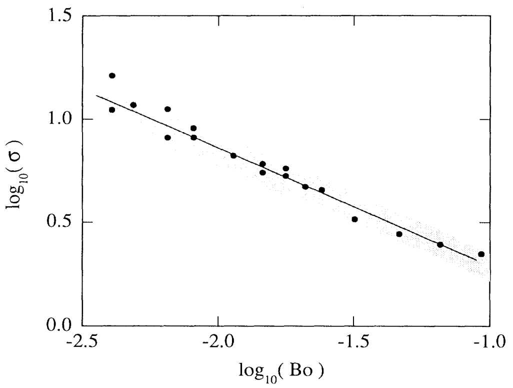

FIG. 2. Front width σ as a function of Bond number ($\mathrm{Bo}=a^2\Delta\rho g\sin\theta/\gamma$). Points represent measured widths of experimental fronts. The shaded area represents ± 1 standard deviation of front width obtained in numerical simulations. The slope of the straight line is $-\nu/(1+\nu)=-0.57$, as predicted by the theory.

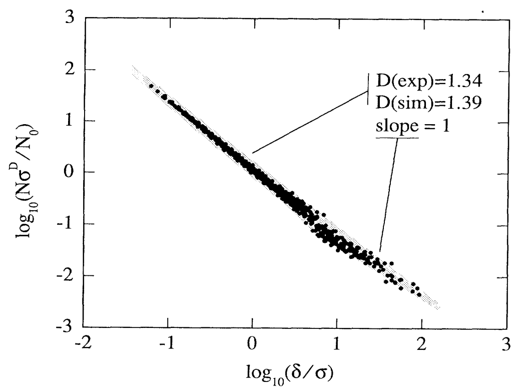

FIG. 3. Data collapse of the box-counting data for twenty uncorrelated fronts from thirteen experiments (black dots), each at different Bond number Bo. The numerical results are given as the shaded area, representing ± 1 standard deviation of box numbers at each box size.

cluster connecting the source to the sink. Slow displacement of an incompressible fluid without buoyancy in 3D produces a cluster that has the same scaling properties as percolation clusters [9,16]. Wilkinson [17] introduced the theory for 3D IP with buoyancy and developed an algorithm to simulate the process. He argued that in 3D the characteristic length ξ of trapped regions scales like a percolation correlation length, depending on the concentration of invaded sites, p, as $\xi\sim(p-p_c)^{-\nu}$, with $\nu\simeq 0.88$ [16]. Interpreting Bo as the gradient of p yields $p-p_c\sim\xi\,\mathrm{Bo}$ and thus $\xi\sim\mathrm{Bo}^{-\nu/(1+\nu)}$. The width L of the transition region where both phases percolate scales differently: $L\sim 1/\mathrm{Bo}$ [17]. The theory was supported by 3D simulations [17].

Sapoval, Rosso, and Gouyet [18] discussed the width of 2D diffusion fronts in terms of gradient percolation, where the occupation probability $p=p(x)$ depends on the position x. With arguments similar to those used by Wilkinson [17], they found that the front width σ scales as $\sigma\sim G^{-\nu/(1+\nu)}$, with the occupation probability gradient G substituting for the Bond number.

In the present paper we extend Wilkinson's theoretical discussion to 2D IP. Although trapping is very important for the behavior of the bulk of the cluster in 2D, we argue that in 2D systems the front of the invading cluster, identified as its external perimeter, is not affected by trapping. This is because the perimeters of the trapped regions do not belong to the external perimeter. Therefore the external perimeter of IP (with and without trapping) is equivalent to that of a percolation cluster.

If the system includes buoyancy, the front becomes spatially limited. The trapped regions inside the invaded structure are generated within the span of the front so that the front width σ sets an upper limit to the linear trapped-region size ξ: $\sigma\simeq\xi$. We conclude that ξ should scale with the same exponent ν of ordinary percolation, in both 2D and 3D.

In IP, at some position of the fluid-fluid front, x_m, the probability of invading pores is $p(x_m)=p_c$. The capillary pressure difference of two arbitrary pores on the front depends on the distance as $\mathcal{P}-\mathcal{P}_m=(x-x_m)\mathrm{Bo}$, where the dimensionless capillary pressure is $\mathcal{P}=(a/\gamma)P_{\mathrm{cap}}$ [17]. The front of the invading cluster is thus equivalent to the connected-cluster front in gradient percolation [18], where the front width scales as Eq. (1). The basic argument for the front width [17,18] depends on the fact that the correlation length scales with the deviation from the critical occupation probability. When the fjords of the front are comparable in size to the correlation length, $x-x_m\sim\xi\sim\sigma$, they are closed and trapped regions result.

The analogy between our invaded sites and occupation in percolation is based on a mapping between $\mathcal{P}$ and the occupation probability p in the following way:

$$\int_{\mathcal{P}_m}^{\mathcal{P}(x=x_m+\sigma)} N(\mathcal{P})\,d\mathcal{P}=p-p_c\sim\xi^{-1/\nu}, \qquad (2)$$

where $N(\mathcal{P})$ is the normalized dimensionless capillary pressure distribution when Bo=0. A uniform distribution gives $\mathcal{P}-\mathcal{P}_c=\Delta\mathcal{P}$. For a nonsingular distribution a Taylor expansion of $N(\mathcal{P})$ around $\mathcal{P}_m$ gives $\mathcal{A}\Delta\mathcal{P}+\mathcal{B}\Delta\mathcal{P}^2+\mathcal{C}\Delta\mathcal{P}^3+\cdots$. Neglecting higher-order terms giving corrections to scaling, and using the results above, we get Eq. (1).

The experiments were performed using transparent two-dimensional porous models consisting of a monolayer of 1-mm glass beads placed at random and sandwiched between two plastic sheets [19,20]. The model had dimensions 400×350 mm, porosity $\phi\simeq 0.7$, and was supported between two 25-mm Plexiglas plates. The plastic sheets were forced into contact with the glass beads by a transparent PVC air pillow inflated between a supporting plate and one of the sheets. The model was placed on a stand that enabled us to set the angle θ between the horizontal and the plane of the model.

In the experiments air invaded a glycerol-water mixture (dyed black with Nigrosine) of viscosity $\mu=280$ cP and surface tension $\gamma=64$ dyn/cm. Air entered through a 7×5-mm duct across the top edge of the model as the glycerol was withdrawn through a similar duct at the bottom edge using a syringe pump. The flow rate was 6 ml/h (mean front velocity $\simeq 24$ mm/h) and low enough to ensure that the experiments were in the IP regime.

In our experiments only the gravity component $g\sin\theta$ is relevant. Therefore we use the modified Bond number: $\mathrm{Bo}=a^2\Delta\rho g\sin\theta/\gamma$. Experiments with Bo in the range 0.004–0.093 ($2.5° \leq \theta \leq 90°$) were photographed and the negatives digitized at a resolution of 120 μm per pixel (corresponding to 11.8 pixels per pore) using a film scanner (Nikon LS 3500) connected to an Apollo work station.

Experimental fronts were identified in digitized photographs using image processing software developed here. Details of the analysis are found in Ref. [20]. The front widths for twenty uncorrelated experimental fronts, from thirteen independent experiments, were determined as follows: The number of pixels belonging to the front as a function of position from the bottom edge was counted. These numbers had an approximately Gaussian distribution centered at the mean position of the front. The front width σ was defined to be the standard deviation of this distribution of perimeter sites. Figure 2 shows the dependence of σ (in the units of typical pore size $\simeq 1$ mm) as a function of Bo. The experimental results (points) and results from simulations (discussed below) are both consistent with Eq. (1) (solid line).

To *simulate* the IP process the porous medium was modeled by a square lattice of size $L_1 \times L_2$ ($L_1=400$ corresponds to the width of the experimental model used, $L_2=1200$) consisting of nodes (pores) connected by bonds. The bonds of the lattice were assigned random numbers r_{i0} drawn from a uniform distribution on [0,1]. Each of these numbers represented the threshold value for the capillary pressure needed to penetrate the bond connecting two pores. In simulations the hydrostatic pressure gradient was introduced [17] by assigning the number $r_i=r_{i0}+G(L_2-x)$ to the ith bond. Here $G=\mathrm{Bo}/2$ is the gradient and x denotes the bond's position measured from the bottom edge of the lattice.

The simulations started with all pores filled by the defending "fluid" except sites along the upper edge which were invaded by air. At each time step the pore that was invaded was the pore connected to the already invaded region by the best bond, i.e., the bond that had the lowest number r_i. The simulation stopped when the lower edge was reached. In the IP process regions of the defending fluid sometimes became completely surrounded by the invading fluid. The surrounded regions could not be invaded since the defending fluid was incompressible in our experiments. Therefore, in simulations, the trapped regions were identified and closed for further invasion. The trapping rule leads to structures that differ from ordinary percolation clusters [9,16].

The effect of gravity was investigated in simulations by a study of the invasion front for ten gradient values, corresponding to Bo in the range 0.001–0.1. For each Bond number fifty statistically uncorrelated fronts were analyzed. The front was defined to be the external perimeter [14,15] of the invasion cluster, and the front width was determined by the method used for experiments. In Fig. 2 the results of the simulated front widths are summarized as the shaded area representing 1 standard deviation around the average width obtained in the simulations. The simulations describe the experimentally observed effect of gravity well. We observe no significant corrections to scaling within the precision of our experimental data. This indicates an approximately uniform capillary threshold distribution in the experimental model within the range $[\mathcal{P}(x_m-\sigma),\mathcal{P}(x_m+\sigma)]$.

The fractal structure of the fronts for experiments and simulations were examined by the box-counting method. The number N of square boxes of side δ needed to cover the front scales as

$$N(\delta)=N_0\sigma^{-D}f(\delta/\sigma)\,, \tag{3}$$

where D is the fractal dimension of the front, $f(x)$ is a function depending only on the combination δ/σ, and N_0 is the number of boxes of size $\delta=a$. Fitting $\log[N(\delta)\sigma^D/N_0]$ by the straight line $D\log(\delta/\sigma)+A$, where D and A are the only two free parameters gave us the best scaling data collapse presented in Fig. 3. Here, the data (at all Bo) were fitted for box sizes $\delta/\sigma<0.3$. In Fig. 3 points are the experimental data, while the shaded area represents ± 1 standard deviation around the mean of the simulation box-counting data. We obtained $D_{\mathrm{exp}}=1.34\pm0.04$ for experimental data and $D_{\mathrm{sim}}=1.39\pm0.02$ for simulations. Crossover to a one-dimensional front is seen at the rescaled box size $\delta/\sigma>10$. We find that our results are consistent with the fractal dimension of external perimeter of the percolation cluster [14,15] $D_e \simeq 1.37$. This result indicates that the external perimeters of percolation and IP with trapped clusters have the same structure.

In summary, the effects of gravity tend to stabilize the drainage front. Invasion fronts have a *finite* width, $\sigma \sim \mathrm{Bo}^{-\nu/(1+\nu)}$, with $\nu=\frac{4}{3}$, consistent with the experimental results and numerical simulations based on the IP algorithm. We have shown that the geometry of the front is well described by a modified IP algorithm with trapping. The front is shown to be fractal with a box-counting dimension $D_{\mathrm{exp}}=1.34\pm0.04$, consistent with the simulations, $D_{\mathrm{sim}}=1.39\pm0.02$. We conclude that two-dimensional gravity drainage in a porous medium may be quantitatively modeled by IP with a gradient.

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[1] R. Lenormand, E. Touboul, and C. Zarcone, J. Fluid Mech. **189**, 165 (1988).

[2] V. Frette, K. J. Måløy, F. Boger, J. Feder, T. Jøssang, and P. Meakin, Phys. Rev. A **42**, 3432 (1990).

[3] K. J. Måløy, J. Feder, and T. Jøssang, Phys. Rev. Lett. **55**, 2688 (1985).

[4] J. D. Chen and D. Wilkinson, Phys. Rev. Lett. **55**, 1892 (1985).

[5] J. Nittmann, G. Daccord, and H. E. Stanley, Nature (London) **314**, 141 (1985).

[6] P. Meakin, in *Phase Transitions and Critical Phenomena,* edited by C. Domb and J. L. Lebowitz (Academic, New York, 1987), p. 336.

[7] P. G. de Gennes and E. Guyon, J. Mec. **17**, 403 (1978).

[8] R. Lenormand and C. Zarcone, Phys. Rev. Lett. **54**, 2226 (1985).

[9] D. Wilkinson and J. F. Willemsen, J. Phys. A **16**, 3365 (1983).

[10] L. Furuberg, J. Feder, A. Aharony, and T. Jøssang, Phys. Rev. Lett. **61**, 2117 (1988).

[11] B. B. Mandelbrot, *The Fractal Geometry of Nature* (Freeman, New York, 1982).

[12] J. Feder, *Fractals* (Plenum, New York, 1988).

[13] E. Clément, C. Baudet, and J. P. Hulin, J. Phys. (Paris), Lett. **46**, L1163 (1985); E. Clément, C. Baudet, E. Guyon, and J. P. Hulin, J. Phys. D **20**, 608 (1987); J. P. Hulin, E. Clément, C. Baudet, J. F. Gouyet, and M. Rosso, Phys. Rev. Lett. **61**, 333 (1988).

[14] T. Grossman and A. Aharony, J. Phys. A **19**, L745 (1986); J. Phys. A **20**, L1193 (1987).

[15] H. Saleur and B. Duplantier, Phys. Rev. Lett. **58**, 2325 (1987).

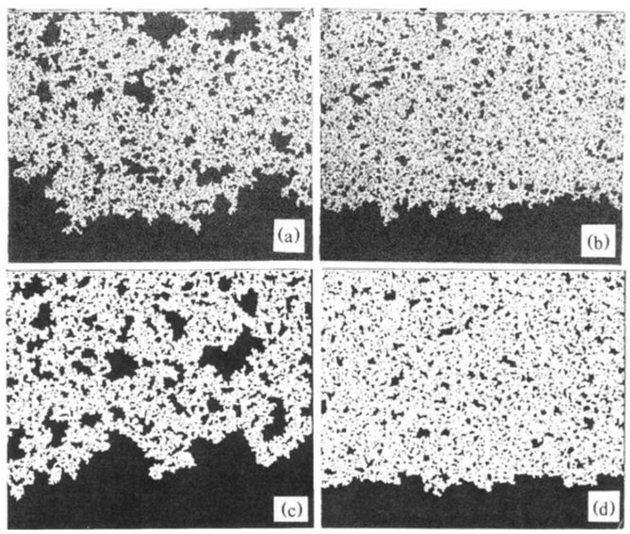
[16] D. Stauffer, *Introduction to Percolation Theory* (Taylor & Francis, London, 1985).

[17] D. Wilkinson, Phys. Rev. A **30**, 520 (1984); **34**, 1380 (1986).

[18] B. Sapoval, M. Rosso, and J. F. Gouyet, J. Phys. (Paris), Lett. **46**, L149 (1985).

[19] K. J. Måløy, F. Boger, J. Feder, and T. Jøssang, in *Time-Dependent Effects in Disordered Materials,* edited by R. Pynn and T. Riste (Plenum, New York, 1987), p. 111.

[20] A. Birovljev, Master's thesis, Department of Physics, University of Oslo, 1990 (unpublished).



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