Transient-Gain-Assisted Noise Reduction in Photodetection of Nonclassical Light

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The effect of transient gain in photodetection of light is studied. By treating the photoelectron multiplication as a classical, stochastic process, a generalization of the quantum Mandel formula is given. To illustrate the theory developed, the detection of sub-Poisson light from single-atom resonance fluorescence is studied. It is shown that the results may substantially differ from those predicted from the Burgess variance theorem. In particular, a reduction of noise of the photoelectric counts can be achieved even if the value of the gain rate is increased.

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In the standard theory of photoelectric detection of light the statistics of the photoelectrons primarily emitted by an illuminated photosurface is calculated [1-8]. In quantum optics, the result is well-known Mandel formula, in which the photoelectric counting distribution is expressed in terms of normally and time-ordered correlation functions of the electric-field strength of the light field to be detected. In this way, the statistics of the photoelectric counts is closely related to that of the light. In particular, nonclassical light with sub-Poisson statistics gives rise to a noise reduction of the photoelectric counts below the Poisson level, which just corresponds to the case when the photodetector is illuminated with nonfluctuating classical light or quantum light in a coherent state.

In many practical cases, however, photodetector devices are used that operate by multiplying the photoelectrons primarily generated. Clearly, this multiplication process introduces, in general, additional noise, so that the sensitivity of the receiver is determined by the noise of both the light and the gain process. However, appropriately tuning the two noise sources to each other with regard to their time characteristics may substantially improve the sensitivity, as we will show.

There is a series of articles dealing with the amplification process within the framework of deterministic rather than stochastic description [9-11]. The concepts of stochastic treatment commonly used invoke the assumption of instantaneous multiplication [12-15]. Roughly speaking, this assumption means that the transit time through the multiplication region is required to be much smaller than the detector integration time, which may become crucial in high-bit-rate optical communication. Calculations without this assumption were performed for the case of a single photocarrier entering the multiplication region [16].

The aim of the present Letter is to generalize the quantum Mandel formula for the case of an additional stochastic multiplication process being present, without the assumption of instantaneous multiplication. This renders it possible to relate the statistics of the amplified photoelectric counts to those of the light and of the multiplication, as well as to study the time behavior of the "cooperative" action of the two noise sources. To illustrate the theory developed, the noise reduction in the detection of single-atom resonance fluorescence is discussed, the simplified model of an exponential gain process being used.

We are interested in the number of carriers $M(t,\Delta t)$ detected during the time interval $t,t+\Delta t$. Subdividing the detector integration time Δt into small time elements $\Delta\tau$ according to $\tau_j=t+j\Delta\tau$, with $j=0,1,\ldots,J$ and $J\Delta\tau=\Delta t$, we may represent $M(t,\Delta t)$ as follows:

$$M(t,\Delta t)=\sum_{i=1}^{N_a}\sum_{j=0}^{J-1}n_i(\tau_j,\Delta\tau)m_i(t+\Delta t-\tau_j)\,. \qquad (1)$$

Here, $n_i(\tau_j,\Delta\tau)$ is the number of photoelectrons that are primarily emitted by the ith atom of the photosensitive part of the detector during the time interval $\tau_j,\tau_j+\Delta\tau$, and $m_i(t+\Delta t-\tau_j)$ is the gain factor describing the subsequent multiplication process, N_a being the total number of detector atoms. We now regard $M(t,\Delta t)$ as being a stochastic process composed, according to Eq. (1), of the stochastic processes $n_i(\tau_j,\Delta\tau)$ and $m_i(t+\Delta t-\tau_j)$. Instead of explicitly determining the probability distribution function $P_N(t,\Delta t)$ for detecting N electrons during the time interval $t,t+\Delta t$, we prefer to calculate the generating function

$$y(x;t,\Delta t)=\langle(1+x)^{M(t,\Delta t)}\rangle\,, \qquad (2)$$

from which the probability distribution function can be derived by differentiations:

$$P_N(t,\Delta t)=\frac{1}{N!}\frac{\partial^N}{\partial x^N}y(x;t,\Delta t)\bigg|_{x=-1}\,. \qquad (3)$$

Substituting in Eq. (2) for $M(t,\Delta t)$ the result of Eq. (1), we may write

$$y(x;t,\Delta t)=\left\langle\prod_{j=0}^{J-1}\prod_{i=1}^{N_a}[(1+x)^{m_i(t+\Delta t-\tau_j)}]^{n_i(\tau_j,\Delta\tau)}\right\rangle. \qquad (4)$$

To perform the n_i and m_i averagings we make the following assumptions and approximations.

(i) Following the standard photodetection theory [1-3] we assume the probabilities for multielectron emissions from a detector atom during the time interval $t,t+\Delta t$ are much smaller than the probability for single-electron emission, so that we may let $n_i(\tau_j,\Delta\tau)=0,1$, $\sum_j n_i(\tau_j,\Delta\tau)=0,1$.

(ii) We suppose the photoelectrons undergo multiplication processes of equal stochastic character and the processes for different photoelectrons are not correlated with each other $[\langle m_i(\tau) m_{i'}(\tau')\rangle = \langle m_i(\tau)\rangle\langle m_{i'}(\tau')\rangle$ if $i \neq i'$, $\langle m_i(\tau)\rangle = \langle m_{i'}(\tau)\rangle = \langle m(\tau)\rangle]$.

Equation (4) then simplifies to

$$y(x;t,\Delta t) = \left\langle \prod_{j=0}^{J-1} [z(x;t+\Delta t-\tau_j)]^{n(\tau_j,\Delta\tau)} \right\rangle, \tag{5}$$

where

$$z(x;\tau) = \langle (1+x)^{m(\tau)} \rangle \tag{6}$$

is the generating function for the multiplication process, and

$$n(\tau_j,\Delta\tau) = \sum_{i=1}^{N_a} n_i(\tau_j,\Delta\tau) \tag{7}$$

is the stochastic variable representing the total number of photoelectrons (primarily) emitted during the time interval $\tau_j, \tau_j+\Delta\tau$. Applying standard photodetection theory [1-3] we may perform the remaining n averaging in Eq. (5) to obtain

$$y(x;t,\Delta t) = \left\langle :\exp\left[\alpha \int_t^{\Delta t} d\tau\, \hat{I}(\tau)[z(x;t+\Delta t-\tau)-1]\right]:\right\rangle, \tag{8}$$

where

$$\hat{I}(\tau) = \hat{\mathbf{E}}^{(-)}(\tau)\hat{\mathbf{E}}^{(+)}(\tau), \tag{9}$$

$\hat{\mathbf{E}}^{(+)}(\tau)$ and $\hat{\mathbf{E}}^{(-)}(\tau)$, respectively, being the positive and negative frequency parts of the operator of the electric-field strength (taken at the position of the photosensitive entrance plane of the detector with efficiency α). Further, the :: notation introduces the familiar normal and time orderings. Note that in Eq. (8) the j sum is represented as an τ integral.

From Eq. (8) together with Eq. (6) both the probability distribution function $P_N(t,\Delta t)$ [Eq. (3)] and the (factorial) moments may be derived by straightforward differentiations. Clearly, if there is no gain process, that is to say $z(x;\tau) = 1+x$ [cf. Eq. (6)], $P_N(t,\Delta t)$ reduces to the well-known quantum Mandel formula [1-3].

Let us consider the variance $\langle [\Delta M(t,\Delta t)]^2\rangle$, which is derived to be

$$\langle [\Delta M(t+\Delta t)]^2\rangle = \alpha \int_t^{t+\Delta t} d\tau\, \langle m^2(t+\Delta t-\tau)\rangle\langle :\hat{I}(\tau):\rangle + \alpha^2 \int_t^{t+\Delta t}\!\!\int d\tau\, d\tau' \langle m(t+\Delta t-\tau)\rangle\langle m(t+\Delta t-\tau')\rangle\langle :\Delta\hat{I}(\tau)\Delta\hat{I}(\tau'):\rangle, \tag{10}$$

where

$$\langle m(\tau)\rangle = \frac{\partial}{\partial x} z(x;\tau)\bigg|_{x=0}, \tag{11}$$

$$\langle m^2(\tau)\rangle = \langle m(\tau)\rangle + \frac{\partial^2}{\partial x^2} z(x;\tau)\bigg|_{x=0}. \tag{12}$$

If the time dependence of the gain process can be disregarded, Eq. (10) reduces to the Burgess variance theorem [17] widely applied to so-called instantaneous amplifications.

To illustrate the theory developed let us adopt the simple model of a Markovian multiplication process with exponential amplification and study the effect of finite amplification time on the noise reduction in the number of output carriers for the case of the detector being illuminated by sub-Poisson single-atom resonance fluorescence light. For this purpose it is useful to represent the variance $\langle [\Delta M(t,\Delta t)]^2\rangle$ [Eq. (10)] in the form

$$\langle [\Delta M(t,\Delta t)]^2\rangle = [1+\phi(t,\Delta t)]\alpha \int_0^{\Delta t} d\tau\, \langle m^2(\tau)\rangle\langle :\hat{I}(t+\Delta t-\tau):\rangle, \tag{13}$$

where

$$\phi(t,\Delta t) = \frac{\alpha \int_0^{\Delta t}\int d\tau\, d\tau' \langle m(\tau)\rangle\langle m(\tau')\rangle\langle :\Delta\hat{I}(t+\Delta t-\tau)\Delta\hat{I}(t+\Delta t-\tau'):\rangle}{\int_0^{\Delta t} d\tau\, \langle m^2(\tau)\rangle\langle :\hat{I}(t+\Delta t-\tau):\rangle} \tag{14}$$

may be regarded as being a natural measure of the departure of the variance of the number of output carriers from the variance for the case of the radiation field being in a coherent state. In this case we have $\phi(t,\Delta t)=0$. A reduction of noise below this level may be achieved by using nonclassical sub-Poisson light (see below). Note that in the case without amplification ($\langle m^2\rangle = \langle m\rangle = 1$) the ϕ function reduces to Mandel's Q function [18].

Following Ref. [19] we may represent the generating function $z(x_i\tau)$ as follows:

$$z(x;\tau) = 1 + K(\tau)x/\{1-[K(\tau)-1]x\}, \tag{15}$$

where

$$K(\tau) = \exp[a \min(t_T,\tau)], \tag{16}$$

a and t_T being the gain rate and the time of duration of the gain process (transit time through the multiplication region), respectively.

Making use of the results given in Ref. [18], the stationary intensity and intensity correlation function needed read as

$$\langle :\hat{I}(t+\Delta t-\tau):\rangle = \langle :\hat{I}(t):\rangle = \langle :\hat{I}:\rangle = \alpha' \frac{\frac{1}{2}\Omega_R^2/\beta}{\frac{1}{2}\Omega_R^2/\beta^2+1}, \tag{17}$$

$$\begin{aligned}\langle :\Delta\hat{I}(t+\Delta t-\tau)\Delta\hat{I}(t+\Delta t-\tau'):\rangle &= \langle :\Delta\hat{I}(t)\Delta\hat{I}(t+\tau-\tau'):\rangle \\ &= -\langle :\hat{I}:\rangle^2 e^{-3\beta(\tau-\tau')/2}\{\cos[\Omega'\beta(\tau-\tau')]+(3/2\Omega')\sin[\Omega'\beta(\tau-\tau')]\},\end{aligned} \tag{18}$$

where

$$\Omega' = (\Omega_R^2/\beta^2 - \tfrac{1}{4})^{1/2}, \tag{19}$$

Ω_R and β, respectively, being the Rabi frequency and the radiation damping parameter. The unspecified constant α' will be included in a modified detector efficiency $\tilde{\alpha}=\alpha\alpha'$, where $0\le\tilde{\alpha}\le 1$ (cf. Ref. [20]).

Combining Eqs. (11), (12), (15)-(18), and (14), after some algebra we eventually arrive at the rather lengthy formula ($\Delta t \ge t_T$)

$$\begin{aligned}\phi(t,\Delta t) &\equiv \phi(\Delta t;a,t_T) \\ &= -\tilde{\alpha}\frac{a(\Delta t-t_T)}{a(\Delta t-t_T)(2-e^{-at_T})+1-e^{-at_T}}\frac{\frac{1}{2}\Omega_R^2/\beta^2}{(\frac{1}{2}\Omega_R^2/\beta^2+1)^2}\frac{1}{\beta(\Delta t-t_T)} \\ &\quad\times\Bigg\{6\beta(\Delta t-t_T)+\frac{\Omega_R^2/\beta^2-7}{\frac{1}{2}\Omega_R^2/\beta^2+1}+e^{-3\beta(\Delta t-t_T)/2}\Bigg[\Bigg(\frac{(9/2\Omega')(1-\Omega_R^2/\beta^2)}{\frac{1}{2}\Omega_R^2/\beta^2+1}+f_1\Bigg)\sin[\Omega'\beta(\Delta t-t_T)] \\ &\qquad -\Bigg(\frac{\Omega_R^2/\beta^2-7}{\frac{1}{2}\Omega_R^2/\beta^2+1}+f_2\Bigg)\cos[\Omega'\beta(\Delta t-t_T)]\Bigg] \\ &\qquad +e^{-(3\beta\Delta t/2+at_T)}\{f_2\cos(\Omega'\beta\Delta t)-f_1\sin(\Omega'\beta\Delta t)\}+(f_2+f_3)\{1-e^{-(a+3\beta/2)t_T}\cos(\Omega'\beta t_T)\} \\ &\qquad +(f_1+f_4)e^{-(a+3\beta/2)t_T}\sin(\Omega'\beta t_T)+f_5(1-e^{-2at_T})\Bigg\},\end{aligned} \tag{20}$$

where

$$f_1=\frac{2}{\Omega'}\frac{(a/\beta)(\Omega_R^2/\beta^2-\frac{5}{2})+\frac{9}{2}(\Omega_R^2/\beta^2-1)}{(a/\beta+1)(a/\beta+2)+\Omega_R^2/\beta^2}, \tag{21}$$

$$f_2=\frac{2(3a/\beta+7-\Omega_R^2/\beta^2)}{(a/\beta+1)(a/\beta+2)+\Omega_R^2/\beta^2}, \tag{22}$$

$$f_3=4\frac{(a^2/\beta^2+\Omega_R^2/\beta^2-7)(\frac{1}{2}\Omega_R^2/\beta^2+1)}{[(a/\beta-1)(a/\beta-2)+\Omega_R^2/\beta^2][(a/\beta+1)(a/\beta+2)+\Omega_R^2/\beta^2]}, \tag{23}$$

$$f_4=-\frac{(18/\Omega')(\Omega_R^2/\beta^2+\frac{1}{3}a^2/\beta^2-1)(\frac{1}{2}\Omega_R^2/\beta^2+1)}{[(a/\beta-1)(a/\beta-2)+\Omega_R^2/\beta^2][(a/\beta+1)(a/\beta+2)+\Omega_R^2/\beta^2]}, \tag{24}$$

$$f_5=6\frac{(\frac{1}{2}\Omega_R^2/\beta^2+1)(\beta/a-\frac{1}{3})}{(a/\beta-1)(a/\beta-2)+\Omega_R^2/\beta^2}. \tag{25}$$

Typical examples of the behavior of the ϕ function are shown in Fig. 1.

As expected, for $a=0$ or $t_T=0$ the ϕ function reduces to the Q function of Ref. [18] [$\phi(\Delta t;0,t_T)=\phi(\Delta t;a;0)=Q(\Delta t)$]. It can further be proved that in the approximation of the Burgess variance theorem being used the amplification process is predicted to make the relative noise reduction worse by the factor

$$\frac{\phi(\Delta t;a,t_T)}{Q(\Delta t)}=\frac{1}{2-e^{-at_T}}, \tag{26}$$

which tends to $\frac{1}{2}$ as the value of at_T goes to infinity. If the value of Δt is not large compared with that of t_T, application of Eq. (26) may lead to substantial error (cf. Fig. 1). From the correct result of Eq. (20) we see that the asymptotic behavior of $\phi(\Delta t;a,t_T)/Q(\Delta t)$ for large values of a ($a\gg\beta,\Omega_R$) is given by the relation

$$\frac{\phi(\Delta t;a,t_T)}{Q(\Delta t)}=\frac{a(\Delta t-t_T)}{a(\Delta t-t_T)(2-e^{-at_T})+1-e^{-at_T}}\times\frac{Q(\Delta t-t_T)}{Q(\Delta t)}. \tag{27}$$

Whereas for $\Delta t\gg t_T$ Eqs. (26) and (27) obviously lead to equal results, Eq. (27) shows that $\phi(\Delta t;a,t_T)/Q(\Delta t)$

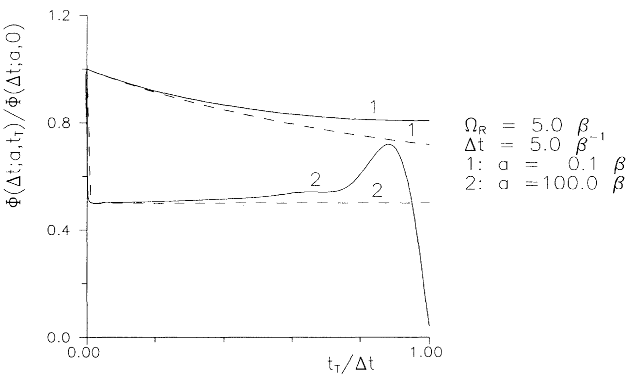

FIG. 1. The relative noise reduction (solid line) as function of $t_T/\Delta t$ for weak and strong amplification, the value of the Rabi frequency being larger than that of the radiative decay rate. The result is compared with that of the Burgess variance theorem (dashed line).

tends to zero as Δt goes to t_T, that is to say, the amplification process tends to prevent the reduction of noise (cf. Fig. 1).

It is worth noting that Eq. (27) reveals the following surprising fact. Let us choose Δt in such a way that $a(\Delta t - t_T) \gg 1$. In this case we have

$$\frac{\phi(\Delta t;a,t_T)}{Q(\Delta t)} = \frac{1}{2-e^{-at_T}} \frac{Q(\Delta t - t_T)}{Q(\Delta t)} . \tag{28}$$

Now suppose the absolute value of the Q function does not monotonically increase with time. In other words, there are regions of time in which the inequality $|Q(\Delta t - t_T)| > |Q(\Delta t)|$ may be expected to be fulfilled. In these regions of time the factor $Q(\Delta t - t_T)/Q(\Delta t)$ compensates partly the factor $(2-e^{-at_T})^{-1}$, and the value of the relative noise reduction tends to increase. This demonstrates that, dependent upon the internal dynamics of the (nonclassical) light and the multiplication and the time scale chosen, increasing gain rate may be assisted by an improvement of the relative noise reduction in the output signal. To observe the effect in the case under study the values of the times t_T, Δt, $\Delta t - t_T$ are desired to be comparable with the value of the duration of the Rabi period Ω_R^{-1}, which for its part should be smaller than the value of the decay time β^{-1} $[\Omega_R^{-1} = (0.2\text{–}0.1)\beta^{-1}]$. Since the value of the transit time is typically $t_T = 10^{-9}\text{–}10^{-10}$ s, relatively strong atomic transitions are favoring. For example, assuming the transition dipole moment is $d = 10^{-29}\text{–}10^{-28}$ Asm, we estimate $\beta = 10^8\text{–}10^9$ s^{-1}, so that in the case when $\Omega_R^{-1} = (0.2\text{–}0.1)\beta^{-1}$ (laser intensity $P_L = 4\text{–}16$ W cm^{-2}), the Rabi period becomes comparable with the transit time. Note that for a typical gain rate $a = 10^{11}$ s^{-1} the conditions $a \gg \Omega_R, \beta$ are fulfilled.

We finally note that in the case of optical receivers, such as avalanche photodiodes with two-carrier multiplication or dead-space-modified avalanche photodiodes, the more complicated multiplication statistics may give rise to additional effects on the output noise. In particular, dead space affects the statistics of the output signal by introducing an additional sub-Poisson effect [21].

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