## Chiral-Odd Parton Distributions and Polarized Drell-Yan Process

R. L. Jaffe and Xiangdong Ji

Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139 (Received 4 April 1991)

We study the chiral-odd spin structure functions of the nucleon,  $h_1(x)$  and  $h_2(x)$ , their physical significance, sum rules, and model estimates. We show that they can be measured in the Drell-Yan process with polarized beams at order  $Q^0$  and  $Q^{-1}$ , respectively.

PACS numbers: 12.40.Aa, 13.85.Qk, 13.88.+e

The nucleon's parton distributions characterize its properties in hard scattering processes. Measurement of these distributions, which has been undertaken for over twenty years, provides us with considerable insight into the quark-gluon substructure of the nucleon. The spinindependent quark and gluon distributions have been measured in a variety of experiments and with high accuracy. The longitudinal quark-spin distribution  $g_1(x)$  has been measured at both SLAC and CERN, and the data have prompted much theoretical work on the spin structure of the nucleon. The aim of this paper is to characterize a new class of nucleon structure functions, the chiral-odd spin structure functions  $h_1(x)$  and  $h_2(x)$ , to study their properties, and to describe how they can be measured in lepton pair production with polarized beam and target ("polarized Drell-Yan process").

The structure function  $h_1(x)$  was defined first by Ralston and Soper [1] in their systematic study of the polarized Drell-Yan process, where it is called  $h^T(x)$ . [Kodaira *et al.* and Bukhvostov, Kuraev, and Lipatov [2] also mentioned  $h_1(x)$ .] More recently, Artru and Mekhfi [3] apparently rediscovered  $h_1(x)$ —called  $\Delta_1 q(x)$  by them -calculated its QCD evolution, and mentioned its place in the polarized Drell-Yan process. Collins [4] and Cortes, Pire, and Ralston [4] have recently discussed  $h_1(x)$ . Some of our discussion of  $h_1(x)$  overlaps the work of Refs. [1-4]. We cannot find any mention of the structure function  $h_2(x)$  in the literature, nor can we find any systematic exploration of sum rules. Regge behavior. model dependence, etc., for either  $h_1(x)$  or  $h_2(x)$ . Finally, distinctions between the chiral-even spin-dependent structure functions  $g_1(x)$  and  $g_2(x)$ , on the one hand, and the chiral-odd spin-dependent structure functions  $h_1(x)$  and  $h_2(x)$ , on the other hand, have never been carefully drawn in the literature and there is considerable confusion surrounding (especially transverse) spin effects in hard processes. We hope to clarify the matter in this Letter.

The parton distributions in QCD are defined by the light-cone Fourier transformation of field operator products. The simplest quark-parton distributions are related to the target matrix elements of bilinear quark operators,

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \overline{\psi}(0) \gamma_{\mu} \psi(\lambda n) | PS \rangle = 2[f_1(x)p_{\mu} + M^2 f_4(x)n_{\mu}], \qquad (1)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0) \gamma_{\mu} \gamma_{5} \psi(\lambda n) | PS \rangle = 2 \{ g_{1}(x) p_{\mu}(S \cdot n) + [g_{1}(x) + g_{2}(x)] S_{\perp \mu} + M^{2} g_{3}(x) n \cdot S n_{\mu} \},$$
<sup>(2)</sup>

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0) \psi(\lambda n) | PS \rangle = 2Me(x), \qquad (3)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0) \sigma_{\mu\nu} i \gamma_5 \psi(\lambda n) | PS \rangle = 2 \{ h_1(x) (S_{\perp\mu} p_\nu - S_{\perp\nu} p_\mu) / M + [h_2(x) + h_1(x)/2] M (p_\mu n_\nu - p_\nu n_\mu) (S \cdot n) + h_3(x) M (S_{\perp\mu} n_\nu - S_{\perp\nu} n_\mu) \},$$
(4)

where *n* and *p* are null vectors of mass dimension -1 and 1, respectively  $(n^2 = p^2 = 0, n^+ = p^- = 0, n \cdot p = 1)$ . *P* and *S* are the nucleon momentum and spin vectors  $[P^2 = M^2, P = p + (M^2/2)n, S^2 = -M^2, S_\mu = S \cdot np_\mu + S \cdot pn_\mu + S_{\perp\mu}, P \cdot S = 0]$ . For a target moving in the  $\hat{\mathbf{e}}_z$  direction:  $p = (1/\sqrt{2})(\mathcal{P}, 0, 0, \mathcal{P}), n = (1/\sqrt{2})(1/\mathcal{P}, 0, 0, -1/\mathcal{P})$ . The order in  $Q^{-1}$  (twist) in which these distributions enter hard processes is determined by a simple algorithm: A generic quark distribution in (1)-(4) contributes like  $Q^{1-d}$ , where *d* is the number of powers of *M* or  $S_{\perp}$  in its coefficient.

According to (1)-(4), the complete specification of the light-cone quark correlation function requires nine distribution functions: three twist 2  $(f_1,g_1,h_1)$ , three twist 3  $(e,g_2,h_2)$ , and three twist 4  $(f_4,g_3,h_3)$ . At each twist there is one spin-average distribution  $(f_1,e,f_4)$ , one chiral-even spin distribution  $(g_1,g_2,g_3)$ , and one chiral-odd spin-dependent distribution  $(h_1,h_2,h_3)$ . This simple pattern can be understood in the light-cone formalism of Kogut and Soper [5], in which the quark field  $\psi$  is decomposed into "good" and "bad" components,  $\psi_+$  and

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 $\psi_{-}$  ( $\psi_{\pm} = P_{\pm}\psi$ ,  $P_{\pm} = \frac{1}{2}\gamma^{\mp}\gamma^{\pm}$ ). Correlators of the form  $\psi_{+}^{\dagger}\psi_{+}$  are twist 2,  $\psi_{+}^{\dagger}\psi_{-} \pm \psi_{-}^{\dagger}\psi_{+}$  are twist 3, and  $\psi_{-}^{\dagger}\psi_{-}$  are twist 4. For each light-cone projection, the counting of the helicity amplitudes  $A_{h_{1}H_{1},h_{2}H_{2}}$  for forward quark-nucleon scattering is the same. (*h* and *H* are helicities of quark and nucleon, respectively.) Using parity and time-reversal invariance, we find that there are three independent helicity amplitudes,  $A_{\uparrow\uparrow,\uparrow\uparrow\uparrow}$ ,  $A_{\uparrow\downarrow,\uparrow\downarrow}$ , and  $A_{\uparrow\downarrow,\downarrow\uparrow}$ . For the appropriate light-cone components, ( $f_{1}$ ,  $e, f_{4} \sim A_{\uparrow\uparrow,\uparrow\uparrow\uparrow} + A_{\uparrow\downarrow,\uparrow\downarrow}$ , ( $g_{1},h_{L},g_{3}$ )  $\sim A_{\uparrow\uparrow,\uparrow\uparrow\uparrow} - A_{\uparrow\downarrow,\uparrow\downarrow}$ , and ( $h_{1},g_{T},h_{3}$ )  $\sim A_{\uparrow\downarrow,\uparrow\uparrow\uparrow}$ , where  $g_{T} = g_{1} + g_{2}$ ,  $h_{L} = h_{2} + h_{1}/2$ . Only chiral-even distributions (f's,g's) contribute to deep-inelastic scattering when small quark-mass effects are ignored. The chiral-odd distributions (*e*, *h*'s) can be measured in certain physical processes such as Drell-Yan production of muon pairs.

At the level of leading twist, a complete quark-parton model of the nucleon requires *three* quark distributions:  $f_1$ ,  $g_1$ , and  $h_1$ . Their physical meaning can be made clear by introducing projection operators for the "good" component of the Dirac spinor. The usual choice is  $P_{L,R}$  $\equiv \frac{1}{2} (1 \mp \gamma^5)$ , which project on chirality [1]. After performing a momentum decomposition of the Dirac field and a projection with  $P_{L,R}$  [5], we obtain

$$f_{1}(x) = \frac{1}{x} \langle P | R^{\dagger}(xP) R(xP) + L^{\dagger}(xP) L(xP) | P \rangle,$$
  

$$g_{1}(x) = \frac{1}{x} \langle P \hat{\mathbf{e}}_{z} | R^{\dagger}(xP) R(xP) - L^{\dagger}(xP) L(xP) | P \hat{\mathbf{e}}_{z} \rangle, \qquad (5)$$
  

$$h_{1}(x) = \frac{2}{x} \operatorname{Re} \langle P \hat{\mathbf{e}}_{\perp} | L^{\dagger}(xP) R(xP) | P \hat{\mathbf{e}}_{\perp} \rangle,$$

for x > 0. Here R(xP) [L(xP)] annihilates a right-[left-] handed quark with  $k^+ = xP^+$  and any  $k_{\perp}$ . For x < 0 we find  $f_1(-x) = -\bar{f}_1(x)$ ,  $g_1(-x) = +\bar{g}_1(x)$ , and  $h_1(-x) = -\bar{h}_1(x)$ , where the overbar denotes the replacement of R and L by  $\bar{R}$  and  $\bar{L}$  which annihilate right- and left-handed antiquarks, respectively. It is clear from (5) that  $f_1(x)$  and  $g_1(x)$  count right- and left-handed quarks. The interpretation of  $h_1(x)$  is obscure in a chiral basis. It is revealed by using the projection operators  $Q_{\pm} = \frac{1}{2} (1 \mp \gamma^5 \gamma^{\perp}) = \frac{1}{2} (1 \pm \gamma^5 \mathscr{S}_{\perp})$  instead of  $P_L, P_R$ . In terms of the  $Q_{\pm}$  basis,

$$f_{1}(x) = \frac{1}{x} \langle P | a^{\dagger}(xP) a(xP) + \beta^{\dagger}(xP) \beta(xP) | P \rangle,$$

$$h_{1}(x) = \frac{1}{x} \langle P \hat{\boldsymbol{e}}_{\perp} | a^{\dagger}(xP) a(xP) - \beta^{\dagger}(xP) \beta(xP) | P \hat{\boldsymbol{e}}_{\perp} \rangle,$$
(6)

and  $g_1(x)$  is off diagonal. Here  $\alpha(\beta)$  annihilates a quark with  $Q_{+}\alpha = \alpha$   $(Q_{-}\beta = \beta)$ . Apparently,  $h_{1}(x)$  measures the probability to find a quark in an eigenstate of the transversely projected Pauli-Lubanski operator  $S_{\perp}\gamma_5$  in a nucleon likewise polarized.  $S_{\perp}\gamma_5$  commutes with the free-quark Hamiltonian and is a light-cone good operator. For that reason, a simple parton model can be made to interpret the distribution  $h_1(x)$ . This basis was introduced in hadron-hadron scattering, where it is known as the "transversity" basis [6]. Hence we name  $h_1(x)$  the quark transversity distribution. From (6), it is clear that  $|h_1(x)| \le f_1(x)$ . It should be stressed that  $h_1(x)$  does not measure the quark's transverse spin distribution. The quark-spin operator projected along the nucleon spin,  $\hat{\Sigma}_{\perp} = \gamma_0 \gamma_5 \mathscr{G}_{\perp}$ , does not commute with the free-particle Hamiltonian, i.e., there exists no energy eigenspinor  $U(p_z)$  such that  $\hat{\Sigma}_{\perp}U(p_z) = \lambda_{\perp}U(p_z)$ . In the light-cone formalism, the transverse spin operator is a bad operator and depends on dynamics. Nevertheless, a transversespin average can still be defined in the nucleon state, and, according to (2), it is just  $g_T$ .  $g_T(x)$  is twist 3 and is sensitive to the quark-gluon interactions, a clear sign that no simple parton interpretation can be made for it [7]. The Burkhardt-Cottingham sum rule [8],  $\int g_2(x) dx = 0$ , guarantees that the quark-spin contribution to the nucleon spin be the same for any polarization. [Note: As discussed in Ref. [7],  $\int g_2(x) dx = 0$  may be realized formally by a cancellation of the integral over data (|x| > 0) by a  $\delta$  function at x = 0.]

To derive sum rules for  $h_1(x)$ , we introduce a set of twist-2 operators,

$$O^{\sigma\mu_1,\ldots,\mu_n} = \mathscr{S}_n \overline{\psi} \sigma^{\sigma\mu_1} i \gamma_5 i D^{\mu_2} \cdots i D^{\mu_n} \psi - \text{trace, for } n = 1, 2, \ldots,$$
(7)

and their matrix elements,

$$\langle PS | O^{\sigma\mu_1,\ldots,\mu_n} | PS \rangle \equiv 2a_n \mathscr{S}_n (S^{\sigma} P^{\mu_1} - S^{\mu_1} P^{\sigma}) P^{\mu_2} \cdots P^{\mu_n} / M - \text{trace}, \qquad (8)$$

where  $\mathscr{S}_n$  symmetrizes the indices  $\mu_1, \mu_2, \ldots, \mu_n$ . Then, from (4) and using  $h_1(x) = 0$  for  $|x| \ge 1$ , we derive

$$\int_{-\infty}^{\infty} dx \, x^{n-1} h_1(x) = \int_0^1 dx \, x^{n-1} [h_1(x) - (-1)^{n-1} \bar{h}_1(x)] = a_n \tag{9}$$

if the above integral is convergent. A simple Regge analysis shows that as  $x \to 0$ ,  $h_1(x) \to x^{-\alpha}$ , where  $\alpha$  is the relevant Regge intercept. Like  $g_1$ , the Pomeron does not contribute to  $h_1(x)$ , so we expect the moments of  $h_1(x)$  to be convergent even for n = 1.

Consider the n=1 sum rule in some detail. To clarify the flavor content, we rewrite the relevant matrix element  $a_1$ , the "tensor charge," as  $\delta q$ , where q = u, d, s, etc. To understand its physical meaning, we compare  $\delta q$  with  $\Delta q$ , the quark helicity contribution to the nucleon spin. In the rest frame of the nucleon,  $P_{\mu} = (M, 0, 0, 0)$  and  $S_{\mu} = (0, \mathbf{S})$ , so (8)

becomes  $\langle PS | \bar{q} \Sigma_i q | PS \rangle = 2 \, \delta q \, S_i$ . From (6) and (9), we find

$$\int_0^1 dx [h_1(x) - \bar{h}_1(x)] = \delta q = \int_0^\infty \frac{dk^+}{k^+} \langle P \hat{\mathbf{e}}_\perp | a^\dagger(k) a(k) - \bar{a}^\dagger(k) \bar{a}(k) - \beta^\dagger(k) \beta(k) + \bar{\beta}^\dagger(k) \bar{\beta}(k) | P \hat{\mathbf{e}}_\perp \rangle.$$
(10)

Therefore,  $\delta q$  counts valence quarks (quarks *minus* antiquarks) of opposite transversity. The sea quarks do not contribute because the operator  $O^{\mu\nu} \equiv \overline{\psi} i \sigma^{\mu\nu} \gamma_5 \psi$  is odd under charge conjugation. In contrast, the quark spin operator,  $A^{\mu} = \overline{\psi} \gamma^{\mu} \gamma^5 \psi$ , is even under charge conjugation. The corresponding equations for  $\Delta q$  are  $\langle PS | q^{\dagger} \Sigma_i q | PS \rangle = 2\Delta q S_i$  and

$$\int_{0}^{1} dx [g_{1}(x) + \bar{g}_{1}(x)] = \Delta q = \int_{0}^{\infty} \frac{dk^{+}}{k^{+}} \langle P \hat{\mathbf{e}}_{z} | R^{\dagger}(k) R(k) + \bar{R}^{\dagger}(k) \bar{R}(k) - L^{\dagger}(k) L(k) - \bar{L}^{\dagger}(k) \bar{L}(k) | P \hat{\mathbf{e}}_{z} \rangle.$$
(11)

Obviously,  $\Delta q$  includes the helicity of the sea. The opposite charge conjugation and chiral properties of  $A^{\mu}$  and  $O^{\mu\nu}$  make it clear that  $h_1(x)$ , while spin dependent, does not measure quark spin. (Formally, the spin tensor density  $M_{\text{spin}}^{\mu\nu\lambda}$  is just  $\frac{1}{2} \epsilon^{\mu\nu\lambda\sigma}A_{\sigma}$ .) The name *transversity distribution* for  $h_1(x)$  avoids possible confusion with the *transverse spin distribution*  $g_T$ .

Returning to the sum rule, if one writes  $\Delta q = \Delta q^{v} + \Delta q^{s}$ , the sum of valence and sea contributions, one might speculate that  $\delta q = \Delta q^{v}$ . This is indeed the case in the nonrelativistic quark model, which predicts  $\delta q = \Delta q^{v}$  $= \Delta q^{NR}$ , where  $\Delta u^{NR} = \frac{4}{3}$ ,  $\Delta d^{NR} = -\frac{1}{3}$ , and  $\Delta s^{NR} = 0$ . However, in a relativistic valence-quark model such as the MIT bag model, the speculation is false. Instead,  $\Delta q^{v} = c \int (f^{2} - \frac{1}{3} g^{2}) r^{2} dr$ ,  $\delta q = c \int (f^{2} + \frac{1}{3} g^{2}) r^{2} dr$ , where *c* is a constant and *f* and *g* are upper and lower components of the quark wave function (in a  $\gamma^{0}$ -diagonal basis). Thus, relativity introduces a deviation of  $\Delta q$  and  $\delta q^{v}$  from  $\Delta q^{NR}$  and a splitting between  $\Delta q$  and  $\delta q$ :  $\delta q - \Delta q^{v} = c \frac{2}{3} \int g^{2} r^{2} dr$ .

Of course, these estimates about  $\delta q$  are made in the context of rather naive models of the nucleon. However, it does appear that the tensor charge  $\delta q$ , free from sea quarks, provides a middle ground between the experimental data on  $\Delta q$  and the nonrelativistic-valence-quark-model estimate  $\Delta q^{\rm NR}$ , and a measurement of  $\delta q$  may pro-

vide us with some knowledge about the relevant importance of the relativistic and sea-quark effects on  $\Delta q$ . To illustrate the relativistic effect, we have calculated  $h_1(x)$ for a single quark in a nucleon in the MIT bag model. The result is shown in Fig. 1 where it is compared to  $g_1(x)$ .

High-twist structure functions cannot be interpreted as simple quark distributions. For this reason, they are potentially useful to understand the quark-gluon dynamics of confinement in QCD [7,9]. There are two important advantages in studying twist-3 structure functions compared with twist 4 or beyond. First,  $g_2(x)$  and  $h_2(x)$ contribute to certain spin asymmetries at leading order in 1/Q and, therefore, they can be extracted straightforwardly from data. Second, although twist-3 structure functions couple to complicated quark-gluon correlation functions under QCD evolution, they have no explicit dependence on gluon fields. This second feature renders it possible to calculate them in valence-quark models without dynamical gluons.

As it is defined in (4),  $h_2(x)$  is not completely determined by matrix elements of twist-3 operators. Instead,  $h_2(x)$  receives a contribution from the same operators as  $h_1(x)$ . The same phenomenon was recognized in the case of  $g_2(x)$  by Wandzura and Wilczek [10]. We find [11]

$$h_2(x) = -\frac{h_1(x)}{2} + 2x \left[ \theta(x) \int_x^1 \frac{h_1(y)}{y^2} dy - \theta(-x) \int_{-1}^x \frac{h_1(y)}{y^2} dy \right] + \bar{h}_2(x) , \qquad (12)$$

where  $\bar{h}_2(x)$  is defined as follows. Define

$$\mathcal{O}_{n,l} = -\frac{1}{2} \mathscr{S}_{n-1} \overline{\psi} \sigma^{a\mu_1} i \gamma_5 i D^{\mu_2} \cdots (ig F_a^{\mu_l}) \cdots i D^{\mu_{n-1}} \psi,$$
(13)
$$R_{n,l} = \mathcal{O}_l - \mathcal{O}_{n-l} \quad (l = 2, \dots, n-1),$$

and its matrix elements,

$$\langle PS|R_{n,l}|PS\rangle = 2Mb_{n,l}\mathfrak{S}_{n-1}S^{\mu_1}P^{\mu_2}\cdots P^{\mu_{n-1}}.$$
 (14)

Then, the sum rules for  $\bar{h}_2$  are

$$\int x^{n-1} \bar{h}_2(x) dx = -\sum_{l=2}^{\left[ (n+1)/2 \right]} \left[ 1 - \frac{2l}{n+1} \right] b_{n,l} .$$
 (15)

The appearance of the combinations of  $\mathcal{O}_l$  in  $R_l$  ensures that each sum rule has definite charge conjugation. The first two terms in (12) give a trivial or kinematic contribution to  $h_2(x)$ . The last term,  $\bar{h}_2(x)$ , contains the truly



FIG. 1. Comparison between  $g_1(x)$  (dotted line) and  $h_1(x)$  (dashed line) in the bag model;  $h_2(x)$  (solid line) in the bag and its twist-3 part (dot-dashed line).

new and dynamical information in  $h_2(x)$ . Like  $g_2(x)$ ,  $h_2(x)$  is hard to model without insight into the quarkgluon correlations which determine  $\bar{h}_2(x)$ . In Fig. 1 we show  $h_2(x)$  and  $\bar{h}_2(x)$  in a bag model where boundary effects replace explicit gluon degrees of freedom [7].

Now, we quote the lepton pair-production asymmetries from the collision of polarized nucleons through order  $Q^{-1}$ . For simplicity, we consider only tree processes and ignore all radiative corrections which, though important, do not change the basic structure of the cross section. To study twist 3 it is necessary to include explicit gluon effects, but they do not introduce any new distribution functions into the cross section. Their only role is to render the first-order tree diagram gauge invariant. Some technical details and further discussion can be found in Ref. [11]. The differential cross section in the center of mass of the lepton pair is

$$\frac{d\sigma}{d^4 Q \, d\Omega} = \frac{\alpha^2}{2(2\pi)^4 s Q^2} (\delta_{ij} - \hat{l}_i \hat{l}_j) W_{ij} , \qquad (16)$$

where  $\hat{l}$  is the unit vector in the lepton-momentum direction and  $Q^2$  is the squared mass of the lepton pair. The hadron tensor  $W_{\mu\nu}$  is defined by

$$W_{\mu\nu} = \int e^{i\xi \cdot Q} d^4 \xi \langle P_A S_A P_B S_B | J_\mu(0) J_\nu(\xi) | P_A S_A P_B S_B \rangle,$$
(17)

$$A_{LT} = \frac{2\sin 2\theta \cos\phi}{1 + \cos^2\theta} \frac{M}{Q} \frac{\sum_a e_a^2 [g_1^a(x) y g_T^{\bar{a}}(y) - x h_L^a(x) h_1^{\bar{a}}(y)]}{\sum_a e_a^2 f_1^a(x) f_1^{\bar{a}}(y)} ,$$

which is a new result. The asymmetry  $A_{LT}$  depends on both twist-3 structure functions  $g_T(x)$  and  $h_L(x)$  and is down by a factor of M/Q. In deriving (21), we assumed the factorization is valid at twist-3 level [13].

We would like to thank J. Collins, J. Qiu, and J. Ralston for stimulating and helpful conversations and correspondence. X.J. is grateful to J. Collins for his invitation to the Polarized Collider Workshop at Penn State. This work was supported in part by funds provided by the U.S. DOE under Contract No. DE-AC02-76ER03069.

- [1] J. Ralston and D. E. Soper, Nucl. Phys. B152, 109 (1979).
- [2] J. Kodaira, S. Matsuda, K. Sasaki, and T. Uematsu, Nucl. Phys. B159, 99 (1979); A. P. Bukhvostov, E. A. Kuraev, and L. N. Lipatov, Zh. Eksp. Teor. Fiz. 87, 37 (1984) [Sov. Phys. JETP 60, 22 (1984)].
- [3] X. Artru and M. Mekhfi, Z. Phys. C 45, 669 (1990).
- [4] J. Collins, Penn State report, 1990 (to be published); (private communication); J. L. Cortes, B. Pire, and J. P.

where  $J_{\mu}$  is the electromagnetic current and  $(P_A, S_A)$  and  $(P_B, S_B)$  are momenta and spins of nucleons A and B, respectively. The results can be simply expressed in terms of the spin asymmetries defined by

$$A_{S_A S_B} = \frac{\sigma(S_A, S_B) - \sigma(S_A, -S_B)}{\sigma(S_A, S_B) + \sigma(S_A, -S_B)}, \qquad (18)$$

where  $-S_B$  means the spin of nucleon B is flipped. For longitudinal-longitudinal collisions, the spin asymmetry is well known [12]:

$$A_{LL} = \frac{\sum_{a} e_{a}^{2} g_{1}^{a}(x) g_{1}^{\bar{a}}(y)}{\sum_{a} e_{a}^{2} f_{1}^{a}(x) f_{1}^{\bar{a}}(y)} .$$
(19)

Here x and y are the longitudinal momentum fractions of the annihilating quarks. The sum over a covers all quark and antiquark flavors. We suppress the beam and target labels A and B: By convention the structure function with argument x (y) refers to hadron A (B). For transverse-transverse collision,

$$A_{TT} = \frac{\sin\theta\cos2\phi}{1+\cos^{2}\theta} \frac{\sum_{a} e_{a}^{2}h_{1}^{a}(x)h_{1}^{\bar{a}}(y)}{\sum_{a} e_{a}^{2}f_{1}^{a}(x)f_{1}^{\bar{a}}(y)},$$
 (20)

which was first obtained by Ralston and Soper [1]. And for longitudinal-transverse collision,

Ralston, in Proceedings of the Penn State Polarized Collider Workshop, November 1990 (to be published).

- [5] J. Kogut and D. E. Soper, Phys. Rev. D 1, 2901 (1970).
- [6] G. R. Goldstein and M. J. Moravcsik, Ann. Phys. (N.Y.)
   98, 128 (1976); 142, 219 (1982); 195, 213 (1989).
- [7] R. L. Jaffe and X. Ji, Phys. Rev. D 43, 724 (1991); R. L. Jaffe, Comments Nucl. Part. Phys. 14, 239 (1990).
- [8] H. Burkhardt and W. N. Cottingham, Ann. Phys. (N.Y.) 56, 435 (1970).
- [9] R. L. Jaffe and M. Soldate, Phys. Lett. 105B, 467 (1981);
  A. Vainshteyn and E. Shuryak, Nucl. Phys. B201, 141 (1982);
  R. K. Ellis, W. Furmanski, and R. Petronzio, Nucl. Phys. B212, 29 (1983);
  J. Qiu, Phys. Rev. D 42, 30 (1990).
- [10] W. Wandzura and F. Wilczek, Phys. Lett. B 172, 195 (1977).
- [11] R. L. Jaffe and X. Ji (to be published).
- [12] F. E. Close and D. Sivers, Phys. Rev. Lett. 39, 1116 (1977); J. C. Collins and D. E. Soper, Phys. Rev. D 16, 2219 (1977).
- [13] J. Qiu and G. Sterman, Nucl. Phys. B353, 137 (1991).