# Leptonic $CP$ Violation in $Z^0$ Decays

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An $SU(2)_L \otimes U(1)_Y$ model containing exotic charged leptons can allow sizable leptonic flavor violations (LFV) and hence rare $Z$ decays into leptons of different flavors. The rates as well as $CP$-violating asymmetries are constrained using available limits on LFV and data from the CERN $e^+e^-$ collider LEP on $Z \to \bar{l}_i l_i$, $l_i = e, \mu, \tau$. Sizable $CP$-violating rate asymmetries are theoretically possible in the model and are allowed by the former data. In contrast, the measured leptonic $Z$ widths severely constrain these asymmetries. As a result, an experimental observation of the $CP$ asymmetries in the model seems unlikely with less than $10^9$ $Z$'s.

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Observing $CP$ violation in systems other than the $K^0$-$\bar{K}^0$ mesons is of vital importance for its theoretical understanding. This has motivated many studies of $CP$ violation in the $B^0$-$\bar{B}^0$ system [1] and at higher energies in the decays of $W$ and $Z$ bosons [2,3]. It has been suggested [2,3] that the rare $Z$ decays into fermion ($f_i$) and antifermion ($\bar{f}_j$) pairs with different ($i \neq j$) flavors may be ideally suited for studying $CP$ violations since such violations can lead to asymmetries in rates involving $\bar{f}_i f_j$ and $\bar{f}_j f_i$ which can be measured relatively easily. The existing studies [2,3] have concentrated on decays with quarks in the final states since decays with $f$ = leptons do not occur in the standard model (SM) or its generalizations which preserve lepton number of each flavor separately. It is, however, important to study other generalizations of the SM which contain leptonic flavor violations (LFV) since detecting rare $Z$ decays, $Z \to \bar{l}_i l_j$ ($i \neq j$), as well as any $CP$-violating asymmetry in them is far easier compared to the corresponding hadronic decays. Thus, unless the asymmetries are much larger in the latter case as compared to the former, the leptonic decays would be better candidates for the study of $CP$ violation.

Leptonic flavor violations can occur in extensions of the standard model (i) if the ordinary neutrinos have (Majorana) masses or if they mix with heavier exotic neutrinos, or (ii) if the ordinary charged leptons mix with additional leptons that transform differently compared to the former under the gauge group. The Glashow-Iliopoulos-Maiani mechanism [4] does not occur in case (ii) and flavor nondiagonal couplings of $Z$ to the leptons arise. The LFV are normally suppressed by (small) neutrino masses in case (i) [5]. In contrast, LFV can be generated in case (ii) even in the absence of neutrino masses or of additional neutrinos. The LFV could arise [6] in such models at rates limited only by experimental constraints. A recent experiment [7] has put direct limits of $\sim 10^{-4}$–$10^{-5}$ on the branching ratios for $Z \to \bar{l}_i l_j$ ($i \neq j$). If branching ratios are in this range, it may be possible to detect relatively large $CP$ violation in these modes with increased luminosity at the CERN $e^+e^-$ collider LEP. It therefore becomes interesting to discuss $CP$ violation in models of type (ii) which allow large LFV.

We shall consider here an $SU(2)_L \otimes U(1)_Y$ model containing $SU(2)_L$-singlet vectorlike charge $-1$ leptons $E_\alpha$ ($\alpha = 4, 5, \ldots$) in addition to the normal fermions. The most general couplings of leptons to $Z$ can be parametrized as

$$\mathcal{L}_Z = -\frac{g}{\cos\theta_W} (\bar{l}_{aL} \gamma_\mu l_{bL} F^L_{ab} + L \leftrightarrow R) Z^\mu , \qquad (1)$$

where $l_a \equiv (e_i, E_\alpha)^T$ denote the mass eigenstates of the charged leptons. $F_{L,R}$ are related to the unitary matrices $U_{L,R}$ which diagonalize the charged-lepton mass matrix:

$$F^L_{ab} = a_L \delta_{ab} + d_L U^L_{a\alpha} U^{L*}_{b\alpha} , \qquad (2)$$

and similarly for $F^R_{ab}$. The couplings $a, d$ depend upon the transformation properties of fermions under the gauge group. In the present case

$$a_L = -\tfrac{1}{2} + \sin^2\theta_W, \quad d_L = \tfrac{1}{2} ,$$
$$a_R = \sin^2\theta_W, \quad d_R = 0 .$$

The absence of flavor-changing couplings in $F^R_{ab}$ is a consequence of the fact that both $e_{iR}$ and $E_{\alpha R}$ transform identically under the gauge group.

Equations (1) and (2) give rise to flavor-violating decays $Z \to \bar{l}_i l_j$ ($i \neq j$) at the tree level but the $CP$-violating asymmetries in these decays do not arise at this level. The latter can be generated at the one-loop level if the relevant amplitudes develop an absorptive part. The diagram of Fig. 1(b) gives rise to such an absorptive part and its interference with Fig. 1(a) generates the asymmetry

$$A_{l_i l_j} = \frac{\Gamma(Z \to \bar{l}_j l_i) - \Gamma(Z \to \bar{l}_i l_j)}{\Gamma(Z \to \bar{l}_j l_i) + \Gamma(Z \to \bar{l}_i l_j)} . \qquad (3)$$

Suppressing the Lorentz and the flavor indices, one could write the contribution of Fig. 1 to the $Z \to l_i^- l_j^+$ amplitude as

$$M \approx \xi + \alpha \eta_m F_m ,$$

 

where $\xi$ and $\eta_m$ contain the phases (to be called "weak phases") generated through the couplings $F_{L,R}$. $F_m$ are form factors arising from Fig. 1(b), and $\alpha = e^2/4\pi$. Up to $O(\alpha)$,

$$A \sim -2\alpha\,\mathrm{Im}(\xi^* \eta_m)\,\mathrm{Im}(F_m)/|\xi|^2 .$$

This is suppressed by $\alpha$ in comparison to the analogous result [2,3] for $A_{q_i q_j}$ in the SM where both the rates as well as the asymmetries arise at the one-loop level. However, in the latter case, the $A_{q_i q_j}$ depend upon the relative phases between the absorptive parts involving different quark intermediate states. These phases are nearly the same due to the near degeneracy (at the $m_Z$ scale) of various quarks contributing to the absorptive part. Since this is not the case with Eq. (1), despite the suppression by $\alpha$, $A_{l_i l_j}$ could be larger than $A_{q_i q_j}$ allowed in the SM. As we shall see, $A_{l_i l_j}$ turns out to be comparable to $A_{q_i q_j}$ in models with four generations [2].

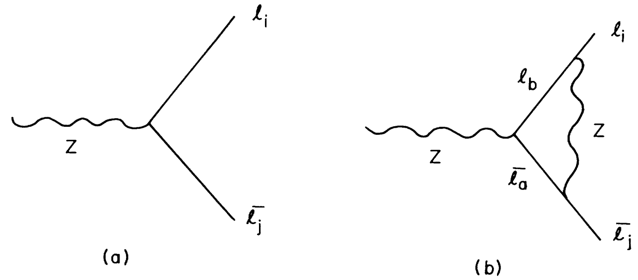


FIG. 1. Diagrams contributing to $Z \to l_i \bar{l}_j$ (a) at tree level and (b) at one-loop level.

Evaluation of the diagrams in Fig. 1 in the unitary gauge directly leads to

$$A_{l_i l_j} = \frac{\alpha}{\sin^2\theta_W \cos^2\theta_W |F_{ij}^L|^2} \sum_{a,b} \{\mathrm{Im}(F_{ij}^{L*} F_{ib}^L F_{ba}^L F_{aj}^L) f(b,a) + \mathrm{Im}(F_{ij}^{L*} F_{ib}^L F_{ba}^R F_{aj}^L) g(b,a)\} . \quad (4)$$

The functions $f(a,b)$ and $g(a,b)$ arise from the absorptive parts and vanish for $m_a + m_b > m_Z$. In the converse case,

$$f(a,b) = [5 - (y_a + y_b)]\epsilon(a,b) + [-4 + 2(y_a + y_b) - \tfrac{3}{2} y_a y_b] \ln \frac{\Delta(a,b)+1}{\Delta(a,b)-1} , \quad (5)$$

$$g(a,b) = \tfrac{1}{2}(y_a y_b)^{1/2} \left[ (y_a + y_b - y_a y_b - 3) \ln \frac{\Delta(a,b)+1}{\Delta(a,b)-1} + (1 - y_a - y_b)\epsilon(a,b) \right] , \quad (6)$$

where

$$\epsilon(a,b) = \tfrac{1}{2}[1 - 2(y_a + y_b) + (y_a + y_b)^2]^{1/2} ,$$

$$\Delta(a,b) = [2\epsilon(a,b)]^{-1}(3 - y_a - y_b) ,$$

and

$$y_{a,b} = m_{a,b}^2 / m_Z^2 .$$

The magnitude of $A$ is constrained by limits on $F_{ab}^L$ following from experimental results on LFV. The recent direct limits on $Z \to \bar{l}_i l_j$ by the OPAL group [7] at LEP imply

$$|F_{\tau\mu}^L| \le 2.62 \times 10^{-2}, \quad |F_{\tau e}^L| \le 1.17 \times 10^{-2} . \quad (7)$$

These limits are comparable to limits coming from bounds on $\tau \to \mu\mu\mu$ and $\tau \to eee$ decays [6]. Hence these bounds allow $Z \to \tau\mu, \tau e$ to occur at rates on the threshold of observability at LEP. In contrast, the limit on $F_{e\mu}^L$ as obtained from $\mu \to eee$ being $\le 1.18 \times 10^{-6}$ does not allow observable $Z \to \mu e$ rates. $A_{e\mu}$ is therefore of no practical significance and we discuss only $A_{\tau e}$ and $A_{\tau\mu}$.

Kinematically, many physical states can contribute as intermediate states in the absorptive part of Fig. 1(b). But, not all contributions give rise to relative weak phase between Figs. 1(a) and 1(b). Using the couplings in Eqs. (1) and (2), it follows that the contributions of the ordinary leptons $(e,\mu,\tau)$ to $A_{\tau e}$ and $A_{\tau\mu}$ vanish in the limit of vanishing $F_{e\mu}^L$. This also applies to the remaining contributions involving exotic leptons as the intermediate states in Fig. 1(b) as long as there exists only one species of such leptons. Hence, one must introduce at least two species. Not all of these need to contribute to the absorptive part and have masses $< m_Z$. Their presence is needed to generate relative weak phases in Fig. 1 in contributions involving exotic leptons. We consider the case of two leptons $E_a$, one $(a=4)$ lying below and one $(a=5)$ above the $Z$ mass. In case $E_4$ has mass $m_4 < m_Z/2$, nonzero contributions to Eq. (4) are obtained when the intermediate state has either a pair of $E_4$'s or one $E_4$ and one normal charged lepton. In case $m_Z/2 < m_4 < m_Z$, only the latter intermediate states contribute. If $m_4 < m_Z/2$, $E_4$'s should be pair produced at LEP but such events have not been seen. An unstable charged lepton with mass less than $m_Z/2$ is already ruled out by LEP, whereas a stable fourth-generation left-handed doublet is ruled out for mass in the range 18.5–42.8 GeV [8]. Similar limits should also hold for a singlet $E_4$. We present results for $m_4 < m_Z/2$ as well as for $m_Z/2 < m_4 < m_Z$. Performing the required sum in Eq. (4), we obtain, neglecting $F_{e\mu}^L$,

$$A_{\tau l} \approx \left[ \frac{\alpha}{\sin^2\theta_W \cos^2\theta_W} \right] \left[ \frac{\mathrm{Im}(F_{l\tau}^{L*} F_{l4}^L F_{4\tau}^L)}{|F_{l\tau}^L|^2} \right] G(y) . \quad (8)$$

The dependence of the function $G(y)$ on $y = m_4^2/m_Z^2$ is calculated in the approximation $|U_{45}^L| \ll |U_{44}^L| \approx 1$ and is shown in Fig. 2. In terms of the elements $U^L_{ab}$ [Eqs. (1) and (2)], the magnitude of the factor in the second brackets in Eq. (8) can be written, under the same approximation, as

$$\left|\frac{d_L a_{4l} a_{5l}\sin\delta}{a_{4l}^2+a_{5l}^2+2a_{4l}a_{5l}\cos\delta}\right|,$$

where

$$a_{al}\equiv|U^L_{\tau a}U^L_{la}|,\ \delta=\arg(U^L_{\tau 4}U^{L*}_{l4}U^{L*}_{\tau 5}U^L_{l5}).$$

Hence, for $a_{4l}\approx a_{5l}$,

$$|A_{\tau l}|\approx\frac{1}{4}\frac{\alpha}{\sin^2\theta_W\cos^2\theta_W}\left|\tan\frac{\delta}{2}\right|G(y)$$

could be large if $\delta$ is near $\pi$. If, on the other hand, $a_{5l}/a_{4l}\ll 1$, following the hierarchy seen in the quark-sector mixings, the asymmetry is given by

$$|A_{\tau l}|\approx\frac{1}{2}\frac{\alpha}{\sin^2\theta_W\cos^2\theta_W}\frac{a_{5l}}{a_{4l}}\sin(\delta)G(y),$$

which is much smaller. It is worth stressing that the direct experimental constraints of Eq. (7) limit the combination

$$|F^L_{\tau l}|^2=(a_{4l}^2+a_{5l}^2+2a_{4l}a_{5l}\cos\delta)d_L^2$$

rather than $a_{4l}$ or $a_{5l}$ individually. Hence both the rates (determined by $|F^L_{\tau l}|^2$) as well as $A_{\tau l}$ are independently allowed to be large; e.g., for $a_{4l}=a_{5l}=\frac{1}{2}$ and $\delta\gtrsim 175°$ one gets $|F^L_{\tau l}|\gtrsim 2.2\times10^{-2}$ and $A_{\tau l}\gtrsim 2.1\times10^{-2}G(y)/G(0)$. $|F^L_{\tau l}|\approx 2.2\times10^{-2}$ corresponds to a branching ratio $B\simeq 2.5\times10^{-4}$. The asymmetry in this situation can be observed with $(A^2B)^{-1}\sim 9\times10^6$ $Z$ events. It should be emphasized that these values depend sensitively on the equality of $a_{4l}$ and $a_{5l}$. When these differ from each other by 10%, e.g., when $a_{4l}=a_{5l}/0.9=\frac{1}{2}$ and $\delta=175°$, $|F^L_{\tau l}|$ increases by a factor of about 1.5, $A_{\tau l}$ decreases by a factor of about 2.5, and the required number of $Z$ events increases by a factor of about 4.

It is possible to derive more stringent limits by using data on lepton-number-conserving processes, more specifically $Z\to\bar{l}_i l_i$ widths. These data can be used [9] to constrain diagonal $F^L$'s and hence $|U^L_{la}|^2$ and $|U^L_{\tau a}|^2$ through Eq. (2). Using the Schwarz inequality these give limits on $|U^L_{la}U^{L*}_{\tau a}|$ and hence the off-diagonal $F^L_{l\tau}$. These constraints in turn can be converted into limits on the product $A_{\tau l}B(Z\to\tau\bar{l}+\bar{\tau}l)$. Using the recent data [10] of DELPHI at LEP on $\Gamma_{ll}$, we obtain

$$|U^L_{la}|^2\lesssim(4.3,2.3,4.4)\times10^{-2}, \tag{9}$$

respectively, for $l=e$, $\mu$, and $\tau$. These numbers should be regarded as illustrative since there is considerable difference in results of $Z\to\bar{l}_i l_i$ width as reported by the various experimental groups at LEP [10,11]. Rather than averaging over these results, we have chosen one specific set to derive the following limit which can be strengthened with improved data.

Assuming $|U^L_{45}|\ll|U^L_{44}|\approx 1$ as before, we obtain from Eqs. (8) and (9)

$$A_{\tau l}B(Z\to\tau\bar{l}+\bar{\tau}l)\lesssim\left(\frac{\alpha}{\sin^2\theta_W\cos^2\theta_W}\right)^2\frac{m_Z}{12\Gamma_Z}G(y)\times|U^L_{\tau\alpha}|^2|U^L_{l\beta}|^2d_L^3$$

$$\lesssim(1.1,0.6)\times10^{-7}\frac{G(y)}{G(0)}, \tag{10}$$

for $l=e$ and $\mu$, respectively.

The values obtained in Eq. (10) are somewhat larger than those obtained by Rius and Valle [5] and are comparable to the corresponding quantities for the most favorable final state of quarks in the SM with four generations [2,3]. The chances of observing such asymmetries at statistically significant levels at LEP in the near future (with $\sim10^7$ $Z$ events) is not very bright; e.g., the number $N_Z$ required to observe $A_{\tau e}$ can be worked out from Eqs. (8) and (9). Assuming $a_{4e}\approx a_{5e}$,

$$N_Z\gtrsim\frac{1}{A^2_{\tau e}B(Z\to\tau\bar{e}+\bar{\tau}e)}\gtrsim 2.4\times10^9\left(\frac{G(0)}{G(y)}\right)^2.$$

The required $N_Z$ in the case of $\tau\mu$ final state is even higher than this.

We need to have at least one exotic lepton lighter than $Z$ in order to generate nonvanishing asymmetries. $Z$ could therefore decay into ordinary-exotic lepton pairs. The branching ratios for such decays are constrained by Eq. (9). Using Eqs. (1) and (2), one finds

$$B(Z\to\bar{l}E+\bar{E}l)\leq(5.5,3.0,5.7)\times10^{-3}$$

for $l=e$, $\mu$, and $\tau$, respectively. These limits are much larger than the corresponding limits [6,7] on decays into ordinary leptons. Such decays may therefore be more readily detectable and it becomes interesting to discuss $CP$ violation in them.

Take, for example, the asymmetry $A_{\mu E}$ for $Z$ decaying into $\mu$ and $E$. Diagrams similar to Fig. 1 contribute to

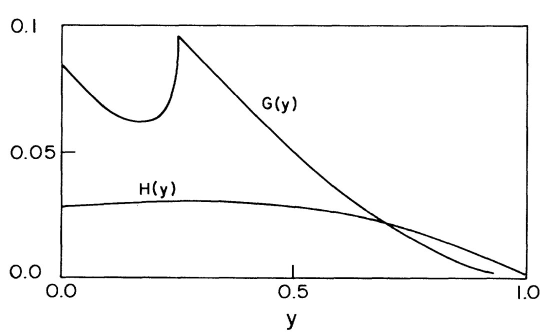


FIG. 2. The functions $G(y)$ and $H(y)$ defined in the text plotted vs $y$.

$A_{\mu E}$ and one finds

$$A_{\mu E}B(Z\to\mu\bar{E}+\bar{\mu}E)\approx\left(\frac{\alpha}{\sin^2\theta_W\cos^2\theta_W}\right)^2\frac{m_Z}{\Gamma_Z}H(y)\times\mathrm{Im}(F_{24}^{L*}F_{23}^{L}F_{34}^{L})$$
$$\lesssim 6.3\times10^{-8}H(y)/H(0)\,.$$

The second relation follows on assuming $|U_{45}^L|\ll|U_{44}^L|\approx1$ and using the constraints in Eq. (9). The function [8] $H(y)$ is displayed in Fig. 2. The relevant product appearing above is similar in magnitude to the earlier case of Eq. (10). Hence, despite the possibility of large branching ratios, the observation of *CP* asymmetry in $Z\to E\mu$ decay seems pessimistic. Similar remarks apply to the other two cases, namely, $Z\to Ee$ and $Z\to E\tau$.

We have concentrated on the model with vectorlike exotic leptons in the singlet representation of $SU(2)_L$. Analogous analyses can be carried out for the cases of exotics which are either vector doublets as in $E_6$ models or mirrorlike as in some grand unified models. The contribution of Fig. 1 to asymmetries in the former case is not expected to be very different from the one considered here. In case of the mirror fermions, the relevant mixings are suppressed [12] severely by the observed limit on the electric dipole moment of the electron. As a result, the asymmetry $A_{e\tau}$ may get severely constrained for mirrorlike exotics.

The rate asymmetries of the type considered here are the simplest among various *CP*-violating signals one could conceive of in the decays of the $Z$. As pointed out here, the exotic charged leptons could generate such asymmetries. The detailed study in a specific case shows that the present limits on LFV still allow asymmetries $A_{\tau e}, A_{\tau\mu}$ of magnitudes comparable to ones obtained for $Z\to\bar{b}s$ in four-generation models. The prospect of observing these asymmetries at LEP does not, however, seem very bright. It seems therefore that the rare $Z$ decays are probably not the ideal candidates for observing *CP* violation at $Z$.


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