Measuring Cabibbo-Kobayashi-Maskawa Parameters with *CP* Asymmetry and Isospin Analysis in $B \rightarrow \pi K$

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Isospin relations are used to eliminate hadronic uncertainties in various *CP* asymmetries in B^0 decays via $b \rightarrow u\bar{u}s$, e.g., $B^0 \rightarrow \pi^0 K_S$. A clean measurement of the angle α of the unitarity triangle is thus made possible. The magnitudes of the tree and the penguin amplitudes can be measured.

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CP asymmetries in B^0 decays into a final *CP* eigenstate are free of hadronic uncertainties if a single Cabibbo-Kobayashi-Maskawa (CKM) combination dominates the decay process. Within the standard model, most processes get contributions from both tree and penguin amplitudes [1]. In $b \rightarrow c\bar{c}s$ processes (e.g., $B^0 \rightarrow \psi K_S$) both amplitudes carry the same CKM phase (to a very good approximation); extracting $\sin(2\beta)$ from this asymmetry is free of hadronic uncertainties [2]. In $b \rightarrow u\bar{u}d$ processes (e.g., $B^0 \rightarrow \pi^+\pi^-$) the two amplitudes carry different CKM phases. It is expected that the contribution from the penguin amplitude is small (a few percent), but it could be larger than the naive expectation if the matrix element for the penguin operator is enhanced; extracting $\sin(2\alpha)$ from this asymmetry may suffer from hadronic uncertainties if this is indeed the case. In $b \rightarrow u\bar{u}s$ processes (e.g., $B^0 \rightarrow \pi^0 K_S$) the two amplitudes carry different CKM phases and are expected to be of the same order of magnitude; it is usually stated that one cannot cleanly extract values of CKM parameters from this asymmetry.

Recently, Gronau and London [3] have shown how to separate the CKM phase of the tree-level $B \rightarrow \pi\pi$ process from any penguin contamination. This is done by means of isospin analysis of various (charged and neutral) B decays into $\pi\pi$. The three relevant amplitudes fulfill a triangle relation: Once their magnitudes are known, the relative phases among them can be calculated. This will allow a determination of α completely free of hadronic uncertainties, independent of how large the penguin amplitude is.

In this work, we study the *CP* asymmetry in $B^0 \rightarrow \pi^0 K_S$. As mentioned above, without isospin analysis this mode does not provide a clean theoretical determination of CKM parameters. Moreover, the analysis of Ref. [3] applies to a case where isospin relates three amplitudes and cannot be applied in a straightforward way to the πK mode, where isospin gives a relation among four amplitudes. However, we show that there is still a way to use isospin relations in order to cleanly measure CKM parameters (specifically, the angle α of the unitarity triangle) from *CP* asymmetries in various $b \rightarrow u\bar{u}s$ modes. Finally, we explain how to measure the magnitude of the tree and penguin amplitudes.

Unfortunately, the branching ratio for $B \rightarrow \pi K$ is expected to be small; unless the penguin contributions are larger than the standard estimates, it will be difficult (though not clearly impossible) even with an asymmetric B factory of luminosity 3×10^{33} cm^{-2}sec^{-1} to make this analysis. To be precise, what is needed is to measure a time-dependent rate for at least one neutral channel (here we assume $B \rightarrow \pi^0 K_S$), as well as integrated rates for all other (neutral and charged) $B \rightarrow \pi K$ channels. It seems that only an asymmetric B factory can make the necessary time-dependent measurements. Hadronic production experiments will have difficulty distinguishing $B_d \rightarrow \pi^0 K_S$ from $B_s \rightarrow \pi^0 K_S$ but may provide useful measurements for charged-B decay rates. The limiting precision will be that of the *CP* asymmetry; the total rates will be known quite accurately.

To better understand the practicality of these measurements, a comparison with the $\pi\pi$ mode (which measures the same CKM phase α) is in order. A detailed analysis of this mode [2] shows that an asymmetric B factory operating at the $\Upsilon(4S)$ will cover a large part of the allowed range for $\sin(2\alpha)$ with the $\pi\pi$ mode. Relative to $\pi\pi$, we expect a factor-of-20 reduction in rate for πK, but this will be partially compensated by good detection efficiency for K_S. (Note that to study ψK_S, good K detection is necessary and so will certainly be part of any detector design.) So it seems reasonable that some use of the *CP* asymmetry in the πK mode can be made, though a smaller range of values of α will be accessible. Moreover, one can easily imagine situations where these measurements will actually be practical and useful: Suppose that the penguin is enhanced by a factor of a few over naive estimates. For $\pi\pi$, this will not change the rate significantly, but isospin analysis will become essential. For πK it would be a blessing: A factor-of-3 enhancement in the matrix element could mean a factor-of-10 enhancement in rate, making this mode competitive with the $\pi\pi$ mode. Another possible situation is that penguins are not enhanced, but that $\alpha \sim \pi/2$. Then the $\pi\pi$ mode has zero asymmetry $[\sin(2\alpha) \sim 0]$, and the bound will be constrained by systematics and statistics. However, the πK mode could still have a very large asymmetry, and the deduction of $\alpha = \pi/2$ will have completely different systematics, and so could be very useful.

B^+ and B^0 decay into final πK states via the quark subprocess $\bar{b} \to \bar{u}u\bar{s}$. The Hamiltonian acting on the B can be written

$$\mathcal{H}|B\rangle = A_0|0,0\rangle + A_1|1,0\rangle . \quad (1)$$

Let us define a decay amplitude A_{ij} by

$$A_{ij} \equiv \langle \pi^i K^j | \mathcal{H} | B \rangle . \quad (2)$$

Then the four amplitudes for B^+ and B^0 decays into final πK states can be written as

$$A_{0+} = U - W, \quad (\tfrac{1}{2})^{1/2} A_{+0} = V + W,$$
$$A_{00} = U + W, \quad (\tfrac{1}{2})^{1/2} A_{-+} = V - W. \quad (3)$$

The amplitudes U, V, and W absorb Clebsch-Gordan coefficients:

$$W \equiv (\tfrac{1}{3})^{1/2} A_0', \quad U \equiv \tfrac{1}{3}(2A_1'' + A_1'), \quad V \equiv \tfrac{1}{3}(A_1'' - A_1') , \quad (4)$$

where A_i' and A_i'' incorporate the change in magnitude as well as the strong-phase-shift corrections to A_i due to hadronization and rescattering effects for final $I = \frac{1}{2}$ and $I = \frac{3}{2}$ states, respectively.

Similarly, $\bar{B}^0$ and B^- decay into final πK states via $b \to u\bar{u}s$. Here, the various amplitudes are given by

$$\bar{A}_{0+} = \bar{U} - \bar{W}, \quad (\tfrac{1}{2})^{1/2} \bar{A}_{+0} = \bar{V} + \bar{W},$$
$$\bar{A}_{00} = \bar{U} + \bar{W}, \quad (\tfrac{1}{2})^{1/2} \bar{A}_{-+} = \bar{V} - \bar{W}, \quad (5)$$

where $\bar{A}_{ij}$ is the amplitude for the CP-conjugated process of A_{ij}, e.g., $\bar{A}_{+0}$ corresponds to $B^- \to \pi^- \bar{K}^0$. The amplitudes $\bar{U}$, $\bar{V}$, and $\bar{W}$ carry weak phases opposite to (but strong phases identical to) those of U, V, and W, respectively.

Measuring the eight time-integrated decay rates gives the various $|A_{ij}|$ and $|\bar{A}_{ij}|$. The time-dependent rate for tagging one B at $t=0$ and observing the other one to decay into $\pi^0 K_S$ at time t is (see Gronau [1])

$$\Gamma(B^0(t) \to \pi^0 K_S) = \tfrac{1}{4} e^{-\Gamma|t|}\{|A_{00}|^2[1-\cos(\Delta m\, t)] + |\bar{A}_{00}|^2[1+\cos(\Delta m\, t)] + 2a_{00}|A_{00}|^2 \sin(\Delta m\, t)\} ,$$
$$\Gamma(\bar{B}^0(t) \to \pi^0 K_S) = \tfrac{1}{4} e^{-\Gamma|t|}\{|A_{00}|^2[1+\cos(\Delta m\, t)] + |\bar{A}_{00}|^2[1-\cos(\Delta m\, t)] - 2a_{00}|A_{00}|^2 \sin(\Delta m\, t)\} , \quad (6)$$

where $-\infty \le t \le \infty$. The CP asymmetry is given by

$$a_{00} = \mathrm{Im}\left(e^{-2i(\phi_M + \phi_K + \phi_T)} \frac{e^{2i\phi_T}\bar{A}_{00}}{A_{00}} \right) . \quad (7)$$

The phases ϕ_M and ϕ_K are the CKM phases in the mixing amplitudes for neutral B and neutral K, respectively $[\phi_M = \arg(V_{td}^* V_{tb}),\ \phi_K = \arg(V_{cs}^* V_{cd})]$. The phase ϕ_T is the CKM phase in the tree diagram $[\phi_T = \arg(V_{ub}^* V_{us})]$.

If $b \to u\bar{u}s$ processes were dominated by tree diagrams (or if ϕ_P, the CKM phase in the penguin diagram, equaled ϕ_T), A_{ij} would be $e^{2i\phi_T}\bar{A}_{ij}$, and the asymmetry in Eq. (7) would reduce to $\sin[2(\phi_M + \phi_K + \phi_T)] = \sin(2\alpha)$. This type of situation holds in $b \to c\bar{c}s$ and, probably, $b \to u\bar{u}d$ processes, which is the reason for the cleanliness in their theoretical interpretation. However, this is not the actual case for $b \to u\bar{u}s$ processes since (i) the penguin diagram depends on $\phi_P = \arg(V_{tb}^* V_{ts})$, so that $\phi_P \ne \phi_T$, and (ii) while the penguin diagram is higher order in couplings [an $(\alpha_S/12\pi)\ln(m_t^2/m_b^2) \sim 0.02$ suppression], the tree diagram is CKM suppressed [a $(\sin\theta_C) \times V_{ub}/V_{cb} \sim 0.02$ suppression]. Thus the two amplitudes are expected to be of the same order of magnitude. In general, then, $e^{2i\phi_T}\bar{A}_{00}/A_{00} \ne 1$ and needs to be determined before α can be calculated from (7). This is done through isospin analysis, as we show below.

For this analysis it is convenient to define new quantities $\tilde{A}_{ij}$:

$$\tilde{A}_{ij} \equiv e^{2i\phi_T}\bar{A}_{ij} . \quad (8)$$

Note that the ratios $\tilde{A}_{ij}/A_{ij}$ are independent of phase conventions. What we need in order to extract α from the asymmetry in (7) is then $\tilde{A}_{00}/A_{00}$. Similarly, we define

$$\tilde{U} \equiv e^{2i\phi_T}\bar{U}, \quad \tilde{V} \equiv e^{2i\phi_T}\bar{V}, \quad \tilde{W} \equiv e^{2i\phi_T}\bar{W} . \quad (9)$$

Let us examine the amplitudes in Eqs. (3) and (5). They fulfill quadrilateral relations:

$$A_{0+} + (\tfrac{1}{2})^{1/2} A_{+0} = A_{00} + (\tfrac{1}{2})^{1/2} A_{-+} ,$$
$$\tilde{A}_{0+} + (\tfrac{1}{2})^{1/2} \tilde{A}_{+0} = \tilde{A}_{00} + (\tfrac{1}{2})^{1/2} \tilde{A}_{-+} . \quad (10)$$

This means that the four A_{ij}'s form a quadrilateral in the complex plane, and similarly the four $\tilde{A}_{ij}$'s. The various decay rates give all eight sides of these two quadrilaterals. However, knowing the lengths of the sides of a quadrilateral does not determine its angles. In $B \to \pi\pi$ decays, there are three (instead of four) amplitudes. The six decay rates give the six sides of two triangles and all angles are consequently determined. It is obvious that the same method cannot be extended in a straightforward way to the present case.

However, there is an additional important piece of information. The penguin operator is purely $I=0$ and, consequently, only tree diagrams contribute to $I=1$ transitions. This gives the following relations among the U and V amplitudes [which are pure $I=1$, as can be seen from their definition in Eq. (4)]:

$$\tilde{U}=U,\quad \tilde{V}=V. \tag{11}$$

Instead of Eq. (5) we can now use

$$\tilde{A}_{0+}=U-\tilde{W},\quad (\tfrac{1}{2})^{1/2}\tilde{A}_{+0}=V+\tilde{W},$$
$$\tilde{A}_{00}=U+\tilde{W},\quad (\tfrac{1}{2})^{1/2}\tilde{A}_{-+}=V-\tilde{W}. \tag{12}$$

This implies the following two relations between the two quadrilaterals of Eq. (10) (see Fig. 1).

(i) One of the two diagonals is common to the two quadrilaterals:

$$A_{00}+(\tfrac{1}{2})^{1/2}A_{-+}=\tilde{A}_{00}+(\tfrac{1}{2})^{1/2}\tilde{A}_{-+}=U+V. \tag{13}$$

(ii) The other (noncommon) diagonals bisect each other:

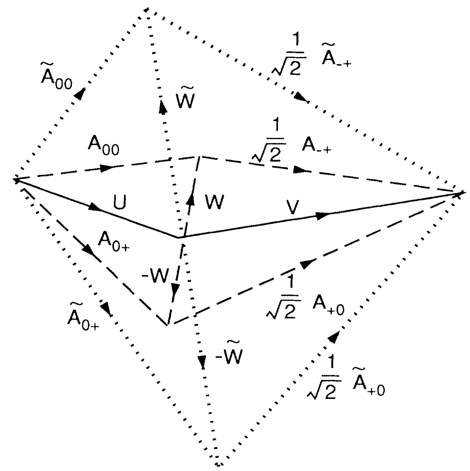

FIG. 1. The two quadrilaterals of Eq. (10). Note that $U+V$ is a common diagonal, while the noncommon diagonals bisect each other.

$$A_{00}+A_{0+}=\tilde{A}_{00}+\tilde{A}_{0+}=2U. \tag{14}$$

The crucial point is that knowing the eight sides of two quadrilaterals that fulfill conditions (i) and (ii) does determine (up to a twofold discrete ambiguity) all the angles. To demonstrate that, we write the equation for the length of the common diagonal, $|U+V|$:

$$h(|A_{00}|,|A_{-+}/\sqrt{2}|,|U+V|)+h(|A_{0+}|,|A_{+0}/\sqrt{2}|,|U+V|)$$
$$=h(|\tilde{A}_{00}|,|\tilde{A}_{-+}/\sqrt{2}|,|U+V|)+h(|\tilde{A}_{0+}|,|\tilde{A}_{+0}/\sqrt{2}|,|U+V|), \tag{15}$$

where $h(a,b,c)/2c$ is (up to sign ambiguity) the height of a triangle of basis c and sides a and b:

$$[h(a,b,c)]^2=2(a^2b^2+b^2c^2+c^2a^2)-a^4-b^4-c^4. \tag{16}$$

Obviously, once $|U+V|$ is known, the angles within each quadrilateral are determined. In particular, $\tilde{A}_{00}/A_{00}$ is determined, allowing us to derive $\alpha=\phi_M+\phi_K+\phi_T$ from the CP asymmetry in $B^0\to\pi^0K_S$ [Eq. (7)].

The above analysis can be applied in the same way to additional hadronic final states for $b\to u\bar{u}s$ processes where the nonstrange meson is an isovector: ρK_S, πK^*, and others. Note that, unlike the case of a very small penguin contribution, the asymmetries in these various modes are not expected to be all equal. The reason is that $\tilde{A}_{00}/A_{00}$ depends on strong-interaction effects and is, therefore, different for different modes. However, a single value of α should, of course, arise from all asymmetries.

We note that there is a discrete twofold ambiguity in the determination of $\tilde{A}_{00}/A_{00}$, which corresponds to reflecting both quadrilaterals along the $U+V$ axis. This will give two solutions, one corresponding to the correct $\tilde{A}_{00}/A_{00}$ and the other to its conjugate. Measuring the asymmetries in various hadronic modes will resolve the ambiguity: Only the true values of $\tilde{A}_{00}/A_{00}$ will give a consistent solution for α. (This ambiguity is not to be confused with another discrete ambiguity which persists even in the case of $\tilde{A}_{00}/A_{00}=1$ [4].)

An isospin analysis carries useful information besides the CP-violating CKM phases. Assuming the standard model, we can actually extract measures of the tree and the penguin contribution to W [5]. To show this, we use

$$W=P_0\exp[i(\delta_{P_{1/2}}+\phi_P)]+T_0\exp[i(\delta_{T_{1/2}}+\phi_T)],$$
$$\bar{W}=P_0\exp[i(\delta_{P_{1/2}}-\phi_P)]+T_0\exp[i(\delta_{T_{1/2}}-\phi_T)], \tag{17}$$

where P_0 and T_0 denote penguin and tree diagrams, respectively, and δ and ϕ denote strong and weak phases, respectively. We obtain the following relations:

$$T_0=\frac{|W-\tilde{W}e^{2i(\phi_P-\phi_T)}|}{\{2[1-\cos 2(\phi_T-\phi_P)]\}^{1/2}},$$
$$P_0=\frac{|W-\tilde{W}|}{\{2[1-\cos 2(\phi_T-\phi_P)]\}^{1/2}}. \tag{18}$$

The quantities W and $\tilde{W}$ can be determined from the isospin analysis. Within the standard model the penguin amplitude depends on the CKM combination $V_{tb}^*V_{ts}$. Consequently, $\phi_T-\phi_P$ is the angle γ of the unitarity triangle, which can be directly measured in CP asymmetries in B_s decays or calculated from CP asymmetries in B_d decays. We conclude that the full isospin analysis allows a determination of P_0 and T_0, and is, therefore, useful not only for our understanding of CP violation but also of hadronic physics. (In the case of $B\to\pi\pi$, comparing the penguin and tree contributions to $I=\frac{1}{2}$ transitions is even simpler. There, $\phi_T-\phi_P=\phi_T+\phi_M=\alpha$, measured by the CP asymmetry in the $\pi\pi$ mode itself.)

In conclusion, CP asymmetries in $B\to\pi^0K_S$ suffer from hadronic uncertainties because they get comparable contributions from penguin and tree diagrams. The simple isospin analysis of Ref. [3] does not help because here the amplitudes fulfill quadrilateral relations. However, because of the isospin properties of the penguin and tree operators, there is still a way to use isospin relations in order to eliminate the hadronic uncertainties and cleanly measure the angle α of the unitarity triangle. The same analysis can be applied to other hadronic modes of the $b\to u\bar{u}s$ process. The list of CP asymmetries in B^0 decays which yield to a clean theoretical interpretation is thus significantly expanded. Experimental application of this analysis will require a high integrated luminosity from an asymmetric B factory.

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[5] We define all processes where final-state interactions change quark flavors to be part of the penguin contribution. We use three-generation CKM unitarity and the approximation that the penguin contributions with u and c quarks in the loop are equal in magnitude to $O(m_c^2/m_b^2)$ apart from their CKM factors. Consequently, terms proportional to $e^{i\phi_P}$ correspond to pure penguin contributions while those proportional to $e^{i\phi_T}$ correspond, to a good approximation, to tree contributions.