CP Violation in $K^+ \to \pi^+ l\bar{l}$ Decays

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We investigate the possibility of observing *CP* violation in the process $K^+ \to \pi^+ l\bar{l}$. We find that, from the point of view of probing the Cabibbo-Kobayashi-Maskawa (CKM) mechanism, the measurement of two-spin correlations of the outgoing leptons is the best recourse. For this purpose, we define a *T*-violating asymmetry $A_{\rm CPV}$, which is the coefficient of the correlation $\hat{\mathbf{p}}_l \cdot \mathbf{s} \times \bar{\mathbf{s}}$ in the differential decay rate. In the standard model we estimate $A_{\rm CPV} \sim 1.9 \times 10^{-2}$ for the muon mode, using the currently allowed CKM parameters and a top-quark mass of 200 GeV/c^2.

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At the present time, the standard model (SM) can describe how various phenomena in the domain of particle physics take place, both qualitatively and (to a large extent) quantitatively. One such phenomenon is *CP* violation (CPV) in kaon decays [1]. In the standard model, CPV can be understood as due to the presence of a phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, the one that describes the mixing among quarks [2]. Although it is "natural" to have CPV in the SM (with at least three generations), a more satisfactory explanation of this phenomenon is desirable. One hindrance in this regard has been that this phenomenon has only been observed in the $K \to \pi\pi$ decay [3,4], and is given in terms of the parameters ε and ε'. Therefore, every avenue where the possibility of observing CPV exists must be explored in order to achieve a better understanding of it. Furthermore, to adequately test a model in a process, the uncertainties in the model's prediction must be under control. Unfortunately, this is not the case for the only process where CPV has been observed. This is essentially because the techniques for the computation in the nonperturbative domain of a model are not yet well developed [5]. In this Letter, we look at the process

$$K^+ \to \pi^+ l\bar{l}\,, \tag{1}$$

and investigate the possibility of observing *CP* (or equivalently *T*) violation in this decay. Here l can be either an electron or a muon. A number of experimental groups have already collected events for the electron mode and data for the muon mode will be available soon [6].

The process $K^+ \to \pi^+ l\bar{l}$ is a rare decay process. In the SM, it is forbidden at the tree level due to the absence of flavor-changing neutral currents. However, this decay occurs at the one-loop level via the electroweak penguin and box diagrams. The one-loop diagrams that contribute to this process are displayed in Fig. 1. Since we are interested in the possibility of observing CPV, which occurs due to the weak interaction, we shall focus on the electroweak contribution to this process and briefly comment on the strong-interaction corrections. We shall see that the *CP*-violating quantities can be calculated fairly reliably using appropriate experimental inputs, when theoretical uncertainties are not under control. Thus the predictions are relatively free from theoretical ambiguities. Hence this process can provide a test of the mechanism of CPV in the SM. To be more specific, we shall calculate the level of *CP*-violating two-spin correlations of the outgoing leptons. One should keep in mind that in practice it will be difficult to measure the polarizations of the leptons, which signal CPV, in the electron decay mode.

We first carry out a general analysis of the process (1) based on Lorentz invariance. We find that the amplitude can be written as [7]

$$\mathcal{M} = \bar{u}(p_l,s)[F_S + iF_P\gamma_5 + F_V p_K{}^\mu \gamma_\mu + F_A p_K{}^\mu \gamma_\mu \gamma_5] v(\bar{p}_l,\bar{s})\,. \tag{2}$$

Here F_S, F_P, F_V, and F_A are scalar, pseudoscalar, vector, and axial-vector form factors, respectively. These form factors are functions of Lorentz-invariant quantities. The p_K, p_π, p_l, and $\bar{p}_l$ are the four-momenta of K^+, π^+, l, and $\bar{l}$, respectively, while the s and $\bar{s}$ are the polarization vec-

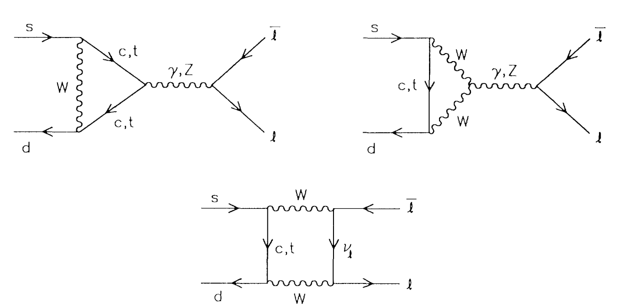

FIG. 1. One-loop diagrams for the short-distance contribution to the $K^+ \to \pi^+ l\bar{l}$ decay.

tors of the l and $\bar{l}$, respectively. Thus to compute a physical quantity for such a process, we have to estimate the contribution of various diagrams to these form factors.

One can easily see using the conventional techniques that the invariant amplitude squared is given by

$$\begin{aligned}
|\mathcal{M}|^2 = {} & |F_S|^2\{\tfrac{1}{2}(L^2-4m_l^2)(1-s\cdot\bar{s})+\bar{p}_l\cdot s\, p_l\cdot\bar{s}\}+|F_P|^2\{\tfrac{1}{2}L^2(1+s\cdot\bar{s})-\bar{p}_l\cdot s\, p_l\cdot\bar{s}\} \\
& +|F_V|^2\{2p_l\cdot p_K\,\bar{p}_l\cdot p_K(1-s\cdot\bar{s})-\tfrac{1}{2}m_K^2L^2(1-s\cdot\bar{s})+2p_K\cdot s\, p_K\cdot\bar{p}_l\, p_l\cdot\bar{s} \\
& \quad +2p_K\cdot\bar{s}\, p_K\cdot p_l\,\bar{p}_l\cdot s-m_K^2\,\bar{p}_l\cdot s\, p_l\cdot\bar{s}-L^2p_K\cdot s\, p_K\cdot\bar{s}\} \\
& +|F_A|^2\{2p_l\cdot p_K\,\bar{p}_l\cdot p_K(1+s\cdot\bar{s})-\tfrac{1}{2}m_K^2(L^2-4m_l^2)(1+s\cdot\bar{s})m_K^2\,\bar{p}_l\cdot s\, p_l\cdot\bar{s} \\
& \quad -2p_K\cdot s\, p_K\cdot\bar{p}_l\, p_l\cdot\bar{s}-2p_K\cdot\bar{s}\, p_K\cdot p_l\,\bar{p}_l\cdot s+(L^2-4m_l^2)p_K\cdot s\, p_K\cdot\bar{s}\} \\
& +2\,\mathrm{Re}(F_SF_P^*)\varepsilon^{\mu\nu\rho\sigma}\bar{p}_{l\mu}\bar{s}_\nu s_\rho p_{l\sigma}-2\,\mathrm{Im}(F_SF_P^*)m_l(\bar{s}\cdot p_l+\bar{p}_l\cdot s) \\
& +2\,\mathrm{Re}(F_SF_V^*)m_l\{p_K\cdot(\bar{p}_l-p_l)(1-s\cdot\bar{s})+\bar{s}\cdot p_K\, s\cdot\bar{p}_l-s\cdot p_K\,\bar{s}\cdot p_l\}+2\,\mathrm{Im}(F_SF_V^*)\varepsilon^{\mu\nu\rho\sigma}p_{K\mu}p_{l\nu}\bar{p}_{l\rho}(s+\bar{s})_\sigma \\
& +2\,\mathrm{Re}(F_SF_A^*)\{\tfrac{1}{2}(L^2-4m_l^2)p_K\cdot(s+\bar{s})-\bar{p}_l\cdot s\, p_l\cdot p_K-\bar{p}_l\cdot p_K\, p_l\cdot\bar{s}\} \\
& +2\,\mathrm{Im}(F_SF_A^*)m_l\varepsilon^{\mu\nu\rho\sigma}p_{K\mu}\bar{s}_\nu s_\rho(\bar{p}_l-p_l)_\sigma+2\,\mathrm{Re}(F_PF_V^*)m_l\varepsilon^{\mu\nu\rho\sigma}p_{K\mu}s_\nu\bar{s}_\rho(\bar{p}_l+p_l)_\sigma \\
& +2\,\mathrm{Im}(F_PF_V^*)\{\tfrac{1}{2}L^2p_K\cdot(\bar{s}-s)-p_K\cdot\bar{p}_l\, p_l\cdot\bar{s}+p_K\cdot p_l\,\bar{p}_l\cdot s\}+2\,\mathrm{Re}(F_PF_A^*)\varepsilon^{\mu\nu\rho\sigma}p_{K\mu}p_{l\nu}\bar{p}_{l\rho}(s-\bar{s})_\sigma \\
& +2\,\mathrm{Im}(F_PF_A^*)m_l\{-\tfrac{1}{2}(m_K^2-m_\pi^2+L^2)(1+s\cdot\bar{s})+\bar{s}\cdot p_l\, s\cdot p_K+s\cdot\bar{p}_l\,\bar{s}\cdot p_K\} \\
& +2\,\mathrm{Re}(F_VF_A^*)m_l\{-2\bar{s}\cdot p_K\, p_l\cdot p_K+2s\cdot p_K\,\bar{p}_l\cdot p_K-m_K^2(s\cdot\bar{p}_l-\bar{s}\cdot p_l)\} \\
& +2\,\mathrm{Im}(F_VF_A^*)\{\bar{p}_l\cdot p_K\,\varepsilon^{\mu\nu\rho\sigma}p_{K\mu}p_{l\nu}\bar{s}_\rho s_\sigma+p_l\cdot p_K\,\varepsilon^{\mu\nu\rho\sigma}p_{K\mu}\bar{p}_{l\nu}\bar{s}_\rho s_\sigma \\
& \quad +s\cdot p_K\,\varepsilon^{\mu\nu\rho\sigma}\bar{p}_{l\mu}p_{l\nu}p_{K\rho}\bar{s}_\sigma+\bar{s}\cdot p_K\,\varepsilon^{\mu\nu\rho\sigma}\bar{p}_{l\mu}p_{l\nu}p_{K\rho}s_\sigma\}\,.
\end{aligned} \tag{3}$$

In the above L is the invariant mass of the dilepton system. Before going on to compute the CP-violating quantity, we shall estimate the width of the decay mode in (1). Comparison of the theoretical calculation with the experimental numbers can help us estimate some form factors also. The penguin and box diagrams do not contribute to the form factor F_S. There is a contribution from a two-photon intermediate state. However, this contribution being of order $G_F\alpha^2$ is expected to be much smaller [8] than the penguin and box contributions to the other form factors which are of order $G_F\alpha$. Therefore we can neglect the terms involving F_S. The penguin and box diagrams, which arise from short-distance physics, have been calculated in Refs. [9–12]. Using their results, we find

$$\begin{aligned}
F_P &= -im_lV_{is}V_{id}^*\frac{G_F}{\sqrt{2}}\frac{e^2}{4\pi}f_+\left(1-\frac{f_-}{f_+}\right)L_I(x_i)\,, \\
F_V &= V_{is}V_{id}^*\frac{G_F}{\sqrt{2}}\frac{e^2}{4\pi}f_+I_L(x_i)\,, \\
F_A &= V_{is}V_{id}^*\frac{G_F}{\sqrt{2}}\frac{e^2}{4\pi}f_+L_I(x_i)\,,
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
I_L(x_i) &= I_L^\gamma(x_i)+I_L^Z(x_i)+I_L^B(x_i)\,, \\
L_I(x_i) &= L_I^Z(x_i)+L_I^B(x_i)\,.
\end{aligned} \tag{5}$$

In the above equations $x_i=m_i^2/M_W^2$, where i can be t or c corresponding to the top or charm quark in the diagrams, respectively. A sum over i is implied in the expressions for the F_P, F_V, and F_A. The V_{ij} are the CKM matrix elements, and f_- and f_+ are the form factors which are related to the ones in K_{l3} decays [13]. The functions I_L and L_I are [9–12]

$$\begin{aligned}
I_L^\gamma(x_i) &= \frac{(25-19x_i)x_i^2}{36\pi(x_i-1)^3} \\
&\quad -\frac{(3x_i^4-30x_i^3+54x_i^2-32x_i+8)\ln x_i}{18\pi(x_i-1)^4}\,, \\
I_L^Z(x_i) &= \frac{4\sin^2\theta_W-1}{\sin^2\theta_W}\frac{x_i}{8\pi} \\
&\quad \times\left[\frac{(x_i-6)(x_i-1)+(3x_i+2)\ln x_i}{(x_i-1)^2}\right], \\
I_L^B(x_i) &= \frac{1}{\sin^2\theta_W}\frac{x_i}{4\pi}\left[\frac{1-x_i+\ln x_i}{(x_i-1)^2}\right], \\
L_I^Z(x_i) &= \frac{I_L^Z(x_i)}{4\sin^2\theta_W-1},\quad L_I^B(x_i)=-I_L^B(x_i)\,.
\end{aligned} \tag{6}$$

After a standard calculation, neglecting the electron mass, the width for the electron decay mode is given by

$$\Gamma_e=(|F_V|^2+|F_A|^2)\frac{m_K^5}{384\pi^3}\left[\frac{1}{8}\left(1-\frac{m_\pi^4}{m_K^4}\right)\left\{\left(1-\frac{m_\pi^2}{m_K^2}\right)^2-6\frac{m_\pi^2}{m_K^2}\right\}+3\frac{m_\pi^4}{m_K^4}\ln\left(\frac{m_K}{m_\pi}\right)\right]. \tag{7}$$

As is well known, the short-distance contribution to the width of $K^+\to\pi^+e^+e^-$ is dominated by the one-photon exchange diagram with the charm quark in the intermediate state [14]. Indeed the effect of a large top-quark mass is negligible due to the CKM matrix suppression. The one-photon exchange diagram only gives a significant contribution to F_V. Therefore, we can neglect F_A in the above expression. Since the one-photon exchange diagram is expected to have large albeit uncertain long-distance corrections, and we are only interested in an estimate of the CP-violating quantity, we shall use the experimental value of Γ_e and Eq. (7) and obtain F_V, denoted by $F_V^{\rm expt}$. Experimentally [13], $B(K^+\to\pi^+e^+e^-)=(2.7\pm0.5)\times10^{-7}$. We find that $|F_V^{\rm expt}|=(9\pm1)\times10^{-15}\ ({\rm MeV}/c^2)^{-2}$. We can use this value to estimate the branching ratio of the muon decay mode. We find [15] $B(K^+\to\pi^+\mu^+\mu^-)=(5\pm1)\times10^{-8}$, which agrees with the value estimated by Beder and Dass [16].

The CP-violating quantities in $K^+\to\pi^+\mu^+\mu^-$ are the transverse polarization of the muon and the two-spin correlation of the outgoing muon-antimuon pair. From Eq. (3), one can see that in the SM, with the form factors given in (4), there is one main CPV term, the two-spin correlation term proportional to ${\rm Im}(F_VF_A^*)$. The contribution of ${\rm Re}(F_PF_V^*)$ to the two-spin correlation is suppressed by m_μ^2/m_K^2 and can be neglected. On the other hand, the single-muon-polarization term is proportional to ${\rm Re}(F_PF_A^*)$, which is small since it depends on ${\rm Im}(f_-/f_+)\simeq0$, as indicated by $K_{\mu3}$ decays [13].

Unlike F_V, short-distance physics takes center stage in F_A. This advantage arises because it contains no large logarithm [11,17] of the form $\ln(M_W^2/\mu^2)$ since the dominant contribution to F_A arises from t-quark exchanges, where μ is the QCD scale. Hence there are no large strong-interaction corrections to F_A. A closer examination of Eqs. (4)–(6) reveals that a measurement of F_A is sensitive to m_t, ρ, and η, where ρ and η are the yet unknown CKM parameters in the Wolfenstein parameterization [18].

To estimate the strength of the CP-violating two-spin correlations, we define a quantity $A_{\rm CPV}$. This quantity is the relative strength of the coefficient of the $\hat{\mathbf{p}}_l\cdot\mathbf{s}\times\bar{\mathbf{s}}$ in the distribution $d\Gamma/d^3p_l\,d^3\bar{p}_l$ in the rest frame of the kaon. We find

$$A_{\rm CPV}=\left|\frac{2\bar{E}_l\,{\rm Im}(F_VF_A^*)|\mathbf{p}_l|}{(|F_V|^2+|F_A|^2)(E_l\bar{E}_l+\mathbf{p}_l\cdot\bar{\mathbf{p}}_l)-m_l^2(|F_V|^2-|F_A|^2)}\right|. \tag{8}$$

We can similarly calculate the relative strength of the coefficient of the $\hat{\bar{\mathbf{p}}}_l\cdot\mathbf{s}\times\bar{\mathbf{s}}$ term in the distribution. As we argued before, the denominator of this quantity will be dominated by F_V. Therefore, neglecting F_A in the denominator, we obtain

$$A_{\rm CPV}\simeq\left|\frac{{\rm Im}(F_A)}{F_V^{\rm expt}}\right|\left(\frac{2\bar{E}_l|\mathbf{p}_l|}{E_l\bar{E}_l+\mathbf{p}_l\cdot\bar{\mathbf{p}}_l-m_l^2}\right). \tag{9}$$

The correction to this will be of $O(|F_A|^2/|F_V|^2)$, which is several orders of magnitude smaller than the above leading term. Because of four-momentum conservation, this asymmetry depends on two variables which can be taken to be E_l and $\bar{E}_l$.

Using the Wolfenstein parametrization [18] of the CKM matrix, and the values $\lambda=0.22$, $|F_V^{\rm expt}|=(9\pm1)\times10^{-15}\ ({\rm MeV}/c^2)^{-2}$, and $f_+=1$, we find to the leading order

$$R\equiv\left|\frac{{\rm Im}(F_A)}{F_V^{\rm expt}}\right|=(3.4\pm0.3)\times10^{-3}A^2\eta L_t(x_t)\,, \tag{10}$$

where $A=1.0\pm0.1$.

Thus we see that $A_{\rm CPV}$ is directly proportional to η. We emphasize that this quantity is relatively free of hadronic uncertainties provided one can measure precisely the momentum dependence in F_V. In Fig. 2 we show the dependence of R on the top-quark mass taking into account 1 standard deviation on the parameters of the

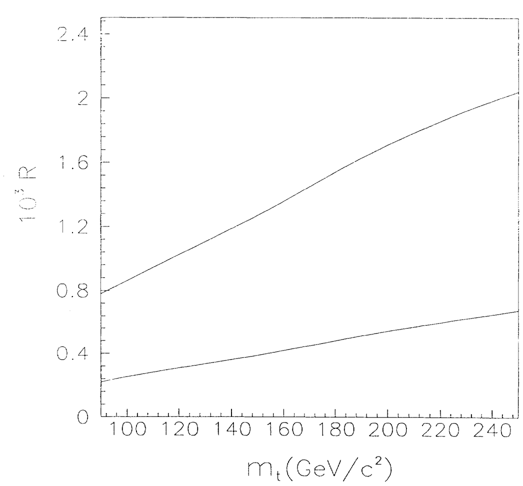

FIG. 2. The allowed region of $R\equiv|{\rm Im}(F_A)/F_V^{\rm expt}|$ as a function of m_t.

CKM matrix [19] and on $F_V^{\rm expt}$. If we choose $m_t = 200$ GeV/c^2, we find the maximum value for the parameter $R \simeq 1.7\times10^{-3}$, leading to an asymmetry for the muon decay mode of $A_{\rm CPV} \simeq 1.9\times10^{-2}$, when $E_\mu = \bar{E}_\mu = m_K/3$. The fixing of the CKM parameters is in itself an important issue. Once we know m_t, the above asymmetry can then be used to determine the parameter η. Note that there are also two-spin asymmetries corresponding to the last two terms in Eq. (3) [20]. These asymmetries are expected to be of the same order of magnitude as the above since they are also proportional to $\mathrm{Im}(F_A)$. The final-state electromagnetic interactions between the pion and the muons do not contribute significantly to the quantity $A_{\rm CPV}$ above.

The measurement of this asymmetry requires the measurement of the polarizations of two final-state leptons. This makes the measurement of this asymmetry harder. However, this may not be completely out of the reach of future experiments. Since the measurement of the polarization of a fast-moving particle is relatively harder, such an experiment is not currently feasible for the electron mode. On the other hand, such an experiment may be feasible for the muon mode at BNL or a future kaon factory. For example, at the proposed KAON, one expects to have about 10^7 K^+ per second. This means that one can expect about 5×10^6 $K^+ \to \pi^+\mu^+\mu^-$ events in a typical one-year run. We find that in the phase space around $E_\mu(\bar{E}_\mu) = m_K/(3\pm2$ MeV), which corresponds to 0.2% of the $K^+\to\pi^+\mu^+\mu^-$ width, integrated $A_{\rm CPV}\simeq 2\times10^{-1}$. This translates into about 2000 CPV events for such a run. However, when one will fold in the experimental efficiency factor, this number will get smaller. This latter factor will depend on specific experimental arrangements. The main experimental issue will be the ability to measure the μ^- polarization, which has been done in the past [21,22].

To conclude, we suggest that an asymmetry in the process $K^+\to\pi^+\mu^+\mu^-$ due to *CP* violation is large enough so that it can be an interesting measurement. Such measurements are especially useful because the predictions are relatively clean (assuming the momentum dependence in F_V can be measured). In the above we have considered only the possible SM contributions to $A_{\rm CPV}$. The implications of alternative models on the size of this asymmetry, and the other signals of CPV in this decay mode, will be explored more completely in a future publication.

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