Interaction of Spirals in Oscillatory Media

Recent studies of topological defects in oscillatory media (spirals) within the framework of the complex Ginzburg-Landau equation (CGLE) [1,2] claim that the asymptotically dominant interaction term is a long-range radial interaction falling off like r_{12}^{-1} (r_{12} denotes the separation of spirals). This was obtained by assuming a direct superposition of isolated spiral solutions far away from the core. The ansatz must fail in general, because the waves emitted from the cores collide and the resulting sources or sinks (shocks) do not vanish for $r_{12} \to \infty$.

We show that the shocks lead to an exponentially decaying interaction by starting from the nonlinear phase equation in the long-wave limit valid far from the cores,

$$\partial_t \kappa = \omega - c + (b-c)(\nabla\kappa)^2 (1+bc)\nabla^2\kappa . \tag{1}$$

Here $\nabla\kappa$ is the slowly varying local wave number, $\omega = -c + (b-c)Q^2$ is the frequency of the asymptotic plane wave with wave number Q, and b,c are the linear dispersion and the nonlinear frequency shift [3]. The Hopf-Cole transform $\kappa = -\ln(W)/\alpha$, with $\alpha = (b-c)/(1+bc)$, leads to the linear equation

$$\partial_t W = (1+bc)(\nabla^2 W - \beta^2 W) , \tag{2}$$

with $\beta^2 = (b-c)(\omega-c)/(1+bc)^2$. The isolated one-arm spiral solutions are of the form (r,φ are polar coordinates) $W = e^{\pm\alpha\varphi - \beta r}(\beta r)^{-1/2}[1 + O(r^{-1})]$, $\beta r \gg 1$, with $\beta = |\alpha Q|$. Since (2) is linear and spirals have an exponential decay in this representation, well separated multispiral solutions are given by a superposition of isolated spirals. This superposition *after* the nonlinear Hopf-Cole transformation includes the shocks. Clearly the residual interaction reflects the exponential decay of the isolated-spiral asymptotic behavior. Using the potentiality of (2) one finds for a spiral pair that the (small) radial velocity resulting from the interaction is independent of the topological charges, in contrast to the results of Refs. [1] and [2], and has the form $v_r = \dot{r}_{12} \cong C \times \exp(-\beta r_{12})/(\beta r_{12})^{1/2}$, $\beta r_{12} \gg 1$. The constant C depends on the parameters b and c and can be positive (repulsion) or negative (attraction). The tangential velocity has the same form but the sign depends on the topological charges. Thus oppositely charged spirals drift perpendicularly to the line connecting the cores and like-charged spirals rotate around their symmetry center.

Very detailed numerical simulations are in quantitative agreement with the analytical results (see inset of Fig. 1). For the case shown ($b=0$ and $c=1$) one has an asymptotic attraction which changes into repulsion for $b=0$ and $c=1.5$. Also shown are the radial and drift velocities for a large range of r_{12}. One sees the existence of a drifting stable bound state with $v_r = 0$ and $dv_r/dr_{12} < 0$. For like-charged spirals we observed that the drift is replaced by rotation. The equilibrium distance is close to the wavelength $2\pi/Q$.

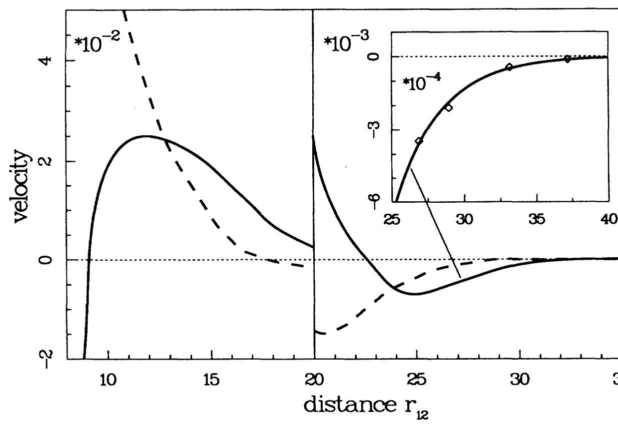

FIG. 1. Radial (solid) and drift (dashed) velocities of oppositely charged pair for $b=0$ and $c=1$. Note the blown-up scale for $r_{12} > 25$. Inset: The asymptotic behavior of the radial velocity. Diamonds give numerical results. Solid line represents theoretical dependence with $C=-3.2$ and $Q=0.3$ (value for free spiral).

The numerical results shown in Ref. [4] represent only the initial (transient) stage of the spiral dynamics where the motion is much faster than under steady-state conditions. Our numerical results for the equilibrium distance are in conflict with the expression given in Ref. [2]. Presumably our results hold quite generally for oscillating and even excitable media because the far field can be described universally by Eq. (1) [3].

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