## Thermal Conductivity of YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7- $\delta$ </sub> in a Magnetic Field: Can $\kappa(H)$ Probe the Vortex State?

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(Received 16 September 1991)

We investigate the behavior of the thermal conductivity  $\kappa(H)$  of both twinned and untwinned single crystals of YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7- $\delta$ </sub> in a magnetic field. Remnant field data are examined in the framework of the Bean critical state model and thermally activated flux creep. In-field data are shown to depart from conventional superconductor models. The  $\kappa(H)$  profile is fitted by a phenomenological form and is suggested to originate from Bloch wave travel of phonons through a vortex array which becomes more regular as the field increases.

PACS numbers: 74.30.Ek, 74.60.Ge

The static and dynamic properties of magnetic vortices in high- $T_c$  superconductors make up a field of study that has been the focus of considerable experimental and theoretical effort in recent years [1]. As of today, there continues to be substantial debate over the nature of the magnetic phase diagram in these systems. We propose that the thermal conductivity may be an interesting new probe of the magnetic state of high- $T_c$  superconductors. In this paper we present remnant field data and time relaxation data which support this claim. We also analyze the thermal conductivity in an increasing field for both twinned and untwinned single crystals (the untwinned data are presented here for the first time) and introduce a phenomenological model of the lattice thermal resistance due to the introduction of vortices. This model suggests that phonons travel through the vortex lattice as Bloch waves, and, as such, the lattice thermal conductivity may probe the degree of regularity of the vortex lattice.

Details of the sample preparation and measurement techniques have been presented elsewhere [2,3]. The data for  $\kappa$  as a function of increasing field for the twinned sample are shown in Fig. 1(a). The heat flow is along the *a-b* plane and the field is directed along the *c* axis. As can be seen from the figure,  $\kappa$  decreases monotonically as *H* is increased. In high- $T_c$  superconductors, the dominant heat carriers are phonons. The magnetic vortices constitute additional phonon scatterers and hence their introduction adds to the lattice thermal resistivity.

Also illustrated in Fig. 1(a) are data (top curve) taken 2 min after the magnetic field has been ramped from an initial value [shown on the x axis in Fig. 1(a)] down to zero. The thermal conductivity relaxes to a metastable value,  $\kappa(B_{\rm rem})$ , which is dependent upon the internal trapped (remnant) field  $B_{\rm rem}$  and hence the number of trapped vortices. The field profile of the metastable value of  $\kappa(B_{\rm rem})$  can be explained in the context of the Bean critical state model [4]. Specifically, the plateau exhibited by the  $\kappa(B_{\rm rem})$  data can be interpreted as first being reached when the initial field is large enough for a full (unnotched) remnant peak to remain in the sample [see insets to Fig. 1(a)]. Since this is the maximum amount of flux that can be trapped in the sample, stronger initial

fields do not lead to a further decrease in  $\kappa(B_{\rm rem})$ . For weaker initial fields the remnant peak is notched, as discussed by Bean in his original paper, and hence  $\kappa(B_{\rm rem})$ is dependent upon the initial flux profile.

The value of  $\kappa(B_{\text{rem}})$  does not remain constant but is observed to decay toward  $\kappa(0)$ . In Fig. 1(b) we illustrate the time decay of  $\Delta \kappa$ , the difference between  $\kappa(0)$  and  $\kappa(B_{\text{rem}})$ , at 40 K. We observe a logarithmic profile which



FIG. 1. (a) Thermal conductivity of a twinned single crystal of YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7- $\delta$ </sub> (normalized to the zero-field value):  $\Box$ , with the field increasing, and  $\blacktriangle$ , 2 min after the field has been ramped to zero. Insets: Schematics of the flux profile in a cross section of the sample. The flux density is plotted vertically. The dashed line indicates flux profile with field on. The shaded region represents trapped remnant flux. (b)  $\Delta\kappa$  (defined in text) normalized to the value at t=0,  $\Delta\kappa_0$ . The solid line indicates logarithmic behavior.

is in good agreement with the predictions of thermally activated flux creep [5].

Returning to the increasing field data, we found that the thermal conductivity decreased with increasing field but not in the manner that we anticipated or has been suggested by previous authors [6] for conventional superconductors. Phonons are the dominant heat carriers in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7- $\delta$ </sub>, especially below  $T_c$ , and therefore the behavior of the thermal conductivity in a magnetic field will depend sensitively upon the interaction between phonons and the vortex array. In the conventional superconductor studies, it was widely proposed that the application of a field will simply add a thermal resistance term which is proportional to the number of vortices and hence to *B*. A dependence of this type leads to an in-field thermal conductivity of the form

$$\kappa(H) = [1/\kappa(0) + cB]^{-1}.$$
 (1)

Here  $1/\kappa(0)$  is the thermal resistivity before a field is applied, and *cB*, where *c* is a constant, is the thermal resistivity due to the scattering of phonons by vortices, which we will denote by  $W_{p}^{\nu}$ .

Attempts to study the validity of this form in conventional superconductors have been limited by the large electron contribution to the heat transport in these materials. This is not a problem in the YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7- $\delta$ </sub> system due to the dominance of the phonon channel for most of the temperature range below  $T_c$ . Another drawback of the conventional materials is the presence of a significant intervortex quasiparticle tunneling term, proposed by Vinen et al. [6], which provides a complicating contribution to the thermal conductivity at higher fields. The corresponding term for  $YBa_2Cu_3O_{7-\delta}$  is obtained by substituting an appropriate value for the upper critical field  $H_{c2}$ (we use 100 T) into the tunneling expression of Ref. [6]. We find this term to be unimportant up to fields of 5 T and this is likely to be an underestimate of the negligible range, because the system we examine has a much lower normal-state carrier density than that studied in Ref. [6]. These features of the high- $T_c$  systems make them uniquely appropriate for the study of the phonon-vortex interaction over a wide range of fields and temperatures.

We have found that the thermal conductivity does not fall away with increasing field in a manner that can be described by Eq. (1) at any of the temperatures investigated. This is illustrated in Fig. 2(b) where a fit by Eq. (1) is indicated by the dashed line. Rather, the data exhibit a marked positive curvature, suggesting that the vortices become less effective at scattering phonons as they become more densely distributed. We find that a phenomenological form for  $W_{\rho}^{\nu}$ ,

$$W_p^v = cBe^{-dB^{1/4}},$$
 (2)

is very successful in describing the data. Here c and d are constants that are adjusted to fit the data. A depen-



FIG. 2. (a)  $\kappa(H)$  for the twinned crystal, and (b) for the untwinned crystal. Solid lines are fits by Eq. (2). The dashed line indicates a fit by Eq. (1).

dence of this type is remarkably robust, being applicable over the entire temperature and field range for both the twinned and untwinned samples even though, as shall be shown below, these materials differ in several qualitative respects. Several other forms were tried, including a simple power law, but only the form of Eq. (2) provided an adequate fit to all of the data. In our fits, we derive B from H according to the Bean model, using the values of the full penetration field  $H^*$  derived from the remnant field data (the initial field for which the remnant plateau is first reached can be shown to be  $\sim 2H^*$ ). This is essential in describing the small low-field bulge exhibited by the data at temperatures below 50 K. In Fig. 2 we display the fits using Eq. (2) at several temperatures for both the twinned and untwinned samples. We can see that the model describes the data extremely well over a broad temperature range.

The most notable feature of the model is the fact that the vortices become less effective as scatterers as the field increases. We propose two possible explanations for this effect. The first possibility is that as the vortices become more dense, they begin to shadow one another from phonons. The second hypothesis suggests that the phonons are able to travel through the periodic potential of a regular vortex array as Bloch waves and the degree to which the heat flow is impeded will depend upon the degree of imperfection in the vortex lattice which diminishes with increasing field. It should be noted that a third possible explanation for our results is that they are the consequence of inhomogeneous flux distributions, including possible vortex clustering around defects. Examinations of the effects of vortex clustering on the thermal conductivity and ultrasonic attenuation in conventional superconductors [7] show qualitatively different behavior from what we observe and for this reason we do not believe this phenomenon is responsible for our findings.

Before discussing the remaining possibilities, it is useful to examine some of the relevant length scales in the system. We have derived [8] the value of the wavelength of the phonons which are contributing most significantly to the heat transport,  $\lambda_{dom}$ , based on an analysis of zerofield thermal-conductivity data for a  $YBa_2Cu_3O_{7-\delta}$ single-crystal sample. We arrived at a value of  $\lambda_{dom}$  $\approx 100$  Å at 77 K, and this value slowly increases as the temperature is decreased. These phonons were found to have a mean free path of  $\sim 5 \ \mu m$ . It should be noted here that these values are surprisingly large and are the result of a frequency-dependent analysis which gives qualitatively different results from the kinetic-theory approximation. As the field is varied from 1 to 6 T, the intervortex spacing  $a_v$  changes from ~500 to 200 Å according to the relation  $a_v \propto B^{-1/2}$ . The zero-temperature value of the vortex diameter is approximately equal to the coherence length,  $\xi \sim 15$  Å [9], and increases with temperature.

The above dimensions cast some doubt on the shadowing hypothesis for the anomalous field dependence of  $W_p^p$ in that, even at the highest fields we obtain, the intervortex spacing is still approximately 200 Å while the vortex diameter is only on the order of 15 Å. It is difficult to picture shadowing having a significant effect given these dimensions. One possible resolution to this objection would be that the phonons are actually scattering off of strain fields surrounding the vortices in which case the effective vortex diameter would be considerably greater than the coherence length. Exactly what the nature of a vortex strain field would be and what influence it might have on phonons remains an open question, however.

Our preferred hypothesis is based on the notion that increasing vortex-lattice regularity leads to relatively less scattering, meaning that, as the field increases, a smaller fraction of the total number of vortices serve to scatter phonons. The first issue such a hypothesis raises concerns whether or not, assuming a perfect vortex lattice, thermal phonons can travel as Bloch waves through the array without scattering. Coherent phonon transport of this type is not a phenomenon often considered since it is difficult to physically realize a situation in which phonons encounter a periodic arrangement of scatterers whose separation is not many orders of magnitude greater than the phonon wavelength. The system under study meets these requirements, and, therefore, it does not seem unreasonable to suppose that a Bloch wave description would apply. We note, however, that this interpretation requires that the phonon-vortex interaction have an elastic character. The nature of this interaction is not well understood at present but is likely composed of a combination of phenomena. The phonons could inelastically interact with the bound quasiparticles in the vortex cores but in the only detailed examination of this process that we know of [10], the author concludes that scattering of this type will only involve phonons for which  $\lambda \leq \xi$ . We believe phonons with wavelengths this short make up only a minor part of the thermal transport in our samples. Other types of interactions involve the relationship between crystal lattice and vortex motion. These would consist of either elastic distortions of the vortex lattice or relative motion of the crystal to the vortices. Interactions of this type have not been well described for thermal phonons but could contain an elastic component.

Assuming, for the moment, the validity of the Bloch wave picture, the next question which arises concerns what thermal-phonon transport can tell us about defects in the vortex lattice. Given the dimensions above, the dominant thermal phonons in these systems should be of an appropriate wavelength to probe features of the vortex array. At present, the nature of the vortex phase in high- $T_c$  materials is a highly contentious issue. Whether the vortices are arranged in a true lattice or in a glass, and the behavior of these phases in different temperatures and fields are not fully settled questions at this time. We propose that our data indicate that the phonons are scattering off of vortex-lattice defects, and that the number of such defects in an N-unit-cell area of the vortex lattice is decreasing in an increasing field as a stretched exponential [Eq. (2)]. It is plausible that the vortex lattice could become more regular with increasing field since, as the vortices get closer together, the intervortex forces strengthen and could start to overcome local pinning forces which serve to distort the lattice. This position is supported by recent decoration experiments [11] on Bi-Sr-Ca-Cu-O which demonstrate that the orientational and translational lattice correlation lengths are increasing as a function of field. The additional factor of Bin the expression for  $W_p^v$  given in Eq. (2) accounts for the fact that the number of vortex-lattice unit cells per unit area of the crystal is increasing linearly with field.

The question remains whether or not the lattice is sufficiently ordered to support Bloch wave travel of phonons. Though there is much disagreement on how ordered the vortex phase is, a recent paper by Houghton, Pelcovits, and Sudbø [12], in which they apply nonlocal elasticity theory to decoration experiments, suggests that in Bi-Sr-Ca-Cu-O the perpendicular correlation length may be as high as  $10^3$  lattice spacings at a field of 1 T. In addition, Farrell, Rice, and Ginsberg [13] have concluded from recent vibrating-reed measurements that, at least in their untwinned material, a vortex lattice exists all the way up to the irreversibility line. Even if the true vortex state is a glass, if there is short-range order over a number of lattice spacings our Bloch wave picture may still be valid, provided that the range of lattice order increases with increasing field.

Another interesting trend which can be observed as one moves through the temperature range is a gradual decrease in the curvature of the  $\kappa$ -vs-H plots as the temperature is increased. This was reflected in a decrease in the coefficient d required to fit the data. This effect is especially apparent in the untwinned crystal, which generally exhibited greater curvature than the twinned material. The curvature decrease can be interpreted in the context of our Bloch wave hypothesis. It is believed [14,15] that the vortex lattice becomes less rigid and is subject to increased thermal fluctuations at higher temperatures. One would therefore expect that any improvements in phonon travel with increasing field which rely on the enhancement of lattice order would begin to get washed out as the temperature is raised.

Concerning the higher degree of curvature present in the untwinned sample plots, we propose that the small defect or intrinsic pinning forces that would be expected to predominate in this material would be more readily overcome as the field is increased than would the stronger forces exerted by twins in the twinned material. This would result in a greater deviation from *B*-linear behavior in  $W_p^c$ , which we suggest results from improvements in lattice regularity, in the untwinned specimen.

The obvious limit of vortex-lattice disorder is the reversible regime where the vortex lattice is either melted or entirely depinned. In this regime, we would expect our data to reflect a *B*-linear dependence of  $W_p^v$  which was not seen at lower temperatures. Though we see a steady approach toward linearity as the temperature is increased, some deviation is still present at the highest temperature investigated (84 K). This is likely due to the increasing importance of a small electronic term in the transport, which has been shown in conventional materials to exhibit a field dependence qualitatively consistent with what we observe [6].

Summarizing the increasing field results, we find that the phenomenological form for  $W_p^v$  given by Eq. (2) provides a highly accurate description of the data for three different samples (the third sample, a twinned crystal, was not discussed in detail above) over a broad temperature and field range. Our model of phonons scattering from defects in the vortex lattice rather than each individual vortex provides a qualitative account for the departure from *B*-linear behavior observed in  $W_p^v$  and suggests that the number of phonon-scattering defects decreases in an increasing field as a stretched exponential. It also provides a natural explanation for the decrease in curvature of the  $\kappa$ -vs-H plots with increasing temperature and the generally greater curvature exhibited by the untwinned sample as compared to the twinned one. We hope that our results motivate further investigations into the use of the thermal conductivity as a probe of the vortex state.

The authors wish to thank Dr. U. Welp for providing the untwinned crystal used in this study. This work was supported in part by a grant from the Kellogg Foundation.

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