Universal Limit of Critical-Current Fluctuations in Mesoscopic Josephson Junctions

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The sample-to-sample fluctuations in the critical current of a disordered Josephson junction are analyzed by means of a transmission-matrix formalism. If the junction becomes small compared to the superconducting coherence length, the fluctuations at $T=0$ become of order $e\Delta_0/\hbar$, dependent only on the energy gap Δ_0 of the bulk superconductors and *independent of junction length or mean free path.* This universal limit is reached in weak links formed from point contacts or microbridges.

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The question addressed in this paper is: Does the phenomenon of "universal conductance fluctuations" have an analog in superconductivity?

In 1985 Al'tshuler and Lee and Stone showed that the sample-to-sample or "mesoscopic" fluctuations in the conductance G of a disordered metal wire at temperature $T=0$ have a root-mean-square value $\mathrm{rms}G \simeq e^2/h$ (up to a numerical coefficient of order unity) [1,2]. This value is called *universal* because, unlike the average conductance, it is independent of both the length L of the wire and the elastic mean free path l (provided $l \ll L$). Universal conductance fluctuations (UCF) have been demonstrated in a variety of experiments, and stand out as one of the most remarkable phenomena in mesoscopic physics [3].

A few years later, Al'tshuler and Spivak studied the mesoscopic fluctuations in the current-phase-difference relationship $I(\phi)$ of a superconductor-normal-metal-superconductor (SNS) Josephson junction [4]. They found that the critical current $I_c \equiv \max I(\phi)$ fluctuates from sample to sample with the rms value

$$\mathrm{rms}I_c \simeq ev_F l/L^2 \tag{1}$$

for $T \ll \hbar v_F l/k_B L^2$. Here v_F is the Fermi velocity and L is the length of the junction, i.e., the separation of the two SN interfaces (it is assumed that the transverse dimension of the junction $\leq L$). The critical-current fluctuations (1) depend on both L and l, and are therefore *not* universal in the sense of UCF.

The theory of Al'tshuler and Spivak applies to a junction which is long compared to mean free path and superconducting coherence length: $L \gg l,\xi$. [The coherence length is given by $\xi=(\xi_0 l)^{1/2}$, in the dirty limit $l \ll \xi_0$, where $\xi_0 \equiv \hbar v_F/\pi\Delta_0$ and $\Delta_0(T)$ is the superconducting energy gap.] The regime $l \ll L \ll \xi$ of a short disordered junction (which is especially relevant for weak links formed from point contacts of microbridges [5]) was not considered. Here we will show that in this short-junction regime one has

$$\mathrm{rms}I_c \simeq e\Delta_0/\hbar \tag{2}$$

for $T \ll T_c$ [$T_c \simeq \Delta_0(0)/k_B$ is the critical temperature]. In contrast to Eq. (1), the magnitude of the critical-current fluctuations has become *independent* of the properties of the junction. This is the analog for the Josephson effect of universal conductance fluctuations in metals.

The research presented here was motivated by work on *ballistic* point contacts ($l \gg L$), which showed that the critical current per transmitted mode takes on the universal value $e\Delta_0/\hbar$ in the limit $L \ll \xi_0$ [6]—but not in longer junctions [7].

Our strategy to arrive at Eq. (2) is to relate the Josephson current through an SNS junction to the scattering matrix of the normal region, and then to use the statistical properties of this scattering matrix which are known from the theory of UCF [1-3]. The model considered is illustrated in Fig. 1. It consists of a disordered normal region (hatched) between two superconducting regions S_1 and S_2. The disordered region may or may not contain a geometrical constriction. To obtain a well-defined scattering problem we insert ideal (impurity-free) normal leads N_1 and N_2 to the left and right of the disordered region. The SN interfaces are located at $x=\pm L/2$. We assume that the only scattering in the superconductors consists of Andreev reflection at the SN interfaces; i.e., we consider the case that the disorder is contained entirely within the normal region. This spatial separation of Andreev and normal scattering is the key simplification of our model. The model is directly applicable to superconductors in the clean limit $\xi_0 \ll l_S$, where l_S is the mean free path in the superconductor. We will argue that the qualitative results are not dependent on whether the disorder extends into the superconductor or not.

Further assumptions are standard within the theory of superconducting weak links [5]. The junction width is assumed to be much smaller than the Josephson penetration depth, so that the vector potential can be disregarded.

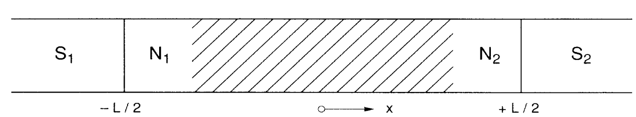

FIG. 1. Superconductor–normal-metal–superconductor Josephson junction containing a disordered normal region (hatched).

The reduction of the order parameter $\Delta(\mathbf{r})$ in the superconducting region on approaching the SN interface is neglected; i.e., we approximate $\Delta=\Delta_0\exp(\pm i\phi/2)$ for $|x|>L/2$. (In the normal region $|x|<L/2$ one has $\Delta\equiv 0$ by definition.) As discussed by Likharev [5], this approximation is justified if the weak link has length and width much smaller than ξ. (It is then also irrelevant whether the weak link is formed out of a superconductor or a normal metal.) This is generally the case when the weak link consists of a constriction. If the weak link is not small compared to ξ, one may still neglect the reduction of the order parameter at the SN interfaces if the resistance of the SNS junction is dominated by the resistance of the normal region, which in the present model occurs when $l\ll L$.

The starting point of our analysis is a general relation between the Josephson current $I(\phi)$ and the quasiparticle excitation spectrum [8]:

$$\begin{aligned} I&=I_1+I_2+I_3\,,\\ I_1&\equiv-\frac{2e}{\hbar}\sum_p\tanh(\varepsilon_p/2k_BT)\frac{d\varepsilon_p}{d\phi}\,,\\ I_2&\equiv-\frac{2e}{\hbar}2k_BT\int_{\Delta_0}^{\infty}d\varepsilon\ln[2\cosh(\varepsilon/2k_BT)]\frac{\partial\rho}{\partial\phi}\,,\\ I_3&\equiv\frac{2e}{\hbar}\frac{d}{d\phi}\int d\mathbf{r}\,|\Delta|^2/|g|\,,\end{aligned} \tag{3}$$

where g is the interaction constant of the BCS theory. The supercurrent is given as the sum of three terms: I_1 is a sum over the discrete spectrum, consisting of the energies $\varepsilon_p(\phi)\in(0,\Delta_0)$; I_2 is an integral over the continuous spectrum, with density of states $\rho(\varepsilon,\phi)$; the third term I_3 vanishes for a ϕ-independent $|\Delta|$.

The excitation spectrum consists of the positive eigenvalues of the Bogoliubov-de Gennes equation [9]

$$\begin{pmatrix}\mathcal{H}_0 & \Delta\\ \Delta^* & -\mathcal{H}_0\end{pmatrix}\Psi=\varepsilon\Psi\,, \tag{4}$$

where $\Psi(\mathbf{r})$ is a two-component wave function and $\mathcal{H}_0=\mathbf{p}^2/2m+V(\mathbf{r})-E_F$ is the single-electron Hamiltonian in a potential V. Energies are measured relative to the Fermi energy E_F. In the normal lead N_1 the eigenfunctions are

$$\begin{aligned}\Psi_{n,\mathrm{e}}^{\pm}(N_1)&=\begin{pmatrix}1\\0\end{pmatrix}(k_n^{\mathrm{e}})^{-1/2}\Phi_n\exp[\pm ik_n^{\mathrm{e}}(x+\tfrac{1}{2}L)]\,,\\ \Psi_{n,\mathrm{h}}^{\pm}(N_1)&=\begin{pmatrix}0\\1\end{pmatrix}(k_n^{\mathrm{h}})^{-1/2}\Phi_n\exp[\pm ik_n^{\mathrm{h}}(x+\tfrac{1}{2}L)]\,,\end{aligned} \tag{5}$$

where $k_n^{\mathrm{e,h}}\equiv(2m/\hbar^2)^{1/2}(E_F-E_n+\sigma^{\mathrm{e,h}}\varepsilon)^{1/2}$ and $\sigma^{\mathrm{e}}\equiv 1$, $\sigma^{\mathrm{h}}\equiv-1$. The labels e and h indicate the electron or hole character of the wave function. The index n labels the modes, $\Phi_n(y,z)$ is the transverse wave function of the nth mode, and E_n its threshold energy. The eigenfunctions in lead N_2 are chosen similarly, but with L replaced by $-L$.

In the superconducting lead S_1, where $\Delta=\Delta_0\exp(i\phi/2)$, the eigenfunctions are

$$\begin{aligned}\Psi_{n,\mathrm{e}}^{\pm}(S_1)&=\begin{pmatrix}e^{i\eta^{\mathrm{e}}/2}\\ e^{-i\eta^{\mathrm{e}}/2}\end{pmatrix}(2q_n^{\mathrm{e}})^{-1/2}(\varepsilon^2/\Delta_0^2-1)^{-1/4}\\ &\quad\times\Phi_n\exp[\pm iq_n^{\mathrm{e}}(x+\tfrac{1}{2}L)]\,,\end{aligned} \tag{6}$$

while $\Psi_{n,\mathrm{h}}^{\pm}(S_1)$ has the label e replaced by h. We have defined $q_n^{\mathrm{e,h}}\equiv(2m/\hbar^2)^{1/2}[E_F-E_n+\sigma^{\mathrm{e,h}}(\varepsilon^2-\Delta_0^2)^{1/2}]^{1/2}$ and $\eta^{\mathrm{e,h}}\equiv\phi/2+\sigma^{\mathrm{e,h}}\arccos(\varepsilon/\Delta_0)$. The square roots are to be taken such that $\mathrm{Re}\,q^{\mathrm{e,h}}\ge 0$, $\mathrm{Im}\,q^{\mathrm{e}}\ge 0$, $\mathrm{Im}\,q^{\mathrm{h}}\le 0$. The function $\arccos t\in(0,\pi/2)$ for $0<t<1$, while $\arccos t\equiv -i\ln[t+(t^2-1)^{1/2}]$ for $t>1$. The eigenfunctions in lead S_2 are obtained by replacing ϕ by $-\phi$ and L by $-L$.

The wave functions (5) and (6) have been normalized to carry the same amount of quasiparticle current, because we want to use them as the basis for scattering (s) matrices. Our goal is to express the excitation spectrum of the SNS junction in terms of the s matrix of the normal region. To this end we will make use of several different s matrices, which we now introduce.

A wave incident on the disordered normal region is described in the basis (5) by a vector of coefficients $c_N^{\mathrm{in}}\equiv(c_{\mathrm{e}}^+(N_1),c_{\mathrm{e}}^-(N_2),c_{\mathrm{h}}^-(N_1),c_{\mathrm{h}}^+(N_2))$. (The mode index n has been suppressed for simplicity of notation.) The reflected and transmitted waves have vector of coefficients $c_N^{\mathrm{out}}\equiv(c_{\mathrm{e}}^-(N_1),c_{\mathrm{e}}^+(N_2),c_{\mathrm{h}}^+(N_1),c_{\mathrm{h}}^-(N_2))$. The s matrix s_N of the normal region relates these two vectors, $c_N^{\mathrm{out}}=s_Nc_N^{\mathrm{in}}$. Because the normal region does not couple electrons and holes, this matrix has the block-diagonal form

$$s_N(\varepsilon)=\begin{pmatrix}s_0(\varepsilon) & \emptyset\\ \emptyset & s_0(-\varepsilon)^*\end{pmatrix},\quad s_0\equiv\begin{pmatrix}r_{11} & t_{12}\\ t_{21} & r_{22}\end{pmatrix}. \tag{7}$$

Here s_0 is the unitary and symmetric s matrix associated with the single-electron Hamiltonian $\mathcal{H}_0$. The reflection and transmission matrices r and t are $N\times N$ matrices, N being the number of propagating modes at energy ε. (We assume for simplicity that the number of modes in leads N_1 and N_2 is the same.)

We will make use of two more s matrices. For energies $\varepsilon<\Delta_0$ there are no propagating modes in the superconducting leads S_1 and S_2. We can then define an s matrix s_A for Andreev reflection at the SN interfaces by $c_N^{\mathrm{in}}=s_Ac_N^{\mathrm{out}}$. The elements of s_A can be obtained by matching the wave functions (5) at $|x|=L/2$ to the decaying wave functions (6). Since $\Delta_0\ll E_F$, one may neglect normal reflections at the SN interface [10]. The result is

$$s_A=\alpha\begin{pmatrix}\emptyset & r_A\\ r_A^* & \emptyset\end{pmatrix},\quad r_A\equiv\begin{pmatrix}e^{i\phi/2}\mathbb{1} & \emptyset\\ \emptyset & e^{-i\phi/2}\mathbb{1}\end{pmatrix}, \tag{8}$$

where $\alpha\equiv\exp[-i\arccos(\varepsilon/\Delta_0)]$. The matrices $\mathbb{1}$ and $\emptyset$ are the unit and null matrices, respectively. For $\varepsilon>\Delta_0$ we can define the s matrix s_{SNS} of the whole junction by $c_S^{\mathrm{out}}=s_{\mathrm{SNS}}c_S^{\mathrm{in}}$. The vectors c_S^{in} and c_S^{out} are the coefficients in the expansion of the incoming and outgoing waves in leads S_1 and S_2 in terms of the wave functions (6). By matching the wave functions (5) and (6) at $|x|=L/2$, we arrive after some algebra at the matrix-product expression

$$s_{\rm SNS}=U^{-1}(\mathbb{1}-M)^{-1}(\mathbb{1}-M^\dagger)s_N U\,,$$
$$U\equiv\begin{pmatrix} r_A & \emptyset \\ \emptyset & r_A^* \end{pmatrix}^{1/2},\quad M\equiv\alpha s_N\begin{pmatrix} \emptyset & r_A \\ r_A^* & \emptyset \end{pmatrix}. \tag{9}$$

One can verify that the three s matrices defined above (s_N,s_A for $\varepsilon<\Delta_0$, $s_{\rm SNS}$ for $\varepsilon>\Delta_0$) are unitary and satisfy the symmetry relation $s(\epsilon,\phi)_{ij}=s(\epsilon,-\phi)_{ji}$, as required by flux conservation and time-reversal invariance.

We are now ready to relate the excitation spectrum of the Josephson junction to the s matrix of the normal region. First the discrete spectrum. The condition $c_{\rm in}=s_A s_N c_{\rm in}$ for a bound state implies $\mathrm{Det}(\mathbb{1}-s_A s_N)=0$. Using Eqs. (7) and (8), and the identity

$$\mathrm{Det}\begin{pmatrix} a & b \\ c & d \end{pmatrix}=\mathrm{Det}(ad-aca^{-1}b)\,, \tag{10}$$

we find the equation

$$\mathrm{Det}[\mathbb{1}-\alpha(\varepsilon_p)^2 r_A^* s_0(\varepsilon_p) r_A s_0(-\varepsilon_p)^*]=0\,, \tag{11}$$

which determines the discrete spectrum. The density of states of the continuous spectrum is related to $s_{\rm SNS}$ by the general relation [11] $\rho=(2\pi i)^{-1}(\partial/\partial\varepsilon)\ln\mathrm{Det}\,s_{\rm SNS}$ plus a ϕ-independent term. Using Eqs. (9) and (10) we find

$$\frac{\partial\rho}{\partial\phi}=-\frac{1}{\pi}\frac{\partial^2}{\partial\phi\partial\varepsilon}\mathrm{Im}\ln\mathrm{Det}[\mathbb{1}-\alpha(\varepsilon)^2 r_A^* s_0(\varepsilon) r_A s_0(-\varepsilon)^*]\,. \tag{12}$$

The determinantal equations (11) and (12) are the key technical results of this paper.

In the short-junction limit $L\ll\xi$, the determinants can be simplified further. The condition $L\ll\xi$ is equivalent to $\Delta_0\ll E_c$, where the correlation energy $E_c\equiv\hbar/\tau$ is defined in terms of the traversal time τ through the junction [12]. The elements of $s_0(\varepsilon)$ change significantly if ε is changed by at least E_c [13]. We are concerned with ε of order Δ_0 or smaller [since $\rho(\varepsilon,\phi)$ becomes independent of ϕ for $\varepsilon\gg\Delta_0$]. For $\Delta_0\ll E_c$ we may thus approximate $s_0(\varepsilon)\approx s_0(-\varepsilon)\approx s_0(0)$. Equation (11) then takes the form

$$\mathrm{Det}[(1-\varepsilon_p^2/\Delta_0^2)\mathbb{1}-t_{12}(0)t_{12}^\dagger(0)\sin^2(\phi/2)]=0\,. \tag{13}$$

Equation (12) reduces to $\partial\rho/\partial\phi=0$, from which we conclude that the continuous spectrum does not contribute to $I(\phi)$ in the short-junction limit [$I_2=0$ in Eq. (3)]. Equation (13) can be solved for ε_p in terms of the eigenvalues T_p of the Hermitian $N\times N$ matrix $t_{12}t_{12}^\dagger$ [14],

$$\varepsilon_p=\Delta_0[1-T_p\sin^2(\phi/2)]^{1/2}\,. \tag{14}$$

Substitution into Eq. (3) finally yields the Josephson current

$$I=\frac{e\Delta_0^2}{2\hbar}\sin\phi\sum_{p=1}^{N}\frac{T_p}{\varepsilon_p}\tanh\left(\frac{\varepsilon_p}{2k_BT}\right). \tag{15}$$

Equation (15) is a generalization to arbitrary transmission matrix (i.e., to arbitrary disorder potential) of a result in the literature [15] for the Josephson current through a tunnel barrier. The generalization is *essential* for determining the sample-specific supercurrent fluctuations. A formula of similar generality for the conductance is the Landauer formula: $G=(2e^2/h)\mathrm{Tr}tt^\dagger\equiv(2e^2/h)\sum_{p=1}^N T_p$. In contrast to the conductance, the Josephson current is in general a *nonlinear* function of the transmission eigenvalues T_p. If the weak link consists of a ballistic point contact ($l\gg L$) with a quantized conductance [3], one has $T_p=1$ for $p\le N_0$, $T_p=0$ for $p>N_0$, for some integer N_0. Equation (15) then yields (at $T=0$) the discretized critical current $I_c=N_0 e\Delta_0/\hbar$ derived in Ref. [6] under the more restricted condition of adiabaticity. In the opposite regime $l\ll L$ of diffusive transport we may approximate $\varepsilon_p\approx\Delta_0$ in Eq. (15), since $T_p=\mathcal{O}(l/L)\ll 1$. Equation (15) then reduces to a *linear* relation between I and T_p,

$$I=\frac{e\Delta_0}{2\hbar}\sin\phi\tanh(\Delta_0/2k_BT)\mathrm{Tr}tt^\dagger\,. \tag{16}$$

In this regime, and at $T=0$, the average supercurrent $\langle I\rangle$ (averaged over an ensemble of impurity configurations) is related to the average conductance $\langle G\rangle$ by $\langle I\rangle=(\pi\Delta_0/2e)\langle G\rangle\sin\phi$. This relation for the supercurrent through a disordered normal region has the same form as the Ambegaokar-Baratoff formula [16] for a tunnel junction. It differs from the result obtained by Kulik and Omel'yanchuk [17] for a point contact in a disordered superconductor, by the absence of higher harmonics in ϕ. (The fundamental $\sin\phi$ term agrees.) We attribute the difference to the fact that we have assumed a clean superconductor ($l_S\gg\xi_0$) containing a disordered region ($l\ll\xi_0$), while in Ref. [17] both the superconductor and the weak link are in the dirty limit ($l\equiv l_S\ll\xi_0$). The difference in the average critical current $\langle I_c\rangle$ is a difference in a numerical coefficient, not in the order of magnitude. [Reference [17] gives $\langle I_c\rangle=C(\pi\Delta_0/2e)\langle G\rangle$ with $C=1.32$ instead of $C=1$.]

The analysis of Kulik and Omel'yanchuk is based on a diffusion equation for the *ensemble-averaged* Green's function, and cannot therefore describe the mesoscopic fluctuations of $I(\phi)$ from the average. In contrast, our Eq. (16) holds for a *specific* member of the ensemble of impurity configurations. The statistical properties of $\mathrm{Tr}tt^\dagger$ in this ensemble are known from the theory of UCF [1-3]. The central result is that $\mathrm{rms}\,\mathrm{Tr}tt^\dagger\equiv C_{\rm UCF}$ is a number of order unity, calculated in Ref. [2], which depends weakly on the shape of the junction [18]. Since the supercurrent (16) is linear in $\mathrm{Tr}tt^\dagger$, we obtain without further calculation the result

$$\mathrm{rms}I(\phi)=\tfrac{1}{2}C_{\mathrm{UCF}}(e\Delta_0/\hbar)\sin\phi\tanh(\Delta_0/2k_BT)\,. \tag{17}$$

We thus find that, for $T \ll T_c$, the critical-current fluctuations have magnitude $\mathrm{rms}I_c=\frac{1}{2}C_{\mathrm{UCF}}e\Delta_0/\hbar$, as advertised in Eq. (2). These results are obtained from the model of a disordered junction between clean superconductors. Just as for $\langle I_c\rangle$ (previous paragraph), we expect that, if the disorder extends into the bulk superconductor, the numerical value of $\mathrm{rms}I_c$ differs—but not its order of magnitude.

Experimentally, sample-to-sample fluctuations are not as easily studied as fluctuations in a given sample as a function of some parameter. In the theory of UCF one has an ergodicity property, which says that averaging over an ensemble of samples is equivalent to averaging a single sample over magnetic field B or Fermi energy E_F [2]. The ergodicity in E_F holds for our problem as well, since Eq. (16) implies that the Josephson current and the conductance have *identical statistical properties* at $T=0$, $B=0$. Josephson junctions consisting of a two-dimensional electron gas (2DEG) with superconducting contacts allow for variation of E_F in the 2DEG by means of a gate voltage [19]. Point-contact junctions can be defined in the 2DEG either lithographically or electrostatically (using split gates) [3]. For such a system the theory presented here predicts that if E_F is varied on the scale of E_c, the critical current (at $T \ll T_c$) will fluctuate by an amount of order $e\Delta_0/\hbar$, independent of the properties of the junction.

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