Exponential, Rather than Power-Law, Temperature Dependence of the Damping of a Vibrating-Wire Resonator in ^{3}He-A at Low Temperatures

S. N. Fisher, A. M. Guénault, C. J. Kennedy, and G. R. Pickett
School of Physics and Materials, Lancaster University, Lancaster LA1 4YB, United Kingdom
(Received 23 July 1991)

We have measured the damping of a vibrating-wire resonator in liquid ^{3}He-A down to temperatures of T_c/T of 7 at zero bar pressure in a field of 500 mT. A simultaneous measurement of the damping in the B phase is also recorded. The response in the A phase is found to follow an exponential temperature dependence and not a power law as might be naively expected. This effect is a consequence of the exclusion of a large fraction of the bulk low-energy excitations from a surface by the bending of the texture.

PACS numbers: 67.50.Fi

The excitation gases in the two phases of superfluid ^{3}He should behave very differently in the limit of very low temperatures. In the B phase, the gap is isotropic and the properties of the liquid at low temperatures which are governed by the quasiparticle gas are also isotropic and show temperature dependences dominated by the Boltzman factor $\exp(-\Delta/kT)$, the leading factor in the quasiparticle density. A number of recent studies of the B phase in the ballistic regime down to T_c/T of around 10 have confirmed this picture. For example, both the damping of a vibrating-wire resonator [1] and the Kapitza boundary conductance show exponential dependences on temperature [2].

The A-phase superfluid, on the other hand, is highly anisotropic. The energy gap has polar nodes where the gap falls to zero parallel to the $\hat{l}$ vector. Such a gap configuration gives rise to a much higher quasiparticle density than that of the B phase at the corresponding temperature. The A-phase quasiparticle density follows a T^3 dependence rather than the exponential dependence of the B phase, and this power-law signature might be expected to manifest in the equivalent A-phase transport properties. Very few measurements on the A phase have been made to temperatures as low as those readily achieved in the B phase, with the result that such power-law dependences have not so far been observed.

We have compared the damping of a vibrating-wire resonator in the B phase with a second similar resonator in a volume of A phase stabilized by a magnetic field of 500 mT to temperatures down to T_c/T of 7 at zero bar pressure. It is the purpose of this paper to point out that, when the A phase is cooled to near the $T=0$ limit, we find that the expected differences between the behavior of the two phases are not in practice observed.

The experimental cell used for the present measurements is shown in Fig. 1. Cooling the A phase to low temperatures is made difficult by the requirement of a relatively large magnetic field to stabilize it. This means that the immersed refrigerant technique [3], used for reaching the lowest temperatures, must be modified to allow the region of magnetic field needed for the ^{3}He-A to be situated remote from the low-field region needed for the copper refrigerant. This is achieved by an extension to the cell to which the stabilizing field of 500 mT can be applied by a small solenoid on the still shield of the refrigerator. (At this field the up- and down-spin A-phase gaps are split by less than 1% and the A phase is essentially undistorted.) Outside this localized stabilizing field the ^{3}He remains in the B phase, allowing direct comparison of the simultaneous response of two vibrating-wire resonators situated one in each phase as shown in the figure. The resonators are made of a single filament of multifilamentary superconducting wire approximately 13 μm in diameter bent into an approximate semicircle with diameter 3 mm. The resonator in the A phase is contained inside a box as shown in the inset. Under the same conditions the two wires have identical responses within the experimental error.

The measurement presented here is a very simple one.

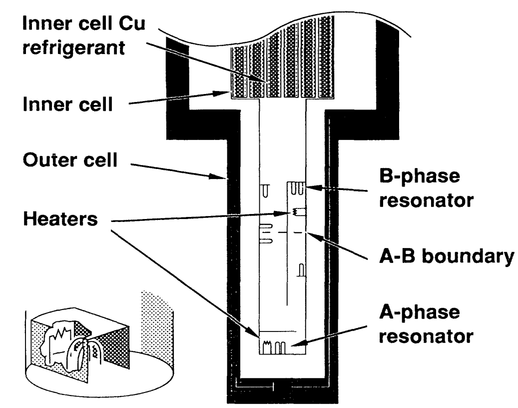

FIG. 1. Schematic view of the experimental cell. The outer cell contains liquid ^{3}He and copper flake refrigerant. The inner cell contains liquid ^{3}He and refrigerant in the form of copper plates coated with sintered silver. The lower extension of the cell projects into a solenoid which produces a 500-mT magnetic field to stabilize the A phase. This part of the cell is furnished with a number of heaters and vibrating-wire resonators as shown. Inset: The arrangement of the A-phase resonator and heater.

The experiment is cooled, with the constant stabilizing field applied, to the lowest temperature, which at zero bar is about 140 μK or T_c/T of 7. As the cell subsequently slowly warms, a continuous record is made of the damping of the vibrating-wire resonator in the B phase and of the resonator in the A phase. Under desktop computer control the resonators are driven through resonance, the Lorentzian line shape is fitted, and the frequency width at half height, Δf_2, proportional to the damping, is measured and recorded.

The data fall into two series, depending on the thermal history before the warming period begins. For the majority of runs, after the cell was cooled, the response was monitored during the normal warmup with no other disturbance. However, in two series of measurements (during an investigation of the response of the resonators to a heat flux), the cell was first exposed to a number of (very small) heating cycles at the lowest temperatures, after which the A-phase–B-phase comparison was made on the subsequent warmup.

The results for several runs are shown in Fig. 2, where we have plotted $\ln(\Delta f_2)$ for the A phase versus $\ln(\Delta f_2)$ for the B phase. Since $\ln(\Delta f_2)$ for the B phase is accurately proportional to $-1/T$ we have added a temperature scale to the B-phase Δf_2 axis. Most importantly, we see that the relationship between the two quantities is linear over most of the temperature range. In other words, the damping of the wire in the A phase is not governed by a power-law temperature dependence but follows an exponential dependence very similar to that of the wire in the B phase. We should emphasize that we do know that the A-phase wire is indeed in the A phase. These are very low velocity measurements with maximum wire velocities of the order of 0.1 mm s^{-1}. If the velocity is increased to the order of 1 mm s^{-1}, then the very different critical velocity seen in each phase indicates unambiguously in which phase the wire is actually operating [4].

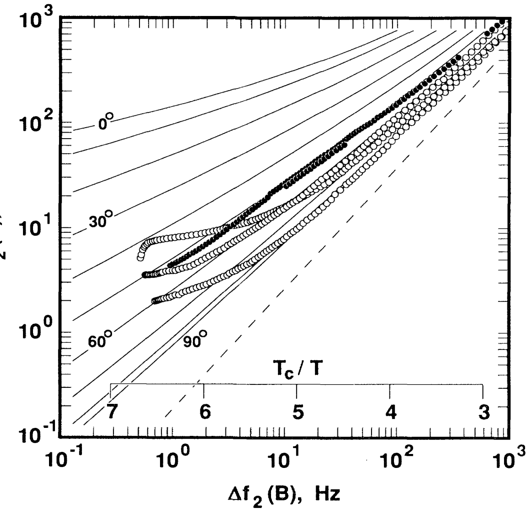

FIG. 2. The frequency width in the A phase plotted against the simultaneously measured width in the B phase. Since the B-phase width is proportional to $\exp(-\Delta/kT)$ below T_c/T of about 3 we can also include a linear T_c/T axis. The filled points represent measurements made after heat pulsing the experiment (see text), whereas the open circles represent runs measured with no other operations applied. The curves labeled 0° to 90° represent simple calculations of the expected response to a uniform texture where the label represents the angle between the nodal line and the direction of motion of the wire. The dashed curve represents a similar calculation for a disordered texture where long-range propagating excitations incident from any direction all have energies greater than the maximum A-phase gap, Δ_0.

Referring to Fig. 2 in more detail, note first the full series of filled points which represent the warmup behavior after a series of heat-flow experiments had been made. (One of the runs is represented by only a few data over a relatively short temperature interval, but with values virtually identical to the similar data taken over the whole temperature range.) We see that, first, the data follow a straight line over the whole temperature range which is accessible to measurement. Second, the slope is less than unity. This means that for these data Δf_2 follows a temperature dependence of the form $\Delta f_2 \propto \exp(-\Delta'/kT)$, where Δ' is less than the zero-temperature B-phase gap. Concentrating now on the open-point data (i.e., taken on the undisturbed warmups) we see that the curves tend to be curved at the lowest temperatures but approach exponential behavior at higher temperatures. The slopes are now much closer to unity, that is to say they indicate a Boltzmann factor behavior with an effective gap very similar to the zero-temperature B-phase value.

To attempt to understand these data, consider how a moving wire exchanges momentum with the liquid in the ballistic (long mean free path) limit. In the case of the B phase, the process is well understood [1]. Put briefly, those quasiparticles which can reach the wire are reflected by normal processes which exchange of order of the Fermi momentum with the wire (the exchange being positive for quasiparticles, negative for quasiholes). When the wire is in motion, quasiholes approaching the forward side of the wire are reflected by Andreev processes from the flow field around the wire, whereas on the rearward side approaching quasiparticles are reflected by the flow. This introduces an asymmetry into the momentum exchange between front and rear, and gives rise to the very large force observed on the moving wire. The force is found to be the product of the net flux of excitations incident on the wire along the direction of motion, taken at zero wire velocity, multiplied by the Fermi momentum p_F and a further "effectiveness" factor $p_F v/kT$ which represents the quasiparticle-quasihole imbalance in the incident flux which gives rise to the force. The important point in the above is that the action of the flow field determines which quasiparticles can reach the wire surface to exchange momentum.

In the case of the A phase, there is a second factor governing which excitations actually reach the wire, namely, the texture. At a solid boundary the $\hat{l}$ vector, that is to say the direction of the gap nodes, is constrained to lie normal to the surface. Consider a plane surface with normal parallel to the nodal direction of a globally uniform texture. In this case, excitations of all energies can reach the surface. If, however, the nodal axis of the uniform texture is not normal to the surface, then only excitations with energies greater than some critical value can penetrate to the surface. This is a consequence of the fact that the minimum energy of quasiparticles is a function of angle θ between the k vector and the nodal line, i.e., $\Delta_0 \sin\theta$, where Δ_0 is the maximum A-phase gap. As the nodal line bends near the surface to satisfy the surface boundary condition, quasiparticle traveling at minimum energy in the liquid parallel to the nodal line find, as they approach the curving texture near the surface, that states with the same k vector have increasing energies. The incoming particles thus experience Andreev reflection and do not reach the surface. If the bulk texture has nodal lines which make an angle of ϕ with the normal to the surface, then it is clear that the minimum energy of quasiparticles reaching the wire is $\Delta_0 \sin(\phi/2)$, which in the extreme case of the texture parallel to the surface yields a minimum of $\Delta_0/\sqrt{2}$. This is illustrated schematically in Fig. 3, where we have plotted the gap as a function of angle for the region of liquid at the surface and also the gap for the bulk liquid with uniform texture. Note that this bending of the texture near a surface excludes the low-energy excitations from the damping mechanism. However, these low-energy excitations are precisely those responsible for the power-law excitation density. If the only excitations involved are those with energies greater than some value E', when we

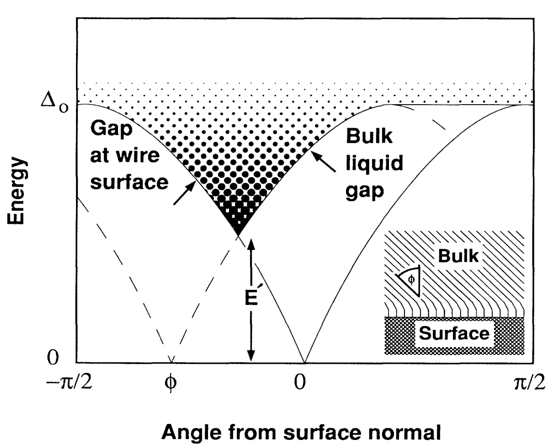

FIG. 3. The effect of a surface on the mobility of excitations in ^{3}He-A at low temperatures. Inset: A surface in contact with a uniform texture with nodal line at a random angle. The main figure shows the gap as a function of angle from the surface normal for the bulk texture and the region near the wire. Only excitations with energies greater than E' can penetrate from the bulk to reach the surface.

should expect the behavior to be governed by an exponential temperature dependence of the form $\exp(-E'/kT)$.

We can extend this argument to provide an estimate of the force on a moving wire in the A phase. As mentioned above, we know that in the B phase the force on a wire is proportional to the flux of incident excitations (taken at zero wire velocity) multiplied by the factor $p_F^2 v/kT$. If we assume that the same holds true for the A phase, we can take into account the different geometries of the excitation gases in the two phases with the assumption that at low velocities the force on the wire will be proportional to the flux of quasiparticles which can reach the wire surface through the texture, multiplied by the factor $p_F^2 v/kT$. For the ideal case of a straight cylindrical wire immersed in liquid with a uniform texture, we can calculate the flux of quasiparticles reaching the wire as a function both of temperature and of the angle between the wire axis and the axis of the texture. This calculation is represented by the solid curves in Fig. 2, where we plot the A-phase flux/T versus the B-phase flux/T as a function of the texture-wire angle at the appropriate temperature. For comparison we have included the dashed curve which represents the response expected from a disordered texture in which the quasiparticles would see the maximum A-phase gap Δ_0 at some distance in *all* directions.

These simple arguments are based on an implicit assumption that the mean free paths of the quasiparticles are long compared with the variation of the texture near the wire. For the isotropic B phase the mean free path is readily calculated and at the lowest temperatures is measured in kilometers. For the A phase a simple back-of-the-envelope calculation is not so easy, since the mean free paths would be expected to be energy dependent, with a much shorter value for the low-energy high-density excitations down in the nodes. However, provided that the mean free paths of the quasiparticles which can reach the wire surface allow them to traverse the region of textural bending near the wire unscattered then the above discussion is valid.

Within the assumption of this simple model we see that the data of Fig. 2 all yield exponential lines (after the initial irreproducible behavior) with magnitudes and slopes in excellent agreement with the calculation. Note that the model has no free parameters. The exponential part of each data set lies within the 50° to 90° range of the calculation, with considerable variation from run to run. The range of widths seen at a given temperature suggests that over the region of the bulk liquid near the active part of the resonator the texture is rather uniform. (A highly disturbed texture would give a reproducible result close to the dashed line.) Two factors would seem to contribute to this apparent textural uniformity. First, the magnetic-field energy constrains the nodal line to lie perpendicular to the field direction. Second, the textural bending term diverges [5] towards low temperatures. Together these terms tend to keep the nodal direction uniform in the horizontal plane. How the texture then orients itself within the horizontal plane would seem to depend on the constraints of the orientation of the surfaces of the surrounding container with competing and relatively weak influence from the various surfaces concerned. Unfortunately, since this experiment was not envisaged when the cell was built, the precise geometry of the container relative to the resonator is not known with any precision. However, with reference to the inset in Fig. 1, we note that the resonator is situated in a rectangular tube of square cross section. The neighboring vertical walls would tend to pull the texture towards a line perpendicular to the motion of the wire corresponding to the 90° line of Fig. 2.

The filled points, taken after the heat-pulse treatment, show a fairly straightforward behavior. The temperature dependence is exponential over the whole temperature range measured and the damping corresponds to that expected from a uniform texture with angle 50° for both sets of data (even though one is taken over a very limited temperature range). The frequency width of the wire is very sensitive to a heat current and may change by a factor of 2 (usually becoming smaller) in a heat flux of order 10^{-13} W (usually but not always), returning approximately to its previous value when the flux is turned off. In other words, through the response of the resonator we can see in real time the heat flux modifying the texture. This disturbing effect of a heat current, we believe, anneals the texture into some stable configuration. Therefore we assume that the filled data points correspond to the response from the stable configuration which has been arrived at by the annealing effects of the previous heating cycles.

The open circles in Fig. 2, representing the passively warming data, are rather different. At the lowest temperatures there is an irreproducible temperature variation above which the data gradually approach exponential behavior. Once the exponential region is reached the data fall close to the 60° to 90° lines. These data have not been annealed by heat flow and we presume that the initial curvature represents the slow spontaneous reorganization of the texture towards a more stable configuration. The data in this region are taken over several days and it is virtually impossible with our present experiment to distinguish between changes with time and changes with temperature although from the behavior of the filled points we believe the former. On this assumption, we are seeing the system settling down with time to some orientation corresponding to the calculated orientation of 60° to 90°. This is significantly lower than that of the heat-treated data and may indicate that the annealing effect of a heat flow (which is inevitably unidirectional from the position of the heater) gives rise to a slightly different textural direction from that when the system is left to itself, which is not unreasonable.

From this discussion we can draw the following conclusion. First, when A-phase ^{3}He is in contact with a solid interface so that the texture bends to satisfy the boundary conditions, this bending prevents the low-energy ballistic quasiparticles from the bulk from reaching the wall. Consequently, any transport property measured via the contact with a wall, such as the damping of a resonator reported here or, for example, the Kapitza conductance, will show an exponential dependence on temperature (with lower magnitude) and not a power law, as might naively be expected. With reference to Fig. 3, we should also emphasize that the texture, or more specifically the nodal direction, acts as a channel making the quasiparticle flow virtually unidirectional at very low temperatures. Since the direction of the quasiparticle flux has a measurable effect on the damping of a mechanical resonator, these devices in one form or another may provide very sensitive textural probes, given the constraints of the bending of the texture at the surface.

[1] S. N. Fisher, A. M. Guénault, C. J. Kennedy, and G. R. Pickett, Phys. Rev. Lett. **64**, 2566 (1989).

[2] C. A. M. Castelijns, K. F. Coates, A. M. Guénault, S. G. Mussett, and G. R. Pickett, Phys. Rev. Lett. **55**, 2021 (1985).

[3] D. I. Bradley, A. M. Guénault, V. Keith, S. G. Mussett, G. R. Pickett, and W. P. Pratt, Jr., J. Low. Temp. Phys. **57**, 359 (1984).

[4] S. N. Fisher, A. M. Guénault, C. J. Kennedy, and G. R. Pickett (to be published).

[5] A. L. Fetter, Phys. Rev. B **20**, 303 (1979).