

Vortex and Fluxon Phase Transitions in Layered Superconductors

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The system of superconducting layers with Josephson coupling J is studied. The ordering temperature T_f for the fluxon loops parallel to the layers is shown to determine the transition temperature T_c of the system, except when J is exponentially small; in the latter case T_c drops towards the $J=0$ vortex transition, accounting for data on $\text{YBa}_2\text{Cu}_3\text{O}_7/\text{PrBa}_2\text{Cu}_3\text{O}_7$ superlattices. A magnetic field parallel to the layers produces a sequence of phases in which the induced fluxon lines are l layers apart. For $l > 8$ and J not too small, phases with two-dimensional fluxon correlations occur near T_c ; this accounts for the observed $V \sim I^{a(T)}$ current-voltage relation in high- T_c superconductors.

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The anisotropic properties of most of the high-temperature oxide superconductors has led to increased interest in the effects of two-dimensional (2D) fluctuations. In particular, data on bulk samples [1-4] and more recent data on thin films [5-7] of $\text{Tl}_2\text{Ba}_2\text{CaCu}_2\text{O}_8$, $\text{YBa}_2\text{Cu}_3\text{O}_7$, and $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$ have shown a relation $V \sim I^{a(T)}$ for the voltage V and current I with $a(T) > 1$ near T_c ; this is consistent with the Halperin-Nelson [8] description of vortex fluctuations in 2D superconductors, based on the Kosterlitz-Thouless (KT) theory. Furthermore, the exponent $a(T)$ yields an effective thickness of the 2D layer [5,7] which is of the order of the unit cell in the c direction. On the other hand, T_c drops dramatically in $\text{YBa}_2\text{Cu}_3\text{O}_7/\text{PrBa}_2\text{Cu}_3\text{O}_7$ superlattices as the separation of the superconducting CuO_2 layers increases [9-11], indicating that T_c with the usual layer separation is much higher than that of an isolated layer.

These data focus on the nature of the phase transition in anisotropic layered superconductors. The effective free energy with Josephson coupling J between layers [12,13] allows for two types of topological excitations: vortex points due to $\pm 2\pi$ phase singularities in each plane and fluxon lines due to $\pm 2\pi$ variations in the relative phase of neighboring layers. The $J=0$ case has been extensively studied [14-17], showing that vortices interact logarithmically in distance and indicating a vortex phase transition at a temperature T_v .

The presence of J allows for a distinct phase transition at a temperature T_f , defined by assuming, for computational convenience, that vortices are absent. This transition is due to fluctuations of fluxon loops parallel to the layers, decoupling the layers at $T > T_f$. The neglect of

vortices is possible in isolated or widely separated junctions, e.g., junctions on twin boundaries [18], or for our system if [19] T_f were lower than T_v so that free vortices are thermally unactivated at T_f . In the latter case the range $T_f < T < T_v$ would be a 2D phase exhibiting $V \sim I^{a(T)}$. However, Korshunov [20] has shown, by using a discrete Gaussian version of the free energy, that in fact $T_f > T_v$ for all the range of parameters, excluding the possibility of a 2D phase. He then concluded that the three-dimensional (3D) transition temperature T_c occurs near T_v with $\ln(T_c - T_v) \sim T_v/J$.

A considerable insight is gained by adding a magnetic field H parallel to the layers which affects the fluxon fluctuations when $H > H_{c1}$, where H_{c1} is the parallel critical field. An isolated junction is thermally destroyed by 2D fluctuations for all $H > H_{c1}$ [18]. For the multilayer system Efetov [14] has proposed that above some critical field ($\gg H_{c1}$) the 3D correlation of the flux lines is lost, leading to a 2D phase; the latter derivation was based on a high-field expansion and neglected the role of vortices.

In the present work I show that indeed $T_f > T_v$; however, the 3D transition temperature T_c is close to T_f for $\ln(T_c/J)$ not too small. Only when J is exponentially small does T_c drop towards T_v , accounting for the data on superlattices [9-11]. I then show that a field $H > H_{c1}$ produces a sequence of phases with the induced fluxon lines l layers apart. For $l \leq 8$ vortices eliminate 2D phases (disproving Efetov's scenario for $l=1$), while for $9 \leq l \leq l_c$ each " l phase" has a 2D regime close to T_f , accounting for the $V \sim I^{a(T)}$ relation [5-7] and for an observed [6,7] H dependence of the exponent $a(T)$.

The effective free energy for a layered superconductor with s -wave pairing is [12,13]

$$\begin{aligned} \mathcal{F} = & \frac{1}{8\pi} \int d^2r dz \left\{ (\mathbf{v} \times \mathbf{A})^2 + \frac{1}{\lambda_e} \sum_n \left[\frac{\phi_0}{2\pi} \nabla \varphi_n(\mathbf{r}) - \mathbf{A}(\mathbf{r}, z) \right]^2 \delta(z - nd) \right\} \\ & - \frac{J}{\xi_0^2} \sum_n \int d^2r \cos \left[\varphi_n(\mathbf{r}) - \varphi_{n-1}(\mathbf{r}) - (2\pi/\phi_0) \int_{(n-1)d}^{nd} A_z(r, z') dz' \right] - E_c \sum_{n, \mathbf{r}} s_n^2(\mathbf{r}), \end{aligned} \quad (1)$$

where $\varphi_n(\mathbf{r})$ is the superconducting phase on the n th layer, \mathbf{r} is the position vector in the layer, $\mathbf{A}(\mathbf{r}, z)$ is the vector potential, $\phi_0 = hc/2e$ is the flux quantum, E_c is the loss of condensation energy in a volume $\xi_0^2 d_0$, and $s_n = \pm 1$ at the vor-

tex sites while $s_n=0$ otherwise. Thus

$$\varphi_n(\mathbf{r}) = \varphi_n^0(\mathbf{r}) + \sum_{\mathbf{r}'} s_n(\mathbf{r}') \tan^{-1}[(y-y')/(x-x')],$$

where $\varphi_n^0(\mathbf{r})$ is a nonsingular function. The length scales are $\lambda_e = \lambda^2/d_0$ with λ the London penetration depth parallel to the layers, d_0 the thickness of each layer, d ($> d_0$) the separation between layers, and ξ_0 the in-plane correlation length; typically $\lambda_e \approx 10^7 \text{ \AA} \gg d_0 \approx 10 \text{ \AA}$. The four terms of (1) describe the 3D magnetic energy, the 2D supercurrents, the Josephson coupling, and the vortex core energies, respectively.

Consider first the $J=0$ case. The vortex-vortex interaction is [14–17] $\sim \ln(r)$ for $r \gg d, \xi_0$, indicating a KT-type transition. A renormalization-group (RG) analysis [21] similar to that of a KT transition [22] confirms this, with the transition temperature

$$T_v = \frac{1}{8} \tau [1 - (1 + 4\lambda_e/d)^{-1/2}] + O(\exp(-E_c/T_v)), \quad (2)$$

where $\tau = \phi_0^2/4\pi^2\lambda_e$. At $T > T_v$ vortices are relevant with a finite density, while at $T < T_v$ vortices are irrelevant and have zero density. Note that $\tau = \tau(T)$ since the effective free energy (1) involves a temperature-dependent λ . Defining T_c^0 as the transition temperature of a corresponding isotropic 3D system and assuming that the relevant temperatures are not too close to T_c^0 , the mean-field form $\lambda(T) = \lambda_0(1 - T/T_c^0)^{-1/2}$ can be used. Hence $\tau(T) = \tau_0(1 - T/T_c^0)$, where $\tau_0 = \phi_0^2 d_0/4\pi^2\lambda_0^2$ (typically $\tau_0 \approx 10^3 \text{ K}$), and from Eq. (2) $T_v = T_c^0 [1 + 8T_c^0/\tau_0]^{-1}$ for $d \ll \lambda_e$.

Consider next the system with $J \neq 0$. I assume first that vortices are absent, i.e., $s_n(\mathbf{r})=0$, and examine later the consistency of this assumption. Equation (1) is then essentially the action for a 2D sine-Gordon system which is dual to the vortex problem [22], and an RG procedure is valid in powers of $\bar{J}_0 = J/T$. The recursion relations which increase the scale ξ from ξ_0 renormalize \bar{J}_0 to $\bar{J}(\xi)$ and $x_0 = (1 + d/2\lambda_e)T/\tau$ to $x(\xi)$ via

$$d \ln \bar{J} = 2[1 - x] d \ln \xi, \quad (3a)$$

$$dx = -2\gamma^2 \bar{J}^2 x^3 d \ln \xi, \quad (3b)$$

where the coefficient γ depends on the procedure of smoothing the cutoff [22] (e.g., with a mass insertion $\gamma = 4\pi\sqrt{6}$). Equation (3) determines a phase transition at the temperature

$$T_f = \frac{\tau}{(1 + d/2\lambda_e)(1 - \gamma\bar{J}_0)}. \quad (4)$$

For $d \ll \lambda_e$, $\gamma\bar{J} \ll 1$, Eq. (4) is factor 8 larger than Eq. (2); in fact for all values of d/λ_e we have $T_f > T_v$. This

confirms Korshunov's result [20]—there is no 2D regime and the transition is intrinsically 3D.

Consider now the complete system of Eq. (1). In the range $T_v < T < T_f$ both vortices and J are relevant and compete for disorder or order, respectively. To study this competition, compare the fluxon correlation length ξ_f at which $J(\xi_f) \approx T$ with the corresponding length ξ_v defined by the vortex density ξ_v^{-2} . The scaling of the separate vortex and fluxon systems is valid up to $\min(\xi_v, \xi_f)$. If $\xi_f < \xi_v$, scaling reaches first $J(\xi_f) \approx T$ and fluctuations in the relative phase of neighboring layers such as free vortices are suppressed, i.e., $\xi_f < \xi_v$ corresponds to a 3D ordered phase. On the other hand, when $\xi_v < \xi_f$, vortices on a scale ξ_v interfere in the cosine term of (1), prevent J from fully renormalizing, and disorder the system. If T is not too close to T_f (i.e., $T_f - T \gtrsim \gamma JT/\tau$) $x(\xi) \approx x_0$ and Eq. (3a) yields $2 \ln(\xi_0/\xi_f) \approx (1 - T/\tau)^{-1} \ln \bar{J}_0$; a similar scaling for vortices yields $2 \ln(\xi_v/\xi_0) \approx E_c/(T - \tau/8)$ and the criterion $\xi_f \approx \xi_v$ determines

$$T_c \approx \frac{\tau [E_c + \frac{1}{8} \tau \ln T_c/J]}{E_c + \tau \ln T_c/J}. \quad (5)$$

Including the effect of J on the vortex system shows [21] that Eq. (5) holds with E_c replaced by E'_c ($> E_c$). The effective vortex energy E'_c can also be determined experimentally by the vortex resistivity ρ , i.e., $\ln \rho \sim (T - T_c)^{-1/2}$ [8]; the limited range for this behavior [2–5] implies $E'_c \gtrsim T_c$. Equation (5) then yields $T_c \rightarrow \tau \approx T_f$ for $\ln(T_c/J) \lesssim E'_c/T_c$, i.e., $J \approx T_c$, while $T_c \rightarrow T_v$ for $\ln(T_c/J) \gtrsim 8E'_c/T_c$, i.e., J exponentially small. Equation (5) thus shows the crossover from fluxon-dominated to vortex-dominated transition; unless J is exponentially small the transition is near T_f .

I consider next the system (1) in the presence of an external magnetic field H parallel to the layers; vortices are first neglected. The magnetization in, say, the x direction corresponds to $-\int d^2r \sum_n \partial[\varphi_n^0(\mathbf{r}) - \varphi_{n-1}^0(\mathbf{r})]/\partial y$, so that H acts as a misfit parameter in the commensurate-incommensurate transition [23–25]. Based on this analogy, it is useful to transform the problem into that of one-dimensional fermions on chains with length L —each time-dependent fermion corresponds to a flux line meandering in space. The algebra involves integrating the Gaussian vector potential fields [20] and transforming into a quantum problem [24]. In terms of annihilation operators $a_n(q)$, $b_n(q)$ for right- and left-moving fermions, respectively, on the n th chain I obtain the equivalent Hamiltonian

$$\begin{aligned} \mathcal{H}_F = & \sum_{n,q} \{V_0 q [a_n^\dagger(q) a_n(q) - b_n^\dagger(q) b_n(q)] - (\pi J/\xi_0 T) [a_n^\dagger(q) b_n(q) + b_n^\dagger(q) a_n(q)] \\ & - (H\phi_0/4\pi T) [a_n^\dagger(q) a_n(q) + b_n^\dagger(q) b_n(q)] + (2\pi/L) U_0 \rho_{1n}(q) \rho_{2n}(-q)\} \\ & + (\pi/L) \sum_{n \neq n'} \sum_q V_{n-n'} [\rho_{1n}(q) + \rho_{2n}(q)] [\rho_{1n'}(-q) + \rho_{2n'}(-q)] + (2\pi/L) \sum_{\pm} \sum_q (U_1 - V_1) \rho_{1n \pm 1}(q) \rho_{2n}(-q), \quad (6) \end{aligned}$$

where $\rho_{1n}(q), \rho_{2n}(q)$ are Fourier transformed densities of the a_n, b_n fermions, respectively, and V_n, U_n have the Fourier transforms $[(4\pi)^2 \beta^2(k) \pm 1]/8\pi\beta(k)$ for $V(k)$ (with $+$) and $U(k)$ (with $-$) and $\beta(k) = \phi_0^2/\{16\pi^3 T d\} [1 + (4\lambda_e/d) \times \sin^2(kd/2)]$.

The on-chain terms [those within curly brackets in (6)] have been extensively studied [23–25]; following the Schulz procedure [24] the fermion spectrum is found by a Bogoliubov transformation to be $\pm [(V_0 + U_0)^2 q^2 + \Delta^2]^{1/2}$ for small q , where $\Delta \sim 1/\xi_f$ is the renormalized gap. When $H > H_{c1} = 4\pi T\Delta/\phi_0$, the upper branch of the fermions becomes occupied with a fermion density n_f which corresponds to flux penetration.

The total on-chain fermion energy has terms linear and cubic in n_f for small n_f , while the interchain coupling has a n_f^2 term [neglecting $U_1 - V_1 \approx V_1(d/\lambda_e)^{1/2}$]. This dominant repulsive force favors a situation with $n_f \neq 0$ on every l th chain and $n_f = 0$ on intermediate chains, i.e., fluxons are l layers apart. Both n_f and l can be considered as variational parameters, determined by minimizing $\langle \mathcal{H}_F \rangle$. A mean-field decoupling yields a sequence of “ l phases” with l decreasing with H ; $l \sim H^{-1}$ for $l \ll (\lambda_e/d)^{1/2}$ with $H \gtrsim 2H_{c1}$, while for higher l with $H \lesssim 2H_{c1}$, $l \sim \ln[H_{c1}/(H - H_{c1})]$. The main feature of the result, namely, the sequence of $l(H)$ transitions, is expected in general due to the discreteness of Eq. (1) in the z direction.

To obtain the long-range behavior I linearize the fermion spectrum of an occupied chain at its Fermi points to obtain fermions with velocities $\pm v_c$ and neglect the states of the filled branch (energies $\leq -\Delta$) which require a finite excitation energy [24]. The interactions between these fermions on chains n, n' are classified [26–28] as either backward scattering $\gamma_1(n - n')$ or forward scattering $\gamma_2(n - n')$ with Fourier transforms $\gamma_i^0(k)$ ($i = 1, 2$). The fermion RG to third order in $\gamma_i(n)$ leads to an integral-differential equation [26–28] for renormalized $\gamma_i(k, \xi)$ with the initial condition $\gamma_i(k, \xi_0) = \gamma_i^0(k)$. The couplings $\gamma_i^0(k)$ can be chosen so that $\gamma_i^0(k) \geq 0$ and vanishes only at $k = \pi$. The latter point is then the most susceptible one to generate a negative $\gamma_1(k, \xi)$ which signals a density wave [27, 28]. The initial direction of flow of $\gamma_1(\pi, \xi)$ is determined by the sign of $\gamma_2^0(\pi)$, which for $l = 1$ and $(\lambda_e/d)^{1/2} \gg 1$ is

$$\gamma_2^0(\pi) = (\gamma_c^2 - \delta_c^2)^2 [(4\pi)^2 \beta^2(\pi) - 1] / 8\pi\beta(\pi)v_c. \quad (7)$$

Here γ_c, δ_c are the coefficients of the Bogoliubov transformation at the Fermi points and depend on H ; as $H \rightarrow H_{c1}$, $\gamma_c, \delta_c \rightarrow 1/\sqrt{2}$. Note that the coupling Eq. (7) vanishes as $H \rightarrow H_{c1}$, justifying the linearization of the fermion spectrum even in this extreme limit.

Equation (7) changes sign at the temperature $T_d^{(1)} = \tau/4$, which remarkably is independent of H . When $T > T_d^{(1)}$, $\gamma_1(k, \xi)$ remains positive, suppressing a density-wave response (at least for ξ not too large, see below); this corresponds to uniform-density fermions, and in terms of fluxons to power-law decay for fluxon position correlations. When $T < T_d^{(1)}$, $\gamma_1(k, \xi)$ develops a negative dip at $k = \pi$ corresponding to a density wave, i.e., a triangular Abrikosov lattice for the fluxons.

For $l > 1$ the initial condition for the RG becomes

$\gamma_i(k, \xi_0) = \sum_{r=1}^l \gamma_i^0(k + 2\pi r/l)/l$, resulting in a density-wave transition for $2 \leq l \ll (\lambda_e/d)^{1/2}$ at

$$T_d^{(l)} = \tau\sqrt{l/8} = T_c^0 [1 + \sqrt{8/l} T_c^0/\tau_0]^{-1}. \quad (8)$$

In an l phase, fluxons penetrate only between groups of l neighboring layers; the layers in each group are phase correlated both above and below $T_d^{(l)}$ and can be considered as an effective layer. For $T < T_d^{(l)}$ fluxons have positional long-range order and all layers are correlated, while for $T > T_d^{(l)}$ fluxon fluctuations decouple the effective layers.

To appreciate this result I reconsider the vortex problem with $J = 0$ between effective layers. If vortices now order at $T_v^{(l)} > T_d^{(l)}$ then the separate fluxon and vortex schemes are consistent and a 2D regime exists. The elementary vortex excitations in an effective layer interact as $\sim l \ln(r)$ for $l \ll (\lambda_e/d)^{1/2}$, leading to a vortex transition at $T_v^{(l)} = \tau l/8$. Comparison with (8) shows a second apparition of a factor 8: For $l \leq 8$, $T_v^{(l)} \leq T_d^{(l)}$ and the transition must be a 3D one between $T_v^{(l)}$ and $T_d^{(l)}$ [as for the $H = 0$ case, it is near $T_d^{(l)}$ unless J is exponentially small]. In particular, the range $T > T_d^{(1)}$, which could correspond to the 2D phase proposed by Efetov [14], is disordered by vortices.

The only possibility of obtaining a 2D phase is for $l_c \geq l \geq 9$ in the range $\tau < T < T_c$; this range exists if $T_c \approx T_f > \tau$ and l_c is determined by $T_d^{(l)} < T_f$, i.e., $\sqrt{8/l} > 1 - \gamma J/T_f$ (Fig. 1). The 2D phases (shaded areas in Fig. 1) are bordered by KT-type transitions—a vortex transition into a normal state at high T and a fluxon transition into a 3D superconducting phase at lower T . Note that fluctuations in l should eliminate the sharp corners in Fig. 1.

The expectation based on the initial flow of $\gamma_1(\pi, \xi)$ is confirmed by numerical solutions of the RG equations [27] up to a large scale ξ_c exceeding typical system sizes ($\xi_c \approx 10^4 \xi_0$ for $\lambda_e/d \approx 10^3$). For longer scales and for $T > T_d^{(l)}$ a density wave with $k \neq \pi$ seems to develop, corresponding to a weakly coupled nontriangular flux lattice. The 2D phases are therefore at least 2D in the finite-size sense, similar to the case of the one-layer system [8].

Consider now experimental data. Controlling d in $\text{YBa}_2\text{Cu}_3\text{O}_7$ by adding $\text{PrBa}_2\text{Cu}_3\text{O}_7$ layers has shown [9–11] a considerable reduction of T_c , by a factor of ~ 4 . If T_c were near T_v such a drop could not be understood. In the present theory, when d increases J becomes exponentially small and T_c shifts from T_f to T_v ; fitting the results for the sharper transitions [11] of well separated 3, 4, and 8 Cu-O bilayers yields $\tau_0 = (1200 \text{ K}) \pm 30\%$, consistent with a direct estimate of τ_0 .

Assuming $\tau_0 \approx 1200 \text{ K}$ for CuO_2 layers in all the compounds, Eq. (4) yields $T_c^0 - T_f \approx 7(1 - \gamma\bar{J}_0) \text{ K}$ while $T_c^0 - T_v$ is a factor ~ 8 higher. Data on $V \sim I^{a(T)}$ determine the shift $T_c^0 - T_c$, which for $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$ is [7] $T_c^0 - T_c \approx 3 \text{ K}$ while for $\text{Tl}_2\text{Ba}_2\text{CaCu}_2\text{O}_8$ is [5] $T_c^0 - T_c \approx 1 \text{ K}$. This is consistent with T_c being close to T_f rather

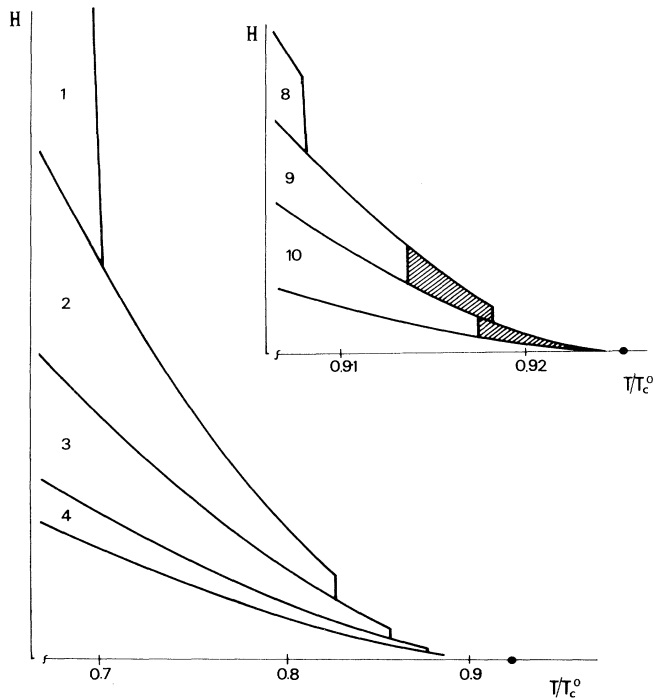


FIG. 1. H - T phase diagram showing the 3D transitions $T_d^{(l)}$ (vertical phase boundary lines) for $l=1-4$. Inset: 3D transitions $T_d^{(l)}$ and 2D transitions $T_c^{(l)}$ for $l=9,10$. The shaded areas are 2D phases with power-law I - V relation. The temperature scale corresponds to $T_c^0/\tau_0=0.1$ and $\gamma J/T_f=0.19$. $T_c/T_c^0 \approx T_f/T_c^0=0.925$ is marked by a dot.

than close to T_c and consistent with the observed vortices having $l > 8$.

I propose therefore that the relation $V \sim I^{a(T)}$ is observed in $l > 8$ phases due to the presence of small fields parallel to the layers; since $H_{c1} \rightarrow 0$ as $T \rightarrow T_c$ these fields are unavoidable. Furthermore, the observed dependence of $a(T)$ on fields perpendicular to the layers [6,7] can be due to a small (unintentional) component of these fields in the parallel direction. H then affects $a(T)$ since the latter is proportional to the effective layer thickness l , in qualitative agreement with experiment. Quantitatively, however, the data [6,7] seem to be fitted by $l \sim -\ln H$ while the mean-field decoupling of (6) gives $l \sim H^{-1}$ for $H \gtrsim 2H_{c1}$.

In conclusion, in this work experimental data have been understood and new insight into the nature of the superconducting transition has been gained.

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- [1] M. A. Dubson, S. T. Herbert, J. J. Calabrese, D. C. Harris, B. R. Patton, and J. C. Garland, *Phys. Rev. Lett.* **60**, 1061 (1988).
- [2] P. C. E. Stamp, L. Forro, and C. Ayache, *Phys. Rev. B* **38**, 2847 (1988).
- [3] N. C. Yeh and C. C. Tsuei, *Phys. Rev. B* **39**, 9708 (1989).
- [4] S. Martin, A. T. Fiory, R. M. Fleming, G. P. Espinosa, and A. S. Cooper, *Phys. Rev. Lett.* **62**, 677 (1989).
- [5] D. H. Kim, A. M. Goldman, J. H. Kang, and R. T. Kampwirth, *Phys. Rev. B* **40**, 8834 (1989).
- [6] M. Ban, T. Ichiguchi, and T. Onogi, *Phys. Rev. B* **40**, 4419 (1989); T. Onogi, T. Ichiguchi, and T. Aida, *Solid State Commun.* **69**, 991 (1989).
- [7] S. N. Artemenko, I. G. Gorlova, and Yu. I. Latyshev, *Phys. Lett. A* **138**, 428 (1989).
- [8] B. I. Halperin and D. R. Nelson, *J. Low Temp. Phys.* **36**, 599 (1979).
- [9] J. M. Triscone, Ø. Fisher, O. Brunner, L. Antognazza, A. D. Kent, and M. G. Karkut, *Phys. Rev. Lett.* **64**, 804 (1990).
- [10] Q. Li, X. X. Xi, X. D. Wu, A. Inam, S. Vadlamannati, W. L. Mclean, T. Venkatesan, R. Ramesh, D. M. Hwang, J. A. Martinez, and L. Nazar, *Phys. Rev. Lett.* **64**, 3086 (1990).
- [11] D. H. Lowndes, D. P. Norton, and J. D. Budai, *Phys. Rev. Lett.* **65**, 1160 (1990).
- [12] W. E. Lawrence and S. Doniach, in *Proceedings of the Twelfth Conference on Low Temperature Physics, Kyoto, 1970*, edited by E. Kanda (Keigaku, Tokyo, 1970), p. 361.
- [13] L. N. Bulaevskii, *Usp. Fiz. Nauk* **116**, 449 (1975) [*Sov. Phys. Usp.* **18**, 514 (1976)].
- [14] K. B. Efetov, *Zh. Eksp. Teor. Fiz.* **76**, 1781 (1979) [*Sov. Phys. JETP* **49**, 905 (1979)].
- [15] S. N. Artemenko and A. N. Kruglov, *Phys. Lett. A* **143**, 485 (1990).
- [16] A. Buzdin and D. Feinberg, *J. Phys. (Paris)* **51**, 1971 (1990).
- [17] L. N. Bulaevskii, D. Feinberg, and S. V. Meshkov (to be published).
- [18] D. Browne and B. Horovitz, *Phys. Rev. Lett.* **61**, 1259 (1988); B. Horovitz, *Physica (Amsterdam)* **165-166B**, 1109 (1990).
- [19] J. Friedel, *J. Phys. Condens. Matter* **1**, 7757 (1989).
- [20] S. E. Korshunov, *Europhys. Lett.* **11**, 757 (1990).
- [21] B. Horovitz (to be published).
- [22] For a review, see J. Kogut, *Rev. Mod. Phys.* **51**, 696 (1979).
- [23] V. L. Pokrovskii and A. L. Talanov, *Zh. Eksp. Teor. Fiz.* **78**, 269 (1980) [*Sov. Phys. JETP* **51**, 134 (1980)].
- [24] H. J. Schulz, *Phys. Rev. B* **22**, 5274 (1980).
- [25] B. Horovitz, T. Bohr, J. M. Kosterlitz, and H. J. Schulz, *Phys. Rev. B* **28**, 6596 (1983).
- [26] L. P. Gorkov and I. E. Dzyaloshinskii, *Zh. Eksp. Teor. Fiz.* **67**, 397 (1974) [*Sov. Phys. JETP* **40**, 198 (1975)].
- [27] L. Mihály and J. Sólyom, *J. Low Temp. Phys.* **24**, 579 (1976).
- [28] P. A. Lee, T. M. Rice, and R. A. Klemm, *Phys. Rev. B* **15**, 2984 (1977).