# Experimental Study of Freely Decaying Two-Dimensional Turbulence

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Freely evolving two-dimensional turbulence is studied experimentally, in thin layers of electrolyte. The density of vortices, their mean sizes, and the average separation between them are measured during the decay phase. Power laws are observed, and the corresponding exponents are found in good agreement with a recent analysis of Carnevale *et al.*

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The problem of two-dimensional turbulence has attracted much attention for twenty years [1–4]; recently, new approaches to this problem have been undertaken: Statistical theories of two-dimensional inviscid turbulence, respecting all conservation laws, have been formulated [5], and new scaling laws for the problem of the evolution towards equilibrium have been proposed [6]. In this Letter, we describe an experimental study of the decay of two-dimensional turbulence, and present a determination of the scaling laws characterizing the geometry of the coherent structures; the results provide strong support to the theory mentioned above (Ref. [6]), and in contrast disagree with previous arguments based on the existence of a single invariant [1], or using an analogy with the kinetic theory of colloidal aggregation [7].

The experimental system is schematically represented in Fig. 1: The flow region takes place in the central part of a container, machined in PVC, of overall dimensions $18\times12\times4.5$ cm; this region is surrounded by two reservoirs, in which two electrodes are immersed. The container is filled with a normal solution of sulfuric acid and, in the central part, the thickness $b$ of the fluid layer is a few millimeters (for the present experiments, the imposed values of $b$ range between 2.5 and 4 mm); in the central region, the flow is confined by barriers, one insulating and two conducting. An electric current $I$ is driven from one electrode to the other; owing to the geometry of the cell, the spatial inhomogeneities of the electric current, in the flow region, are less than $10^{-3}$ in relative value. Just below the flow domain, a two-dimensional array of permanent magnets of alternated poles is formed. Each individual magnet is a samarium cobalt parallelepiped, of dimensions $5\times8\times3$ mm, whose maximum induction is 0.32 T. The interaction of the periodic magnetic field with the electric current produces a system of recirculating flows, which, at low currents, takes the form of a regular array of counter-rotating vortices. In the present study, the dimensions of such vortices are $8\times8$ mm, and we consider systems of $6\times6$ and $10\times10$ vortices. The dynamics of the system is two dimensional on scales much larger than $b$; this represents, in the present experiment, a domain ranging from a few millimeters to more than 5 cm, i.e., more than a decade of variation. The flow is visualized by using neutrally buoyant particles, several tens of microns in size. The particles are made visible, at several levels in the fluid layer, by a sheet of light, produced by an 80-mW argon laser. The images of the flow are taken from above, with a video camera. Averages over several frames are usually performed to reveal the structure of the flow.

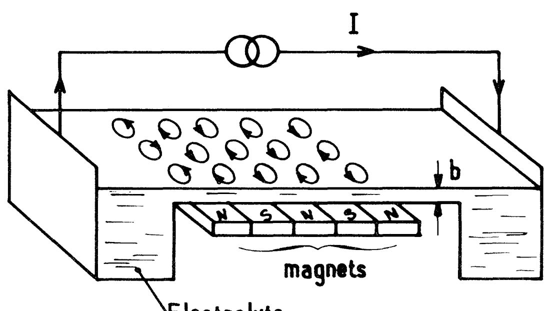


FIG. 1. Schematic representation of the experimental system (not to scale); the walls confining the flow and the lateral limits of the cell are not represented.

The transition to turbulence in this system involves a complicated sequence of bifurcations which we do not discuss here; the regimes which are considered herein correspond to values of $I$ more than 10 times above the onset of the first bifurcation. The corresponding values of the large-scale Reynolds number (calculated on the size of the box) lie between 600 and 2400.

A first series of experiments consists in imposing a steady value of $I$, quenching the system, and following its decay. In this case, the initial regime is a disordered state, where a broad band of wave numbers is excited. Another procedure which has been used consists in increasing rapidly the electric current from zero, holding it for a short time, and then quenching the system. In this case, the initial state is a regular lattice of vortices and there is essentially a single excited wave number. Both initial conditions have been considered. Figure 2 shows one image of the flow taken during the decay phase from an ordered state of 100 vortices, produced after a sudden increase to $I=200$ mA. After the current is quenched, the system undergoes a rapid sequence of mergings of

 

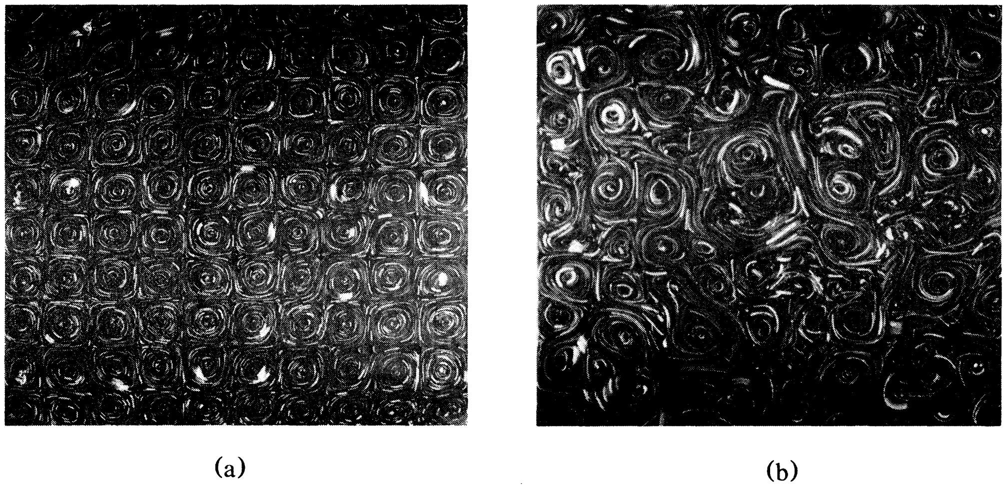


FIG. 2. Freely decaying turbulence from an initially ordered square lattice of 100 vortices, obtained at $I=200$ mA, with a layer thickness $b=3$ mm. The size of the system, 80 mm×80 mm, gives the scale of the figures. (a) Initial state at $t=0$; (b) $t\approx 0.5$ s.

vortices of the same sign until finally, a few large vortices are formed; the latter further decay without any significant deformation. The first period takes less than 2 s, whereas the typical duration of the final decay regime of the large eddies is 5 s. Similar sequences of events are observed under a wide range of experimental conditions. The final state is a system of a few large eddies, whose number ranges between 1 and 4.

Merging generally occurs when two vortices of the same sign come close to each other: If this happens, a large eddy, embedding the pair, is generated; its amplitude increases in time at the expense of that of the initial vortices, until finally there remains a single vortex. This process thus evokes a large-scale instability. We generally have merging of pairs, and less frequently of triplets. Merging takes typically 0.3 s to be achieved. The further evolution of the new eddy is generally an axisymmetrization. Apart from merging, the major events seem to be propagations of dipoles, mutual advection of vortices, and eddy deformations. We have not observed triple configurations. The events which we find are thus similar to those previously observed, both in experiments [3] and in numerical simulations [2].

The various time scales of the system can be discussed by using Fig. 3: On the same graph, we plot the decay of the spatially averaged velocity $U$, measured at the peripheries of the eddies by using a particle tracking technique, and a quantity representing the number of vortices $N$. $U^2$ gives some information about the amount of kinetic energy present in the system. We find that the velocity decreases over a time scale $\tau_v$ of about 4 s; this value roughly corresponds to the viscous time $b^2/3\nu$, calculated by assuming a Poiseuilles profile across the fluid layer (such an expression leads to a decay time of $4\pm 1$ s in this case). On such a time scale, the merging can be considered as instantaneous, so that, to a good approximation, the cascade follows adiabatically the slow viscous damping of the system. Figure 3 also indicates that the sequence of mergings is achieved on a time scale significantly smaller than $\tau_v$: After 0.7 s, the number of vortices present in the system has decreased by 50% whereas the velocity is still at 90% of its initial value (this would represent a level of 80% in the kinetic energy per unit of mass, if the latter is estimated as the square of $U$); a simplified (and rough) picture of the experiment is therefore that the entire cascade develops at constant energy.

We have measured, independently, the evolution of three quantities during the decay phase: the average separation distance $r(t)$ between the vortex centers, the vor-

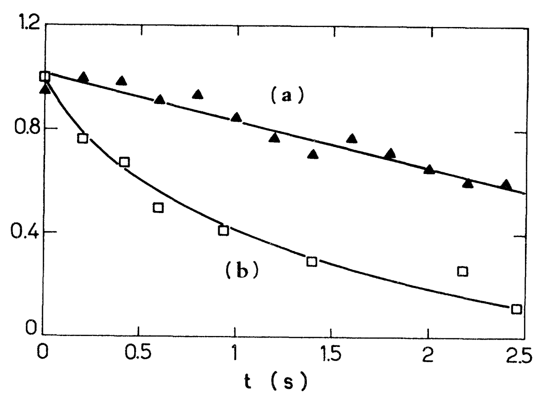


FIG. 3. Temporal evolution of the spatially averaged velocity $U$ (plot $a$) and quantity $N-N_f$ (plot $b$), during the decay phase of flow, with $I=200$ mA and $b=3$ mm (here, $N_f$ is the final number of vortices). All quantities have been renormalized.

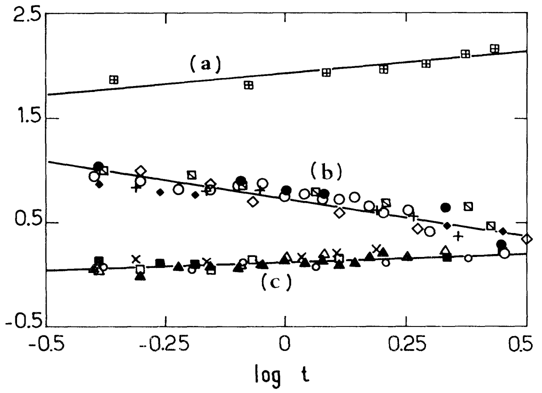


FIG. 4. Temporal evolutions of the separation distance $r(t)$ (plot $a$), rescaled density $I^{0.4}\rho(t)$ (plot $b$), and vortex size $a(t)$ (plot $c$), for various experimental conditions, which are, respectively, ⊞,♦,■: $I=200$ mA, 6×6 array, $b=2.5$ mm (initially disordered); ●,△: $I=400$ mA, 10×10 array (initially disordered); +,×: $I=500$ mA, 6×6 array, $b=4$ mm (initially disordered); ◇,□: $I=800$ mA, 6×6 array, $b=2.5$ mm (initially disordered); ⧅,○: $I=200$ mA, 6×6 array, $b=2.5$ mm (initially ordered); ○,▲: $I=800$ mA, 6×6 array (initially ordered).

tex density $\rho(t)=N(t)/S$ (where $S$ is the surface of the cell), and their mean radius $a(t)$. Such measurements (except the first one) have been performed for three values of the thickness $b$, various initial conditions (the initial values of $I$ range from 200 to 800 mA, and the states of flow are produced according to the two procedures described above), and systems of different sizes (36 and 100 vortices). All the results are collected in Fig. 4. For $r(t)$ and $a(t)$, the data are plotted directly whereas for $\rho(t)$, we introduce a prefactor $I^{0.4}$, in order to obtain reasonable collapse of the data. For all these quantities, we obtain power laws throughout almost one decade in time. The various exponents that we find are the following:

$$r(t)\sim t^{0.4\pm0.1},\quad \rho(t)\sim I^{-0.4\pm0.1}t^{-0.7\pm0.1},\quad a(t)\sim t^{0.25\pm0.1}. \tag{1}$$

It is of interest to compare these results with the predictions of Carnevale *et al.* [6]. These authors assumed the existence of two conserved quantities during the decay phase, the spatially averaged kinetic energy $E$, and the extremum vorticity $\zeta_{\rm ext}$. They found power laws for the evolutions of $r(t)$, $\rho(t)$, and $a(t)$, in the form

$$r(t)\sim E^{1/2}\zeta_{\rm ext}^{\xi/2-1}t^{\xi/2},\quad \rho(t)\sim E^{-1}\zeta_{\rm ext}^{2-\xi}t^{-\xi},\quad a(t)\sim E^{1/2}\zeta_{\rm ext}^{\xi/4-1}t^{\xi/4}, \tag{2}$$

where $\xi\approx0.75$. The predicted exponents concerning the temporal evolution of the system are thus in good agreement with the experimental values shown above. The power laws given by the theory are plotted in Fig. 4: Good agreement with our data is clear. Concerning now the prefactors, the theory is found to be consistent with our results, if one assumes that $\zeta_{\rm ext}$ scales as $I^{1/2}$ and $E$ is a linear function of $I$ (such approximations, which are reasonable at large Reynolds numbers, have also been checked experimentally). The agreement between theory and experiment can therefore be viewed as satisfactory. The fact that energy and extremum vorticity slowly decay while the inverse cascade proceeds (as indicated in Fig. 3) does not appear to have significant influence on the applicability of the theory [8].

One can also compare our results with those obtained under the assumption that $E$ is the single invariant of the system as the viscosity decreases to zero [1]. The corresponding predictions, which are

$$r(t)\sim E^{1/2}t,\quad \rho(t)\sim E^{-1}t^{-2},\quad a(t)\sim E^{1/2}t, \tag{3}$$

are in disagreement with our results. A similar statement holds for the theoretical approach based on an analogy with colloidal formation [7], which leads to the conclusion that $\rho\sim t^{-1}$.

In conclusion, although we have restricted ourselves to the study of the geometry of coherent structures, our results already provide strong support to the validity of the predictions of Carnevale *et al.* [6]. This is the first time that an experiment establishes convincingly the existence of power laws for free-decaying two-dimensional turbulence. The fact that the energy is not strictly constant as the cascade proceeds does not seem to invalidate the conclusions of the theory; another interesting feature is that the three dimensionality of the small scales does not appear to affect the dynamics of the inverse cascade. This is encouraging for the problem of applying two-dimensional theories to the real world.

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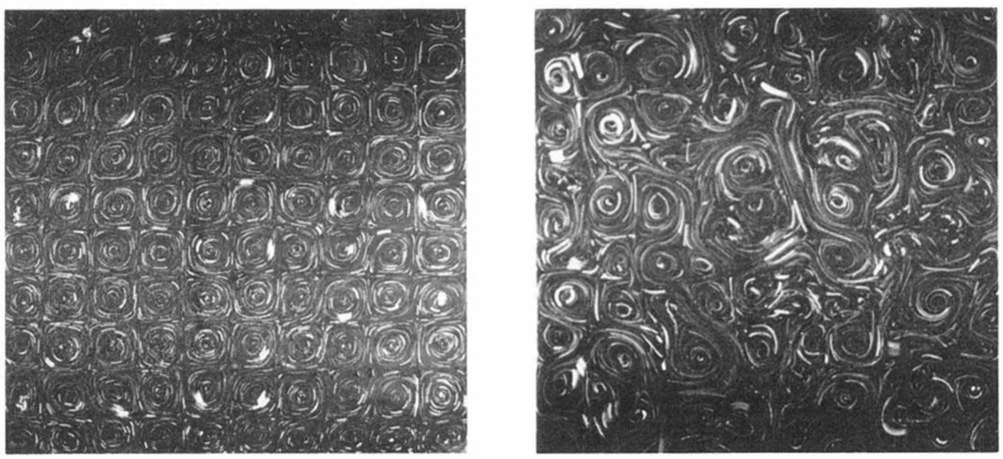
[8] One can introduce in (2) exponentially decaying functions, in order to estimate the effect of the dissipation. Actually, with decay rates of 4 s for $\zeta_{ext}$ and 2 s for $E$, the resulting changes on the predicted values are found to lie within the experimental error.



(a) (b)

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