Probability Distribution of a Passive Scalar in Grid-Generated Turbulence

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The probability distribution of passive temperature fluctuations is experimentally studied in grid-generated wind tunnel turbulence. It is shown that exponential tails occur for a mean temperature gradient but do not in the absence of a mean gradient. The velocity probability distribution is Gaussian.

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In order to develop theories of turbulence, accurate measurement of the probability distribution function (PDF), which defines all the moments of the fluctuations, is required. In simple homogeneous turbulent flows the velocity PDF is Gaussian (see below) and it is generally assumed that the PDF of a passive scalar, advected by the turbulence, is also Gaussian [1]. Experiments [2] show this to be approximately the case but the tails of the PDF, which describe the higher-order moments, have not been examined. Our interest in the subject stems from the Chicago convection experiments [3] which showed that at low Rayleigh numbers (Ra) the fluctuations of temperature have a Gaussian PDF but at $\mathrm{Ra} \sim 4\times 10^7$ the tails of the distribution become exponential in the so-called "hard-turbulence" regime. The structure of that flow is complex, consisting of thermal plumes and therefore large-scale intermittency, so a non-Gaussian PDF could be expected. What is remarkable is the universality of the shape and the fact that the tails are exponential, rather than of a different (non-Gaussian) form. An attempt to explain the PDF has been made in terms of the structure of the convective motion itself [3], yet the experiment has prompted Pumir, Shraiman, and Siggia [4] to ask whether exponential tails of scalar fluctuations exist under more general conditions. Using a one-dimensional model they predict they will occur for a passive scalar subject to a mean gradient. This is an interesting result since Kraichnan [5] has shown that linear theory, for a Gaussian velocity PDF and a linear mean scalar gradient, implies Gaussian tails for the scalar. Calculations using Kerstein's linear model [6] also show an exponential PDF for a mean scalar gradient but recent direct computer simulations [7] find exponential tails in the absence of a mean gradient. Clearly the matter deserves experimental attention and this is the objective of the present work.

Here we examine passive temperature fluctuations, both with and without a mean gradient, in grid turbulence [8,9], the simplest form of laboratory-generated turbulence; it is free of the large-scale coherent structures that occur in free shear flows and convection [10,11]. The velocity fluctuations are close to isotropic [9] and their PDF is Gaussian (Ref. [8] and below). The flow is generated by passing undisturbed (laminar) flow through a square mesh grid which produces isotropic turbulence (by about 30 mesh lengths, M, from the grid) which thereafter decays. Measurements usually extend to a few hundred M and the width of the wind tunnel is many M wide so that there is homogeneity in the transverse direction and wall effects are minimized. Because of its simplicity it has been the experimental basis for statistical theories of turbulence [8,10]. It also has the advantage of being relatively easy to measure, with both temperature and velocity fluctuations resolved well into the dissipation range [9,12]. Its disadvantage is the complexity of the boundary conditions (unlike the convection experiment it is an open system with very complex flow at the grid) and its susceptibility to external disturbance. The Reynolds number is also relatively low (with limited range variation) although it exhibits many of the characteristics of high-Reynolds-number turbulence, including intermittency at the small scales [1,8].

In the present work, two different wind tunnels, both of the same open circuit design, are used. Their test sections are $0.41\times 0.41\ \mathrm{m}^2$ and 4.4 m long and $0.91\times 0.91\ \mathrm{m}^2$ and 9.1 m long. They are described in detail in Refs. [12] and [13]. M was varied from 0.86 to 10.16 cm in the various experiments. In order to produce a mean cross-stream temperature gradient a set of parallel heating ribbons, *a toaster*, was placed at the entrance to the settling chamber of the tunnel and differentially heated [12,13]. The disturbances caused by the toaster are smoothed out in the screened plenum and contraction so that by the time the flow reaches the grid it is laminar. Thus the temperature fluctuations are generated by the action of the grid turbulence against the mean temperature gradient. Temperature fluctuations in the absence of a mean gradient were generated by a *mandoline*, fine heated wires, equally spaced, downstream of the grid [14]. The thermal wakes (the wires are too thin to disturb the flow) coalesce [15], forming temperature fluctuations that are homogeneous in the transverse direction and decay longitudinally. For all experiments the thermal field was shown to be passive by determining that the buoyancy term in the energy equation was negligible. Longitudinal velocity fluctuations (u) and temperature fluctuations (θ) were measured with a 4- and a 1.27-μm-diam hot wire and resistance wire, respectively [12–15]. The data were digitized with 12-bit resolution and 4×10^5 data points were used to determine spectra and PDF's. The specifics of the flow conditions are described with their results.

Figure 1 shows the longitudinal velocity PDF and the velocity and temperature spectra $196M$ from the grid in the 9.1-m tunnel with a passive cross-stream mean temperature gradient, $dT/dy = 5.04$ K/m, generated by the toaster. $M = 2.54$ cm and the mean speed (U) was 6.19

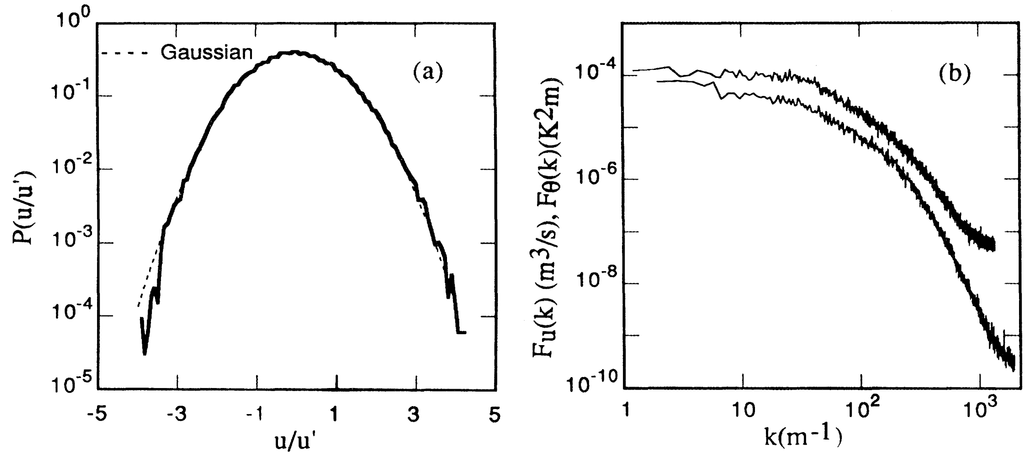

FIG. 1. (a) PDF of velocity normalized by the rms at $x/M=196$ for the 2.54-cm grid in the 9.1-m tunnel. $Re_l=70$. (b) Velocity (lower curve) and temperature spectra for the same conditions as (a) as a function of wave number $k=2\pi f/U$. The mean temperature gradient is 5.04 K/m. Same vertical scale applies to both spectra.

m/s. The frequency spectrum has been converted to a wave-number spectrum using the frozen flow relation [9] $k=2\pi f/U$, where f is the frequency (Hz) and the PDF has been normalized by the rms of the velocity, u'. The PDF is Gaussian and this is typical of all the velocity PDF's measured. The integral scale l of the velocity spectrum occurs at $k_l=35.4\ \mathrm{m}^{-1}$ ($l=1/k_l=2.8$ cm) and the peak of the dissipation spectrum [14] is at $k_\eta=250\ \mathrm{m}^{-1}$ ($l_\eta=4.0$ mm). The velocity variance decay was found to follow the law $\overline{u^2}/U^2=0.0673(x/M)^{-1.3}$, where x is the distance from the grid. Note that in this relatively low-Reynolds-number turbulence an inertial subrange ($-\frac{5}{3}$ power law) does not occur. The turbulence Reynolds number $Re_l \equiv lu'/\nu$ varied from 115 to 70 from $x/M=36$ to 196, respectively, and the Taylor Reynolds number $Re_\lambda \equiv \lambda u'/\nu$ varied from 42 to 33. Here ν is the kinematic viscosity and λ is the Taylor microscale [10]. The mesh Reynolds number $Re_M \equiv UM/\nu$ was 10000. The peak of the temperature dissipation spectrum occurs at $k=406\ \mathrm{m}^{-1}$ and the thermal integral scale l_θ is 2.5 cm. Note that l and l_θ are of the order of M.

Figure 2(a) shows the temperature PDF's, normalized by the temperature rms, θ', for the conditions of Fig. 1. They are clearly non-Gaussian showing exponential tails, and this is our principal finding. Close to the grid ($x/M=116$) the PDF is skewed, reflecting that it has taken a few eddy turnover lengths $[(l/u')U]$ for the effect of initial conditions to disappear for this particular experiment, although PDF's observed (for similar conditions) in the 4.4-m tunnel showed no initial skewness, suggesting it is not germane to the PDF evolution. The kurtosis $[\equiv\overline{\theta^4}/(\overline{\theta^2})^2]$ is 4.41, 4.26, and 3.85 at $x/M=116$, 196, and 216, respectively. For a Gaussian distribution it is 3, and for a purely exponential distribution it is 6. Note that the kurtosis is slowly decreasing and this was a characteristic of other experiments done with different values of U, dT/dy, and M. The PDF's are similar to those observed in the Chicago experiment, at least for the low-Ra range of the hard-turbulence regime. We surmise that the exponential tails would become longer if our

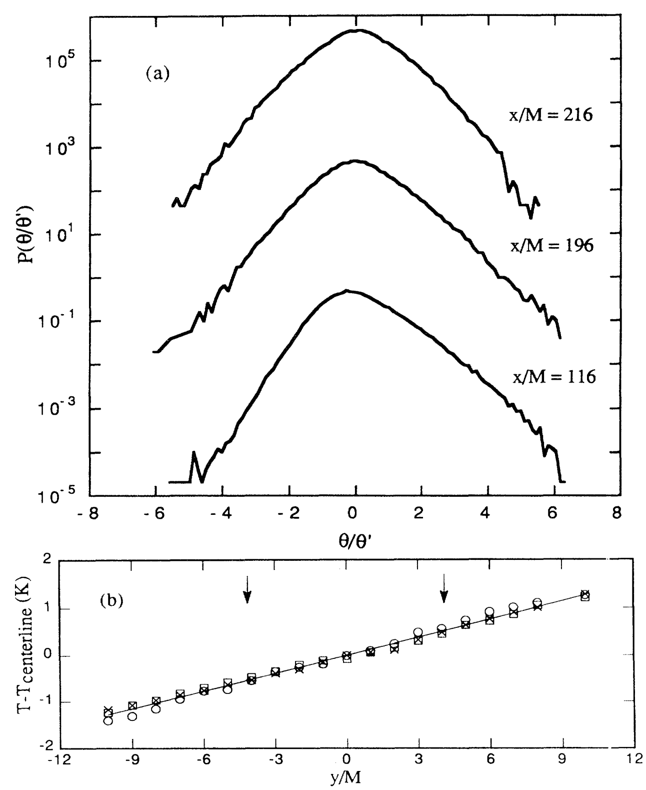

FIG. 2. (a) PDF of temperature for the same flow conditions as Fig. 1, at various x/M. The upper two PDF's have been shifted 3 and 6 decades, respectively, with respect to the lowest one. (b) The mean cross-stream temperature profiles at $x/M=40$ (○), $x/M=120$ (□), and $x/M=240$ (×). Same conditions as (a). The arrows show ± 6 temperature standard deviations (at $x/M=196$).

Reynolds number could be significantly increased. Note that these tails are due to rare, large-amplitude fluctuations. There was concern that they may be affected by the (non-Gaussian) small-scale (internal intermittency) structure since the separation of scales is relatively small (one decade). To check this we low-pass filtered the time series [using a finite impulse response (FIR) digital filter], thereby cutting off the dissipation range, but the exponential tails still remained. We further note that if the non-Gaussian small scales were affecting the PDF, this would also have been reflected in the tails of the velocity PDF, but as we have shown they are Gaussian (and remained so with low-pass filtering).

The mean temperature profiles [Fig. 2(b)] show that there are small departures from linearity (of the order of the rms) in the core of the flow which extends $20M$ in the transverse direction (the tunnel width was $36M$ for these measurements). It is unlikely that these small departures (which varied with x/M, and slowly over time) can be reduced in a tunnel such as this. They do not affect the growth rate of the temperature variance [16] and we suspect that since they are random, if anything they would tend to make the PDF more Gaussian. For the case without a mean gradient (see below) similar departures in the mean were also observed but there the PDF was close to Gaussian indicating that the mean gradient (and not small departures from it) determines the nature of the PDF. Also shown on the profiles are arrows 6 standard temperature deviations (the width of the PDF) from the center of the tunnel where the measurements were made. The excursion of the fluctuations [determined as $(6\theta'/M)/(dT/dy)$] corresponds to $\pm 4y/M$ (at $x/M = 196$). This suggests that the wall boundary layers were not causing interference in this case.

The parameters M, U, and dT/dy were varied in an effort to determine their effects on the PDF. The results showed that while on most occasions exponential tails were present, on some they were not. The reason for this appears to be contamination of the very low frequencies (see below). However, all measurements with a linear profile yielded exponential tails when high-pass filtered (with the one exception of a very-low-Re case where the integral scale was only 3 times the dissipation scale). Since high-pass filtering places more emphasis on the small scales, the data were also low-pass filtered to cut off the dissipation scales, as discussed above.

Figure 3 shows such a bandpass-filtered PDF of temperature and velocity for a 5.0-cm mesh grid with $U = 16.24$ m/s and $dT/dy = 2.8$ K/m, measured in the 4.4-m tunnel at $x/M = 51.2$. Re_l is 968, somewhat higher than for the data of Figs. 1 and 2. The bandpass wavenumber range was from 38.7 to 212.8 m^{-1}. The peak of the energy and the dissipation spectra occurred at 24.4 and 572.3 m^{-1}, respectively; thus the PDF covers the frequency range (of approximately a decade) between the integral and dissipation scales. The temperature PDF has an exponential character similar to the unfiltered PDF (Fig. 2). However, the velocity PDF remains Gaussian (using a similar bandpass filtering procedure) providing confidence that the non-Gaussian nature of the temperature PDF is not an artifact of the filtering, which reduced the unfiltered variance by about half in both cases. It is interesting that the exponential tails of the temperature PDF exist even in the absence of the largest scales (those greater than l_θ), indicating that the linear mean gradient affects the intermediate scales also. We emphasize that the dissipation scales (which are known to be non-Gaussian) have been excluded by the low-pass filtering. Other data yielded results similar to Fig. 3. We suspect that the high-pass filtering was necessary to remove contamination from the wall boundary layer which might obscure the tails of the PDF. Note that for the data of Fig. 3 the tunnel was only $8M$ wide compared to $36M$ for the data of Fig. 2, suggesting the likelihood of wall influence. Another experiment using a 10.16-cm mesh grid in the 9.1-m tunnel ($\mathrm{Re}_l = 860$) also required filtering. Here the tunnel was $9M$ wide.

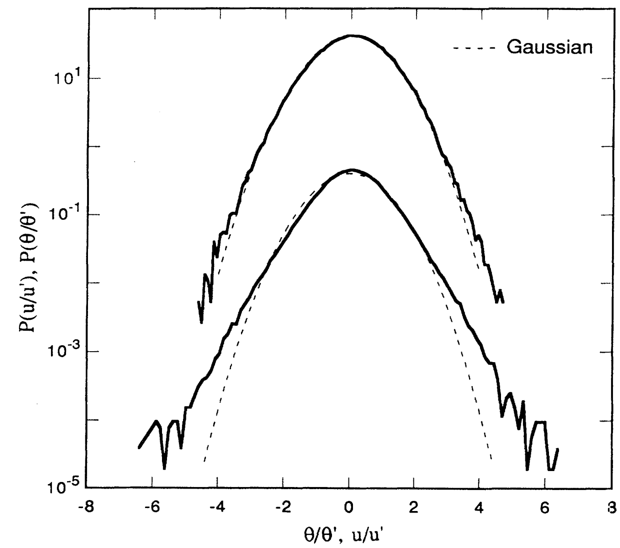

FIG. 3. Velocity (upper curve) and temperature PDF's at $x/M = 51.2$ in the 4.4-m tunnel. $M = 5$ cm, $dT/dy = 2.8$ K/m, and $\mathrm{Re}_l = 968$. Both PDF's are bandpass filtered (see text). The upper curve has been shifted 2 decades with respect to the lower.

On the other hand, when temperature fluctuations were generated without a mean temperature gradient, we did not observe exponential tails even if filtering was employed. Figure 4 shows an unfiltered temperature PDF generated by a mandoline [14]. The spacing of the mandoline wires was $1M$ and it was placed $10M$ from the grid in the 4.4-m tunnel. M and Re_l were respectively 2.5 and 145 cm. The measurement was $92.4M$. Here the tails of the PDF are slightly narrower than a Gaussian (of equal area and variance) rather than wider, which was observed for the linear profile cases (Figs. 2 and 3). The kurtosis was 2.9 and this was typical for other measurements at

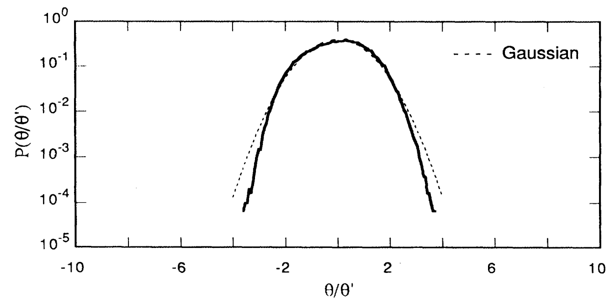

FIG. 4. PDF of temperature in the absence of a mean gradient at $x/M=92.4$ in the 4.4-m tunnel. $Re_l=145$.

different Reynolds numbers and downstream distances (the average kurtosis for all trials was 2.8). Previously published data [17] and numerical simulations [18] also show a kurtosis close to 3 for temperature fluctuations in the absence of a mean gradient. Finally, we note that the opposite extreme, that of a strongly nonlinear temperature gradient formed by a thermal mixing layer [19,20], also yields a kurtosis slightly lower than 3, suggesting here too that there will be no exponential tails.

In summary, our findings are that for a mean temperature gradient in grid turbulence the tails of the temperature PDF are exponential over the observed range $70 < Re_l < 1000$ $(10000 < Re_M < 50000)$. In some instances high-pass filtering was necessary to reveal the exponential nature of the distribution, probably because the very low frequencies were contaminated by wall effects, masking the relatively rare events that contribute to the tails of the PDF. Concurrent measurements of the velocity PDF were Gaussian, whether they were filtered or not. The temperature PDF's are similar to those of the Chicago experiment [3] (for which no velocity statistics are available) and are consistent with recent theory [4]. On the other hand, exponential tails were not observed in the absence of a mean gradient. The results presented here appear to be the first experiments to show universal tendencies for the PDF of passive scalar fluctuations in a simple turbulent flow.

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[1] We note that the PDF of the *derivative* of the turbulent fluctuations (which pertains to the small-scale, dissipative region of the flow) are definitely non-Gaussian due to internal intermittency and this has been extensively discussed in the literature [A. Monin and A. Yaglom, *Statistical Fluid Dynamics* (MIT Press, Cambridge, 1971)]. For the same reason, the PDF's of the derivative of passive scalar fluctuations are also non-Gaussian [R. Antonia, E. Hopfinger, Y. Gagne, and F. Anselmet, Phys. Rev. A **30**, 2704 (1984)]. In the present work it is the PDF of the scalar fluctuations themselves, which relates to the overall flow structure, that is of concern, and not their derivative.

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