Axial-Charge Transitions in Heavy Nuclei and In-Medium Effective Chiral Lagrangians

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It is shown that the recently reported strong enhancement of the axial-charge matrix element in first-forbidden β decay in heavy nuclei ($A=205$–212) can be simply explained in terms of an effective Lagrangian that incorporates approximate chiral and scale invariances of QCD. We suggest this as an evidence for the scaling property of hadrons in dense medium as predicted by effective chiral Lagrangians that are consistent with the symmetries of QCD.

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In a recent Letter, Warburton [1] analyzed first-forbidden β decay in $A=205$–212 nuclei and arrived at an intriguing result, that the axial-charge matrix element in heavy nuclei is enhanced over the impulse approximation by about 100%, with the extracted enhancement factor being

$$\epsilon_{\mathrm{MEC}}=2.01\pm0.05\,, \tag{1}$$

where MEC denotes meson exchange currents. This is a considerably stronger enhancement than what was anticipated theoretically in 1978 ($\sim$50%) [2] and calculated more recently [3]. In this Letter, we offer a very simple explanation of this large enhancement based on an in-medium effective chiral Lagrangian [4] and the exchange-current operator derived therefrom along the line developed in [2], which has recently been justified [5] by means of the Weinberg expansion [6]. For the density corresponding to that of nuclear matter, we predict $\epsilon_{\mathrm{MEC}}=2.0\pm0.2$.

Our argument relies on two recent developments in implementing chiral symmetry in nuclear physics: first, the construction of an effective Lagrangian appropriate for nonzero baryon density consistent with the symmetries of QCD, and second, a consistent chiral expansion with a given in-medium effective chiral Lagrangian. We start with the first.

It was shown in [4] that incorporation of approximate chiral and scale invariances of QCD leads to a chiral Lagrangian of low-energy hadrons in which masses of the hadrons are universally scaled as a function of the matter density ρ,

$$\frac{m_N^*}{m_N}\approx\frac{m_\sigma^*}{m_\sigma}\approx\frac{m_\rho^*}{m_\rho}\approx\frac{m_\omega^*}{m_\omega}\approx\frac{f_\pi^*}{f_\pi}\equiv\Phi(\rho)\,. \tag{2}$$

Here the density dependence is indicated by the asterisk; the symbols without asterisks denote values at zero density. The f_π is a constant related to the pion-decay constant and the subscripts on the masses label the hadrons involved. (The σ is the effective scalar meson of mass $\sim$560 MeV needed in nuclear physics.) In establishing this scaling relation, implementing the trace anomaly of QCD in effective chiral Lagrangians turns out to play a crucial role [7]. The point is that in QCD, the divergence of the dilatation current $\partial_\mu D^\mu$ or, equivalently, the trace of the energy-momentum tensor θ_μ^μ is—apart from the mass term and its anomalous dimension term—equal to $-[\beta(g)/g]\mathrm{Tr}G_{\mu\nu}G^{\mu\nu}$, where $G_{\mu\nu}$ is the gluon field tensor. This can be conveniently implemented by defining an effective scalar field χ,

$$\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}]\sim\chi^4\,. \tag{3}$$

If we define, for any density, the condensate of the scalar field as $\chi_*=\langle0^*|\chi|0^*\rangle$, then the scaling property of the chiral Lagrangian dictates, *at least at small density*, a prescribed dependence on the condensate in medium χ_*, and the redefinition of the pion-decay constant by $f_\pi^*=f_\pi\chi_*/\chi_0$, combined with low-energy relations (e.g., Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation, Weinberg spectral-sum rule, etc.), leads immediately to Eq. (2).

We note that within the scheme adopted in Ref. [4], the pion—and Goldstone bosons in general—scale less rapidly than (2): The scaling predicted at the tree order is $(m_\pi^*/m_\pi)^2\approx\Phi(\rho)$. In fact, experiments indicate little or no scaling of the pion mass in nuclear medium [8] at least at low temperature. [At high temperature, the situation can be entirely different, the pion mass falling even faster than (2) at temperatures reached in relativistic heavy-ion collisions. See [8] for discussions on this matter.] In this paper, we will assume that the pion mass remains unmodified for the range of densities involved.

One can also make definite statements, using the same argument, on the property of other constants of the chiral theory (other than f_π). One can readily see that *at the tree level* of the effective chiral Lagrangian, the axial coupling constant g_A or, equivalently, the gauge coupling g_{VNN} of the vector mesons and the πN coupling $g_{\pi NN}$ remain unchanged in medium. They are renormalized, however, by loop corrections as we will elaborate later.

How to calculate the axial-charge transition matrix elements, given a chiral Lagrangian, was described in Ref. [2]. It was recently shown [5] by means of the Weinberg expansion [6] that to the leading order in chiral expansion only one diagram involving a single soft-pion exchange dominates the exchange axial current amplitude, with short-ranged heavy-meson exchanges and loop contributions suppressed by the higher order of the chiral expansion parameter, justifying the hitherto *ad hoc* notion of the "chiral filter" mechanism used in [2]. It is appealing to consider this as a manifestation of a nonperturbative aspect of QCD. The upshot in the context of the in-medium chiral Lagrangian is that we obtain precisely the same axial-charge one-body and two-body operators as in [2] *with the relevant hadron masses and the pion-decay constant replaced by the in-medium quantities.* Thus we have

$$\begin{aligned} A_0^{\pm} &= A_0^{(1)\pm} + A_0^{(2)\pm}, \\ A_0^{(1)\pm} &= -g_A \sum_i \tau_i^{\pm} (\sigma_i \cdot \mathbf{p}_i / m_N^*) \delta(\mathbf{x}-\mathbf{x}_i) + g_A \sum_i \tau_i^{\pm} (\sigma_i \cdot \mathbf{k}/2m_N^*) \delta(\mathbf{x}-\mathbf{x}_i), \\ A_0^{(2)\pm} &= \frac{m_\pi^2 g_{\pi NN}}{8\pi m_N^* f_\pi^*} \sum_{i<j} (\tau_i \times \tau_j)^{\pm} [\sigma_i \cdot \hat{\mathbf{r}}\, \delta(\mathbf{x}-\mathbf{x}_j) + \sigma_j \cdot \hat{\mathbf{r}}\, \delta(\mathbf{x}-\mathbf{x}_i)] Y(r), \end{aligned} \tag{4}$$

with

$$Y(r) = \left(1 + \frac{1}{m_\pi r}\right) \frac{e^{-m_\pi r}}{m_\pi r}, \tag{5}$$

where $\mathbf{r} = \mathbf{x}_i - \mathbf{x}_j$, $\mathbf{k}$ is the momentum carried by the axial current, and $\mathbf{p}$ is the initial momentum of the nucleon making the transition. Note that in accordance with our strategy of chiral expansion to the leading order [5], both g_A and $g_{\pi NN}$ appear unrenormalized in the axial-charge density operator. Equation (4) is an extremely simple *in-medium* realization of the chiral-expansion strategy. We have not given a rigorous proof that this is the entire story but we suggest that it is the *dominant* one, the validity of which could be easily subjected to further experimental tests. Possible caveats to this result will be mentioned later.

The consequence of the above reasoning on axial-charge transitions in nuclei [9] can be simply stated. Denote the in-medium single-particle axial-charge matrix element by M_1^* and the in-medium exchange-current matrix element by M_2^*, with the asterisk denoting dense matter. Then to the extent that the pion mass remains unchanged, we get

$$M_1^* = \Phi(\rho)^{-1} M_1, \quad M_2^* = \Phi(\rho)^{-2} M_2, \tag{6}$$

where the $M_{1,2}$ without asterisks stand for quantities calculated with $\Phi = 1$. Thus we predict for Warburton's ratio ϵ,

$$\epsilon_{\rm MEC} \equiv (M_1^* + M_2^*)/M_1 = \Phi(\rho)^{-1}[1 + \Phi(\rho)^{-1}\mathcal{R}], \tag{7}$$

where

$$\mathcal{R} = M_2/M_1. \tag{8}$$

Although the operators in $\mathcal{R}$ are defined for $\Phi = 1$, the matrix elements could have hidden Φ dependence (i.e., through the wave functions). We will ignore this in view of the fact that the ratio $\mathcal{R}$ is quite insensitive to details of nuclear models and also to nuclear masses [10,11]: It ranges in all cases between 0.4 and 0.6. This range will be denoted for convenience as $\mathcal{R} = 0.5 \pm 0.1$. Calculations with sophisticated wave functions seem to support this insensitivity. This leaves the scaling factor Φ as the *principal* mechanism for density-dependent enhancement. Strictly speaking, Φ is a parameter of the theory. In nuclear matter, it is clearly a constant, but in finite nuclei, it may be problematic to define it accurately. In finite systems, it may be better to make a local-density approximation to it, in which case one would expect that the single-particle operator in (4), involving a derivative, will be less affected by density than the two-body operator. In any event, we will take in this paper a Φ defined for an average density. For light nuclei, we expect that $\Phi \approx 1$ and hence $\epsilon \approx 1.5 \pm 0.1$, which is consistent with experiments [12], for the range of $\mathcal{R} = 0.5 \pm 0.1$. For nuclear-matter density, a reasonable value is

$$\Phi(\rho \approx \rho_0) \approx 0.8. \tag{9}$$

This gives, for $\mathcal{R} = 0.5 \pm 0.1$,

$$\epsilon_{\rm MEC}(\rho) \approx 2.0 \pm 0.2 \ \text{ for } \rho = \rho_0. \tag{10}$$

This is the main prediction of the theory and agrees with the experimental result (1). The result is only slightly modified if the pion mass is taken to scale as $\sqrt{\Phi}$ everywhere in (4).

A comment is in order concerning the space component of the axial current, in particular, the Gamow-Teller operator $g_A \tau^{\pm} \sigma$. It is stated above that the g_A does not scale in the effective axial-charge operator. It is, however, known that the axial-vector coupling constant g_A in Gamow-Teller transitions gets renormalized in nuclei from its free-space value 1.26 to 1 in a rather precocious way [13]. It may thus seem inconsistent to ignore this modification in calculating the enhancement in the single-particle axial-charge operator. This, however, is not so. The point is that the renormalization of g_A should be calculated *explicitly* with the in-medium Lagrangian as a loop correction *specifically for the Gamow-Teller operator* and must not be present in the mean-field modification that enters in the *charge density operator* relevant to (6). One possible mechanism for the "effective" quenching of g_A (in Gamow-Teller transitions) is the Δ-hole contribution proposed a long time ago [14,15]. This belongs to a class of (density-dependent) loop corrections. We suggest that the quenching of g_A^* to about 1 in medium heavy nuclei is *not a fundamental renormalization that reflects a vacuum change* as it is for (2), contrary to what was previously conjectured in [15]: We suspect that the observed quenching of g_A^* is a fortuitous result involving an interplay between the Δ resonance and/or short-range interactions that figure at a higher order in chiral expansion as defined in [5] and that should not be confused with a "precocious approach" to the Wigner-Weyl mode [16]. The same argument applies to $g_{\pi NN}$ through the in-medium Goldberger-Treiman relation [15].

Our mechanism for the enhancement (1) is simple and unambiguous enough to be testable by further experiments. It is generally consistent with what is observed in such nuclear processes as elastic K-nucleus scattering [17] and proton-nucleus scattering [18] at several hundred MeV and in the longitudinal and transverse response functions in electron scattering off nuclei [19]. These are admittedly indirect indications. A direct test of the theory will come from dedicated measurements of the hadron properties in nuclear medium, such as in-medium masses of the vector mesons, in heavy-ion collisions, or in electron machines [20]. In the meantime, it is clearly legitimate to ask what other mechanisms could explain the observation (1). One immediate explanation suggested by Warburton [1] is the effect of quenched tensor forces. Warburton notes that if the *effective* tensor force in nuclei were weaker by 60% than that used in the analysis, the experimental data could be reconciled with $\epsilon_{\mathrm{MEC}} \approx 1.4$. This is a respectable possibility. As suggested in [21], one consequence of the scaling masses is that as density increases, the attraction in the tensor force in the two-body nucleon-nucleon interaction may be considerably weakened. This is almost inevitable from the point of view of the effective Lagrangian of Ref. [4]. The question, however, remains as to how this weakening will be reflected in the effective tensor force in the G matrices. A detailed calculation is needed to see what actually happens. The strategy adopted in our present work requires that the G matrices used in shell-model calculations be recalculated with the scaled masses. An initial attempt to do just this was recently reported [22], but the calculation for the axial-charge transition is yet to be performed. Our mechanism would clearly be endangered if the tensor-force mechanism were to explain wholly the observed enhancement. In any event, our scheme would make sense only if the scaling (2) were used consistently *throughout* the entire analysis including the extraction of the enhancement factor ϵ [11].

Another possibility is that hadron masses and coupling constants scale in effect differently from ours. It is possible, for instance, with assumptions different from ours on scaling and chiral properties of QCD, to define an effective Lagrangian that incorporates some of the loop effects of our Lagrangian while preserving certain aspects of QCD symmetries. Our scheme relies on a *minimal* implementation of chiral and scale invariances of QCD and as such it is not impossible that it is lacking in some important quantum effects. This point was discussed recently by Banerjee [23], who obtained results that differ from ours in the way coupling constants scale. While we question the validity of such an approach, it cannot be ruled out *ab initio*. Which scheme is correct will have to be settled by experiments.

Finally, it may be possible to start with an effective Lagrangian defined at zero density as in the standard approach and calculate medium effects, taking consistently into account exchange-current and relativistic effects. With a judicious choice of terms based on Ward identities, it may be possible to also explain the enhancement (1). An approach of this type was considered in Ref. [24]. The trouble with this, however, is that it involves correction terms of large magnitude which tend to cancel each other and it is not at all clear how to organize *all* the relevant terms. Ward identities would help but they are in practice difficult to implement. Furthermore, this approach and ours are not necessarily at odds in their physics contents. In fact, there may be a significant overlap between the two.

There is mounting evidence that nuclear processes, strong or electroweak, are better described with the scaled masses (2) in effective chiral Lagrangians [17–19]. Furthermore, implementing the running masses in the G matrix does not seem to upset the previous success in shell-model calculations [22]. This is highly nontrivial in view of the nonlinearity involved in the G-matrix calculations. It was argued in [4] that the nuclear Hamiltonian possesses an approximate scale invariance and the scaling (2) factors out in many nuclear processes. There are, however, cases where such factoring fails to occur as, e.g., in spin-orbit interactions. The case presented here —which has the further merit of combining the scaling property with chiral symmetry of QCD—is, despite its seeming complexity, a particularly elegant one and provides yet additional evidence for the "swelled world" concept of nuclear matter. If confirmed, this would constitute significant progress in our understanding of nuclei from a fundamental point of view. Extrapolated to an extreme condition, say, high temperature or high density, it could also provide valuable insight into the phase structure of the strongly interacting system relevant to relativistic heavy-ion collisions and the early Universe. This exciting possibility was recently suggested by Brown, Bethe, and Pizzochero [25].

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