Surprises from Bose-Einstein Correlations

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An investigation of the space-time extension of particle sources in the current-ensemble formalism shows that under quite general assumptions there exist a quantum-statistical $(\pi^+\pi^-)$ correlation and a difference between $(\pi^0\pi^0)$ and $(\pi^-\pi^-)$ Bose-Einstein correlations. These effects are found to be enhanced for small momenta and radii.

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Identical particles produced in high-energy collisions are correlated in their momenta due to quantum statistics (QS). This effect is used to determine the space-time form and the coherence properties of the source (for recent reviews, cf. [1-3]). Let us consider in particular the normalized two- and three-particle distributions of π mesons,

$$C_2(\mathbf{k}_1,\mathbf{k}_2)=\frac{\rho_2(\mathbf{k}_1,\mathbf{k}_2)}{\rho_1(\mathbf{k}_1)\rho_1(\mathbf{k}_2)},$$
$$C_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=\frac{\rho_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{\rho_1(\mathbf{k}_1)\rho_1(\mathbf{k}_2)\rho_1(\mathbf{k}_3)}. \tag{1}$$

$\mathbf{k}_i$ are three-momenta, and ρ_1, ρ_2, and ρ_3 are the one-, two-, and three-particle inclusive cross sections,

$$\rho_1=\frac{1}{\sigma}\frac{d\sigma}{d\omega},\quad \rho_2=\frac{1}{\sigma}\frac{d^2\sigma}{d\omega_1 d\omega_2},$$
$$\rho_3=\frac{1}{\sigma}\frac{d^3\sigma}{d\omega_1 d\omega_2 d\omega_3},\quad d\omega=\frac{d^3k}{(2\pi)^3 2E_k}. \tag{2}$$

It is commonly assumed that without taking into account final-state interactions and in the absence of coherence, the maximum of the two-particle correlation of identical pions is 2 (for $\mathbf{k}_1=\mathbf{k}_2$; see, however, [4] and [5]). The corresponding maximum value of $C_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ is 6. It is also assumed that there are no similar correlation effects among different kinds of pions because these particles are not identical. This last assumption is sometimes used in normalizing the experimental data on $C_2^{\pm\pm}$ with respect to C_2^{+-}. In the absence of interference effects between π^+ and π^- mesons and in the absence of dynamical correlations, $C_2^{+-}=\text{const}=1$.

In this Letter we show that these assertions are not necessarily true. In particular, the distribution $C_2^{00}(\mathbf{k}_1,\mathbf{k}_2)$ of two neutral pions can be as large as 3 for soft pions and for small sizes of the pion source. The corresponding limit of C_3^{000} is 15. Also, there is, in general, a similar correlation effect between π^+'s and π^-'s, so that the maximum value of C_2^{+-} is 2 (instead of 1) and that of C_3^{++-} is 6 (instead of 2).

We use an extension of the current formalism [4,6], considering the pions as created by some random currents (sources) $J_i(x)$ through the Lagrangian

$$\mathcal{L}_{\text{int}}(x)=J_1(x)\pi_1(x)+J_2(x)\pi_2(x)+J_3(x)\pi_3(x)$$
$$=J_+(x)\pi^-(x)+J_-(x)\pi^+(x)+J_0(x)\pi^0(x). \tag{3}$$

Here x is the space-time coordinate and the indices 1,2,3 refer to isotopic spin components of the pion field and to the corresponding currents. Charged pions

$$\pi^\pm=(1/\sqrt{2})(\pi_1\pm i\pi_2) \tag{4}$$

are created by complex conjugated currents $J_\mp(x)$, and neutral pions $\pi^0=\pi_3$ are created by real currents $J_0(x)=J_3(x)$. Equation (3) represents the simplest ansatz for the interaction of the pion field with an external source.

The currents are assumed to exist in a restricted space-time region and to be chaotic, i.e., Gaussian random functions, with a characteristic internal correlation length L [7]. This prescription corresponds to the most commonly used averaging procedure in optics, where it has been proven to be applicable for the majority of light sources, with the obvious exception of coherent sources (the inclusion of coherent currents is straightforward and will be considered elsewhere). It is also the simplest ansatz and follows from the central limit theorem.

Under these conditions all multiparticle pion distributions $\rho_n(\mathbf{k}_1,\ldots,\mathbf{k}_n)$ can be explicitly calculated. This is due to the decomposition property of Gaussian averaging, i.e., field correlators of even order can be expressed as a sum over products of two-point correlators. For example,

$$\langle J(x_1)J(x_2)J(x_3)J(x_4)\rangle=\langle J(x_1)J(x_2)\rangle\langle J(x_3)J(x_4)\rangle+\langle J(x_1)J(x_3)\rangle\langle J(x_2)J(x_4)\rangle+\langle J(x_1)J(x_4)\rangle\langle J(x_2)J(x_3)\rangle \tag{5}$$

for every component $J_i(x)$, and analogously for higher orders in the $J(x)$.

In momentum space, the two-current correlators with the space-time form of the source taken into account are ($m=1,2,3$)

$$\langle J_m^*(k_1)J_m(k_2)\rangle=\langle J_+^*(k_1)J_+(k_2)\rangle=F(k_1,k_2)\,,$$
$$\langle J_m(k_1)J_m(k_2)\rangle=F(-k_1,k_2)\,, \tag{6}$$

with the brackets $\langle\cdots\rangle$ denoting the ensemble average, and

$$F(k_1,k_2)=\int\frac{d^4k}{(2\pi)^4}f^*(k_1+k)C(k)f(k+k_2),\quad k_r^0=E_r\,. \tag{7}$$

In Eq. (7) the function $f(x)$, or, equivalently, its Fourier transform $f(k)$, describes the space-time form of the source and contains a characteristic size R (for convenience, we use the same symbol for a function and its Fourier transform). The function $C(x-y)$ is a primordial current correlator, containing the correlation length L.

To relate the above to the usual field operator and density-matrix formalism, let us consider the isotopic spin components of the pion field, π_m ($m=1,2,3$). Equivalence and mutual statistical independence of different isotopic spin components imply

$$\langle\pi_m(x_1)\pi_n(x_2)\rangle\propto\delta_{mn}\,. \tag{8}$$

Introducing the corresponding creation and annihilation operators, $a_m^\dagger(k)$ and $a_m(k)$, one has

$$F(k_1,k_2)=(2\pi)^3(4E_1E_2)^{1/2}\langle a_m^\dagger(k_1)a_m(k_2)\rangle\,, \tag{9}$$

$$F(-k_1,k_2)=-(2\pi)^3(4E_1E_2)^{1/2}\langle a_m(k_1)a_m(k_2)\rangle\,. \tag{10}$$

The terms $\langle a_m(k_1)a_m(k_2)\rangle$ are related to the nonstationarity of the field correlator (8); that is to say, they vanish if (8) depends on t_1-t_2 only. The existence of a nonvanishing expectation value of the products $a(k_1)a(k_2)$ is what one would expect from two-particle coherence (squeezing), just as $\langle a(k)\rangle\neq 0$ follows from ordinary (one-particle) coherence. Note that here we have not assumed the latter, but rather Gaussian, i.e., chaotic, statistics for the fields.

In terms of the annihilation operators of π^+, π^-, and π^0,

$$a_\pm=(1/\sqrt{2})(a_1\pm ia_2),\quad a_0=a_3\,, \tag{11}$$

the expectation values of two annihilation operators are

$$\langle a_+(k_1)a_+(k_2)\rangle=\tfrac{1}{2}[\langle a_1(k_1)a_1(k_2)\rangle-\langle a_2(k_1)a_2(k_2)\rangle]=0\,, \tag{12}$$

$$\langle a_-(k_1)a_-(k_2)\rangle=\tfrac{1}{2}[\langle a_1(k_1)a_1(k_2)\rangle-\langle a_2(k_1)a_2(k_2)\rangle]=0\,, \tag{13}$$

$$\langle a_0(k_1)a_0(k_2)\rangle=\langle a_3(k_1)a_3(k_2)\rangle\,, \tag{14}$$

$$\langle a_+(k_1)a_-(k_2)\rangle=\tfrac{1}{2}[\langle a_1(k_1)a_1(k_2)\rangle+\langle a_2(k_1)a_2(k_2)\rangle]=\langle a_0(k_1)a_0(k_2)\rangle\,. \tag{15}$$

Using Eq. (5) and its analog for the six-point correlator, and introducing the normalized current correlators

$$d_{rs}=\frac{F(k_r,k_s)}{[F(k_r,k_r)F(k_s,k_s)]^{1/2}}\,,\qquad \tilde d_{rs}=\frac{F(k_r,-k_s)}{[F(k_r,k_r)F(k_s,k_s)]^{1/2}}\,, \tag{16}$$

we obtain the one-particle spectra

$$\rho_1^i(\mathbf{k})=F(k,k)=\langle|J_i(k)|^2\rangle\quad(i=+,-,0)\,, \tag{17}$$

the two-particle distributions for different pairs of π^+, π^-, π^0 mesons,

$$\begin{aligned}
C_2^{++}(\mathbf{k}_1,\mathbf{k}_2)&=1+|d_{12}|^2\,,\\
C_2^{+-}(\mathbf{k}_1,\mathbf{k}_2)&=1+|\tilde d_{12}|^2\,,\\
C_2^{+0}(\mathbf{k}_1,\mathbf{k}_2)&=1\,,\\
C_2^{00}(\mathbf{k}_1,\mathbf{k}_2)&=1+|d_{12}|^2+|\tilde d_{12}|^2\,,
\end{aligned} \tag{18}$$

and the three-particle distributions for different combinations of π^+, π^-, π^0 mesons,

$$\begin{aligned}
C_3^{+++}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)&=1+|d_{12}|^2+|d_{13}|^2+|d_{23}|^2+2\,\mathrm{Re}(d_{12}d_{23}d_{31})\,,\\
C_3^{++-}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)&=1+|d_{12}|^2+|\tilde d_{13}|^2+|\tilde d_{23}|^2+2\,\mathrm{Re}(d_{12}\tilde d_{23}\tilde d_{31}^*)\,,\\
C_3^{++0}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)&=C_2^{++}(\mathbf{k}_1,\mathbf{k}_2)\,,\\
C_3^{+00}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)&=C_2^{00}(\mathbf{k}_2,\mathbf{k}_3)\,,\\
C_3^{000}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)&=1+|d_{12}|^2+|d_{13}|^2+|d_{23}|^2+|\tilde d_{12}|^2+|\tilde d_{13}|^2+|\tilde d_{23}|^2\\
&\quad+2\,\mathrm{Re}(d_{12}d_{23}d_{31}+d_{12}\tilde d_{23}\tilde d_{31}^*+d_{23}\tilde d_{31}\tilde d_{12}^*+d_{31}\tilde d_{12}\tilde d_{23}^*)\,.
\end{aligned} \tag{19}$$

We refer to the terms marked by a tilde as "new." They are smaller than the "usual" ones. Indeed, let us write the primordial current correlator as

$$C(x-y)=C_0\exp\left[-\frac{(x_0-y_0)^2}{2L^2}-\frac{(\mathbf{x}-\mathbf{y})^2}{2L^2}\right]. \qquad (20)$$

For reasons of mathematical simplicity, we choose the space-time distribution of the source, $f(x)$, also in Gaussian form,

$$f(x)=\exp\left[-\frac{x_0^2}{R_0^2}-\frac{x_\parallel^2}{R_\parallel^2}-\frac{x_T^2}{R_T^2}\right], \qquad (21)$$

where the indices $T,\parallel$ refer to the transverse and longitudinal space directions, respectively, and 0 to the time duration. Then, with $R_T=R_\parallel=R$ one has

$$d_{12}=\exp\left[-\frac{(\mathbf{k}_1-\mathbf{k}_2)^2R^4}{8(R^2+L^2)}\right]\exp\left[-\frac{(E_1-E_2)^2R_0^4}{8(R_0^2+L^2)}\right], \qquad (22)$$

$$\tilde{d}_{12}=\exp\left[-\frac{(\mathbf{k}_1+\mathbf{k}_2)^2R^4}{8(R^2+L^2)}\right]\exp\left[-\frac{(E_1+E_2)^2R_0^4}{8(R_0^2+L^2)}\right]. \qquad (23)$$

The term $\tilde{d}_{12}$ gives rise to an anticorrelation effect due to the first factor in Eq. (23), containing the sum $\mathbf{k}_1+\mathbf{k}_2$, if the second factor, containing E_1+E_2, is not too small. The latter is possible if the time duration of the pion emission process and the pion energies are small enough. We thus expect an enhanced contribution of the new terms for soft pions (the momenta are measured here in the rest system of the source) and for small R_0 (compare with Ref. [7], where a similar effect was found for the coherent contribution to the two-particle correlation function).

From Eqs. (22) and (23) follow, among other things, the results mentioned in the introduction, namely,

$$C_2^{00}\neq C_2^{++},\quad \mathrm{Max}C_2^{00}=3,\quad \mathrm{Max}C_3^{000}=15. \qquad (24)$$

The last two results are the analog of an effect predicted to occur in optics for nonstationary sources [8]. It is amusing to realize that while in optics the nonstationary condition is rather exceptional, in particle physics this is not the case, as can be seen from our derivation of Eqs. (22) and (23) which is based on very simple and general considerations. As a matter of fact, in Ref. [4] where a different space-time approach was used, the new terms can also be found, and they correspond there to an emission of two particles from the same space-time point, which again suggests a kind of two-particle coherence. However, in [4] the correlations for the different pion charge states were not considered and thus the effects (24) did not emerge.

On the other hand, the result implying the specific correlation between positive and negative pions,

$$C_2^{+-}>1, \qquad (25)$$

may appear even more surprising because it represents a quantum-statistical correlation between a particle and an antiparticle. However, this result, too, follows without any exotic assumptions from the same elementary considerations and therefore must correspond to a real and observable effect. It is interesting to note that our result corresponds to a density matrix which contains squeezed states [5]. The possibility that general considerations such as those sketched above may lead to squeezing is also new. One may interpret the new effects as a consequence of QS interference between the identical components (a_1,a_1), (a_2,a_2), and (a_3,a_3) as illustrated by Eqs. (11)-(15).

While the quantitative results displayed here depend on the Gaussian averaging procedure and the concrete parametrizations, the qualitative new effects are a consequence of QS and isospin symmetry only. Their importance lies, among other things, in the facts that (i) they suggest that squeezing, i.e., two-particle coherence, may be a very general property of particle physics (this property seems to be a consequence of the finite size and lifetime of the source); (ii) they prove explicitly, as was shown in [4], that Bose-Einstein two-particle correlations are more complex than would be expected from the two-particle (symmetrized) wave function which does not lead to particle-antiparticle correlations (this may have important implications in intermittency studies [9]); and (iii) the normalization procedure in terms of $(+-)$ pairs which is used in experimental Bose-Einstein correlation studies has to be qualified for soft pions.

A more detailed discussion of the derivation of these results, including the consideration of possible coherent currents, the case of boost-invariant sources, and the implications for multiplicity distributions, will be published elsewhere.

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