Noise-Modulated Propagating Pattern in a Convectively Unstable System

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A noise-sustained and noise-modulated structure of propagating Taylor vortices (PTV) in a convectively unstable regime of Couette-Taylor flow with a superimposed axial flow was observed and studied. The structure differs drastically from PTV in an absolutely unstable regime in its noisy power spectrum and in the noisy temporal dynamics of the front. Interaction of perturbations with the interface causes an amplitude modulation near the front and a noisy phase further to the outlet. The dependence of the front location on the control parameter indicates that perturbations from the inlet are responsible for the PTV existence.

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As was demonstrated in our recent paper [1], Couette-Taylor (CT) flow between cylinders with a superimposed axial flow has proved to be a useful system to study both the spatiotemporal dynamics of propagating patterns at convectively unstable conditions and the transition to an absolutely unstable state and the pattern dynamics there. The axial flow suppresses the basic stationary instability of the CT flow and gives rise to traveling waves in the form of propagating Taylor vortices (PTV) at sufficiently low through flow velocity ($\mathrm{Re} < 4$, where Re is the Reynolds number). The existence of both convectively and absolutely unstable regimes in the system makes it similar in this respect to open flow systems with the obvious advantage of the well controlled environment of a closed system which provides the opportunity to investigate the problem quantitatively and in great detail [1]. The main disagreement between the theoretical predictions [2] and the experiment [1,3] was the observation of PTV near the outlet of the cylinder gap at convectively unstable conditions at $\mathrm{Re} \gtrsim 1$, whereas at small enough $\mathrm{Re} < 1$ the first pattern appears at the transition line from convective to absolute instability in good quantitative agreement with the theory [1,2]. This transition line is set by the marginal stability of the front separating the convectively unstable Couette state and PTV [2]. The front is stable if it outruns any perturbation. Convective or absolute instability is considered in regard to the front [2]. Above the transition line to the absolutely unstable region the perturbations appear and grow ahead of the front, and the latter moves into the cell interior. Thus in an infinitely long system or in a finite cell with no reflection lateral boundaries, and in the absence of a permanent source of noise, the two types of instability conditions lead to a significant difference in pattern behavior. Then the convectively unstable regime is characterized by the absence of a steady pattern in the interior of the cell because it will be convected away, while in the absolutely unstable regime a pattern gradually appears at the outlet [1,2]. In a finite geometry cell with reflective lateral boundaries or with the existence of continuous noise, pattern behavior is essentially different [4]. Both these reasons can lead to the appearance of patterns in the convectively unstable region. As was shown theoretically [5] and experimentally [4] in a convective binary mixture the existence of counterpropagating waves and spatially and temporally modulated ("blinking") traveling waves in the convectively unstable region is the result of the wave reflection at the lateral walls, and the interaction between incident and reflecting waves. The pattern appears also at the experimental value of the control parameter ϵ_s which is higher than the threshold for the convective instability in an infinite system [6].

Secondly, permanent perturbations from the inlet being spatially and selectively amplified form spatially growing waves which saturate due to nonlinear effects. An extreme sensitivity of convectively unstable systems to permanent noise and the possibility to observe a noise-sustained structure sufficiently above the threshold were pointed out a long time ago [7]. In this Letter we present the results of studies of the origin and the properties of the noise-modulated PTV which is sustained in a part of the Couette-Taylor column in the convectively unstable region.

We have described our apparatus elsewhere [1,3]. The CT column is installed horizontally and is modified by an axial flow arrangement. Both the CT apparatus and the superimposed flow are temperature regulated with a stability of ± 10 mK. As a working fluid, a mixture of 32.4% by volume of glycerol in water at 22 °C [$\nu = 3.03$ cS (1 cS$\equiv 10^{-2}$ cm^2/s)] was used. An axial flow was driven by gravity in a closed loop, and its average flow charge was measured both by a precise flow meter in the range of 2×10^{-3} to 1.5 cm/s with a maximal resolution of 1×10^{-4} cm/s and by laser Doppler velocimetry (LDV) in a Poiseuille flow regime. In order to produce an axial flow as uniform as possible in the azimuthal and radial directions, fluid before entering the gap passed on its way an inlet chamber, flow directors, and a stainless-steel net (0.25×0.25 mm^2 mesh size) used as rotating boundaries on both sides of the cylinder gap. The geometrical parameters of the column are the following: radii of inner and outer cylinders are $R_1 = 1.900$ cm and $R_2 = 2.685$ cm, respectively (radius ratio is 0.707), and length aspect ratio is $\Gamma = 47$. We performed two types of measurements: by LDV and by video camera with a consequent computer-based image analysis. In the latter

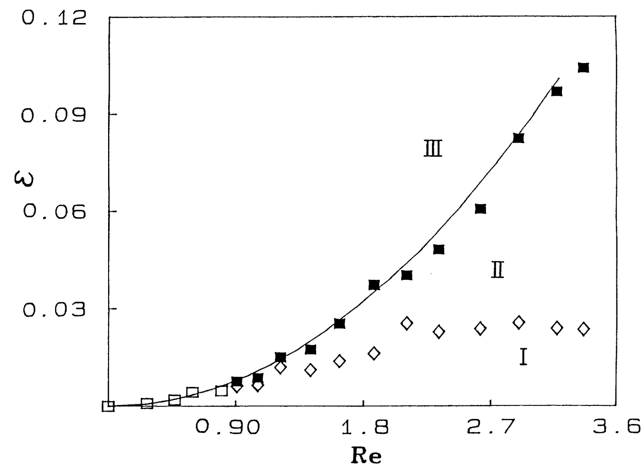

FIG. 1. Stability diagram of CT flow with an axial flow in the ϵ-Re plane. (ϵ is measured from the neutral stability line, Re is taken from the phase velocity $V=\omega/k$ of PTV.) The solid line is the theoretical curve [2] for the transition to absolute instability $\epsilon_{conv}(\mathrm{Re})$. Regions I, II, and III are convectively unstable without pattern observed, convectively unstable with noise-modulated PTV, and absolutely unstable with stationary PTV regions, respectively. Open and solid squares denote transition to time-independent PTV, and diamonds denote an appearance of noise-modulated PTV.

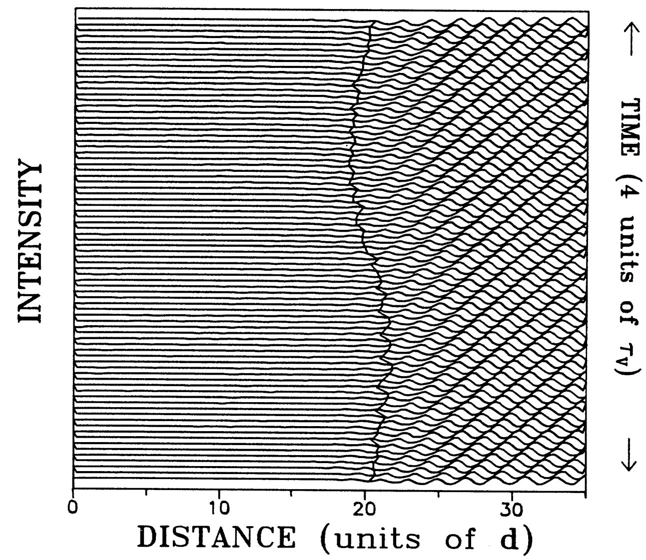

FIG. 2. Space-time contour plot of the optical signal intensity along a CT column for noise-modulated PTV (region II in Fig. 1). The solid line across the plot defines the interface between the Couette and PTV states ($\epsilon=0.04$, Re$=3.2$, $\tau_v=d^2/\nu$ $=20.34$ s).

case visualization by adding 1% Kalliroscope was used.

We present in Fig. 1 the stability diagram of the PTV in the plane of the control parameters $\epsilon=[\Omega-\Omega_c(\mathrm{Re})]/\Omega_c(\mathrm{Re})$ and $\mathrm{Re}=Vd/\nu$ ($V=\omega/k$ is the phase velocity of PTV, Ω is the angular rotation speed of the inner cylinder, and $d=R_2-R_1$ is the gap size). $\Omega_c(\mathrm{Re})$ is the critical value of Ω found experimentally in Ref. [1] which corresponds to the neutral line obtained from linear stability analysis [8]. Three different regions on the stability diagram were found. In the region I above the neutral stability line $\epsilon=0$, PTV are convectively unstable and are carried away from the column. No pattern was observed in this region. At $\mathrm{Re}\lesssim 1$ this region is bounded by the transition to an absolutely unstable CT flow where the pattern grows and expands upstream [1,2]. The solid line is the theoretically predicted transition line to the absolute instability region III which is found to obey the relation [2]

$$\epsilon_{conv}=S^2\tau_0^2/4\xi_0^2(1+c_1^2)\,, \tag{1}$$

where τ_0, ξ_0, S, and c_1 are the coefficients of the Ginzburg-Landau equation with complex coefficients (S is the group velocity, and one gets $S\cong\mathrm{Re}$ in the approximation when the group velocity is about equal to the phase velocity). Then one has the equation $\epsilon_{conv}=(\tau_0/2\xi_0)^2(1+c_1^2)^{-1}\mathrm{Re}^2$, which at $\tau_0=0.0758$, $\xi_0=0.38$ (Ref. [9]), and $c_1\ll 1$ gives $\epsilon_{conv}=0.00995\,\mathrm{Re}^2$. Good quantitative agreement between the theory [2] and the experiment [1] (open squares) was found at $\mathrm{Re}\lesssim 1$.

At $\mathrm{Re}>1$ the PTV commences to appear near the outlet at values of $\epsilon_s(\mathrm{Re})$ denoted by open diamonds, separating regions I and II. Region II bounded by $\epsilon_s(\mathrm{Re})$ and the absolute instability line (solid squares) is characterized by the existence of time-dependent PTV filling part of the column. As ϵ increases at a given value of Re the front separating the Couette and PTV states expands upstream toward the inlet. At fixed values of ϵ and Re the interface fluctuates due to interaction with perturbations coming from the inlet. Figure 2 shows a typical space-time contour plot of PTV in a convectively unstable regime ($\epsilon=0.04$, Re$=3.2$). On the same plot a position of the front separating the Couette and PTV states is denoted by a solid line across the plot. Its location was determined using a demodulation technique and by setting a certain threshold for an amplitude above which PTV is defined. The latter needs to distinguish the PTV state from spontaneous perturbations with amplitudes below the saturated value. We also verified this method by using other techniques, namely, rms amplitude and integral methods. Figure 3 represents the dynamics of the interface [Fig. 3(a)] in comparison with the time dependence of the amplitude of PTV measured by LDV in the vicinity of the interface [Fig. 3(b)]. A remarkable similarity in their temporal behavior is demonstrated by the plots. This temporal amplitude modulation due to the interface noise dynamics transfers into phase modulation of the PTV near the outlet. The latter statement is illustrated by the series of plots of time dependence of the velocity amplitude of PTV at two locations, close to the interface and near the outlet, and their power spectra for two values of ϵ in the convectively unstable regime and for ϵ just at the transition to the absolute instability (Fig. 4). One can make two important conclusions from these re-

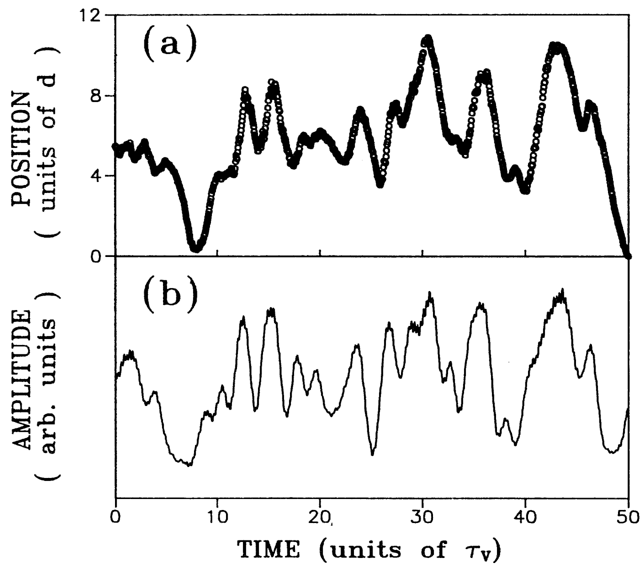

FIG. 3. (a) Interface position and (b) PTV amplitude at a fixed location close to the interface ($\epsilon = 0.034$, Re $= 3.2$).

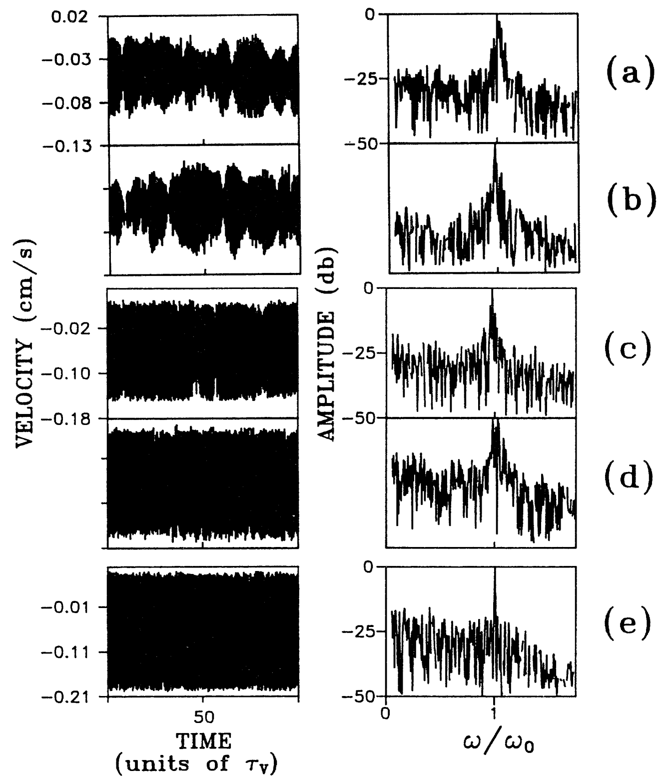

FIG. 4. Velocity of PTV vs time measured at two fixed locations and its power spectra (Re $= 3.2$). Data in (a)-(d) are taken in region II: (a),(b) Close to the interface; (c),(d) close to the outlet. (a) and (c) are at $\epsilon = 0.049$; (b) and (d) at $\epsilon = 0.062$. (e) Data are taken in region III at $\epsilon = 0.1$. The peak frequency in the power spectra $\omega_0 = 10.8$.

sults. In spite of the difference in the time dependence of the signals, their power spectra for both locations are very similar for the two values of ϵ in the convectively unstable region. At the transition to the absolute instability the broad peak collapses to a single sharp peak that witnesses a different response of the structure to spontaneous perturbations in the convectively and absolutely unstable regimes. The interface between the Couette and PTV states in the absolutely unstable region III is steady in time. We use the sharp transition in power spectra together with change in the amplitude modulation and the interface behavior as a criterion for the transition from convective to absolute instability (solid squares in Fig. 1). It is remarkable that the data fall on the theoretical curve [2] given by (1). Thus, we demonstrated now that at Re $\lesssim 1$ (Fig. 1) the PTV are observed first near the outlet at the transition line to the absolute instability, in agreement with the theory, whereas at Re $\gtrsim 1$ (Fig. 1) the PTV appear already in region II, which is convectively unstable. This PTV pattern is characterized by a noisy power spectrum and a time-dependent interface contrary to the PTV in the absolutely unstable region. The noisy spectrum of PTV results from the noisy dynamics of the front which in turn occurs due to interaction with spontaneous perturbation coming from the inlet.

The existence of patterns in the convectively unstable region II may be explained either by one of the following two mechanisms or by their combination. First, the reflection at the outlet boundary if it occurs will stablize PTV, similar to what was found for an oscillatory convection in a binary fluid [4-6]. Any perturbation will grow exponentially $\exp(\epsilon\Gamma/S\tau_0)$ during propagation. The latter feature is direct evidence of the convectively unstable nature of the state. The combined effect of propagation, reflection, and spatial growth cause the onset shift ϵ_s which depends on the boundary condition at the outlet, the aspect ratio, and the group velocity of PTV. A balance between the gain due to the spatial growth over the cell length and the losses due to finite reflection gives the onset shift [6]

$$\epsilon_s = \tau_0 \Gamma^{-1} \mathrm{Re} \ln(1/r) , \tag{2}$$

where r is the reflection coefficient, and we used the approximation $S \simeq$ Re. The linear best fit to the data in Fig. 1, represented by the open diamonds, gives $\epsilon_s = 0.0078\,$Re which corresponds to $r = 0.008$. Its intersection with the absolute instability line $\epsilon_{\mathrm{conv}}(\mathrm{Re})$, given by (1), leads to the crossover value of Re* $= 0.784$.

The second mechanism is related to permanent noise amplification and its nonlinear saturation which leads to a noise-sustained structure [7]. Strong enough disturbances coming spontaneously from the inlet being amplified convectively in a sufficiently long channel reach the saturated amplitude value at large enough ϵ. Thus, the system should be brought above the threshold $\epsilon = 0$ to provide a sufficient gain for noise to reach saturation and to sustain a pattern. This occurs at $\epsilon_s = \tau_0 \Gamma^{-1} \mathrm{Re} \ln \gamma$, where γ is the gain and is defined experimentally as the amplitude ratio of the saturated state and the perturba-

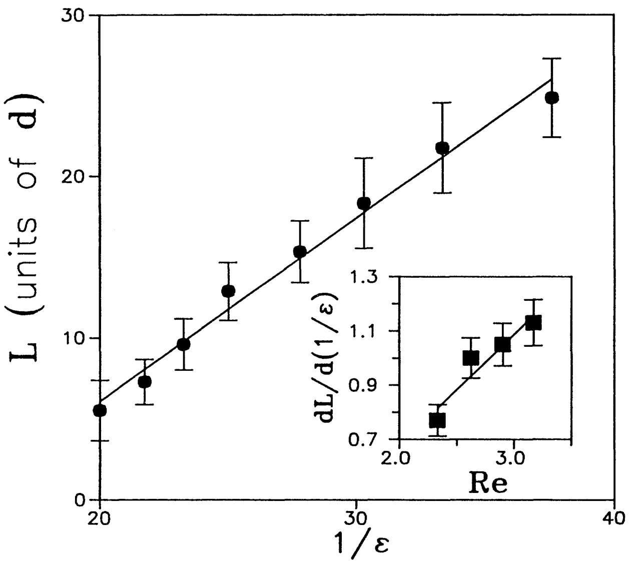

FIG. 5. Averaged interface position vs $1/\epsilon$. Vertical bars show the range of the interface fluctuations (Re = 3.2). The solid line is a linear fit to find the slope $dL/d(1/\epsilon)$ shown in the inset as a function of Re. A linear fit to the data in the inset gives a slope 0.359.

tions at the inlet. Then at $\mathrm{Re} < \mathrm{Re}^*$ one gets $\epsilon_s > \epsilon_{\mathrm{conv}}$, and PTV will appear first at the transition to the absolute instability, while at $\mathrm{Re} > \mathrm{Re}^*$ the pattern will be sustained near the outlet first already in the convectively unstable region. Another remarkable feature of the PTV state shows up differently in regions II and III. As we verified in full agreement with the theory [1,2] the distance of the interface from the inlet as a function of the scaled group velocity is represented by a universal function and diverges at the transition from the absolute to the convective instability. On the other hand, the average length occupied by the convectively unstable Couette state $L = l/d$ in region II at given Re is inversely proportional to ϵ and is defined as $L = \tau_0 \epsilon^{-1} \mathrm{Re} \ln \gamma$.

A typical data set of L vs $1/\epsilon$ at Re = 3.2 is presented in Fig. 5. Vertical bars show the range of the interface fluctuations at each ϵ. The linear dependence of the front location from the inlet is clearly demonstrated. Furthermore, the data at different Re do not fall on the universal curve as in the absolute instability regime [1,2]. In the inset of Fig. 5 the results of measurements of the slope of the plot in Fig. 5 at several Re are shown. From the linear fit of the data one gets $\gamma = 114.3$ in good agreement with the result for $\gamma = 1/r$ obtained above from the $\epsilon_s(\mathrm{Re})$ line. Using the former value of γ one gets the crossover value $\mathrm{Re}^* = 0.77$ in excellent agreement with the value obtained above. Thus both plausible mechanisms of the existence of the sustained PTV describe their properties equally well. We tried in a separate experiment [10] to deliberately create perturbations and to inject them into the flow before the inlet at $\epsilon < \epsilon_s(\mathrm{Re})$. Their amplitude grew downstream exponentially but we were not able to detect reflected pulses within our resolution. Estimates of our visualization technique resolution give an upper bound for the reflection coefficient on the order of $r < 0.008$ which means that the noise amplification mechanism is a more probable reason for the sustained PTV above $\epsilon_s(\mathrm{Re})$.

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