Two-Dimensional Charged Black Holes in String Theory

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Exact string solutions corresponding to two-dimensional electrically charged black holes are constructed.

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Recently, exact solutions to string theory in the form of two-dimensional black holes were found [1-4]. It would be interesting to generalize this result to charged black holes. Previously [5], solutions to the low-order effective action in the form of four-dimensional charged black holes were shown to exist. In this paper, we construct a two-dimensional charged black hole as an exact conformal field theory. The original two-dimensional black hole was constructed by gauging a U(1) subgroup of an SL(2,R) Wess-Zumino-Witten (WZW) model. The electrically charged black hole discussed here is obtained by adding a boson to the model and coupling it to the world-sheet gauge field. After integrating out the world-sheet gauge field, we obtain a solution for the spacetime metric, dilaton, and electromagnetic field which describes a charged black hole. The charge of the black hole is

$$Q=2(e/k)\exp a\,,$$

where e is the coupling between the world-sheet gauge field and the free-boson field, k is the level of the WZW model, and a is an undetermined constant which appears in the dilation background. The mass of the black hole is the same as that of the uncharged black hole:

$$M=\sqrt{2/k}\,\exp a\,.$$

Unlike the Reissner-Nordstrom solution, the singularity is hidden behind a horizon for arbitrary large values of the electric charge. It is also possible to construct other conformal field theories which describe solutions with naked singularities as well as solutions with no singularities at all.

To obtain closed-string theories which have gauge fields in their massless spectrum, one introduces free bosons or, equivalently, free fermions on the world sheet [6]. For example, a bosonic string model with gauge fields can be obtained by starting from a system of bosons, X^μ, describing the embedding of the world sheet in spacetime, together with free bosons, X^I, which are compactified so as to realize the Kac-Moody current of the gauge group. Normally, in the heterotic string one considers chiral bosons, but here we will include both left and right movers. This will then yield a separate gauge symmetry for both the left and right currents. Such a string model in a gravitational, gauge field, and dilation background $(G_{\mu\nu},A^I_\mu,\tilde{A}^I_\mu,\phi)$ is described by a nonlinear sigma model on the world sheet with the following action:

$$\begin{aligned}S=&\frac{1}{2\pi}\int d^2x(\partial X^\mu\bar\partial X^\nu G_{\mu\nu}+\partial X^I\bar\partial X^I)\\&+\frac{1}{2\pi}\int d^2x(A^I_\mu\bar\partial X^\mu\partial X^I+\tilde A^I_\mu\partial X^\mu\bar\partial X^I)\\&+\frac{1}{8\pi}\int d^2x\,R^{(2)}\phi(X)\,.\end{aligned}\qquad(1)$$

Here, we have assumed that the gauge background takes values only in the Cartan subalgebra of the gauge group.

In order to obtain a conformal field theory which describes a charged black hole, we start from an SL(2,R) WZW model with a free compact boson:

$$S=S_{\rm WZW}(g)+\frac{1}{2\pi}\int d^2x\,\partial X\bar\partial X\,.$$

By introducing a world-sheet gauge field, we will induce a spacetime U(1)×U(1) gauge background. As in [1] we gauge the axial U(1) symmetry,

$$\delta g=\epsilon\left[\begin{pmatrix}0&1\\-1&0\end{pmatrix}g+g\begin{pmatrix}0&1\\-1&0\end{pmatrix}\right],\qquad(2)$$

in the SL(2,R) WZW action. We also wish to couple the gauge field to the compact boson. The free-boson system has chiral U(1) currents ∂X and $\bar\partial X$, which are essential to the gauge symmetry of the string theory. There are two ways to couple the gauge fields to these currents. First, we can gauge the U(1) translation symmetry generated by these currents:

$$\frac{1}{2\pi}\int d^2x(\partial X+2eA_z)(\bar\partial X+2eA_{\bar z})\,.\qquad(3)$$

Since the left and right U(1) currents satisfy the U(1) Kac-Moody algebra independently, we can also couple the gauge fields to the U(1) currents in the following way:

$$\frac{1}{2\pi}\int d^2x(\partial X\bar\partial X+2ieA_{\bar z}\partial X-2ieA_z\bar\partial X)\,.\qquad(4)$$

This action is invariant under the world-sheet gauge transformation [7]

$$X\to X,\quad A_\mu\to A_\mu+\partial_\mu\epsilon\,.$$

The first scheme, (3), leads to a naked singularity for

$e^2/2k > 1$ and a black hole with an imaginary gauge field for $e^2/2k < 1$. The second scheme (4), as we now show, leads to a black hole with a real gauge field for all values of e.

Using (4) we have the following two-dimensional action:

$$I = S_{\mathrm{WZW}}(g) + \frac{1}{2\pi}\int d^2x\, \partial X \bar{\partial} X$$
$$+ \frac{k}{\pi}\int d^2x \left\{ A_{\bar{z}} \left[\mathrm{Tr}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} g^{-1}\partial g + \frac{ie}{k}\partial X \right] \right.$$
$$\left. + A_z \left[\mathrm{Tr}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \bar{\partial} g g^{-1} - \frac{ie}{k}\bar{\partial} X \right] + A_z A_{\bar{z}} \left[-2 + \mathrm{Tr}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} g \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} g^{-1} \right] \right\}. \tag{5}$$

Since ∂X and $\bar{\partial} X$ satisfy the U(1) Kac-Moody algebra, the gauge action above is exactly solvable as in the usual gauged WZW models [8]. Since the compact boson has central charge $c=1$ and the gauging reduces c by 1, the central charge of this conformal field theory is just that of the WZW model:

$$c = 3k/(k-2).$$

We now show that this gauged action describes a string theory in a charged black hole background. Going to unitary gauge as in [1],

$$g = \cosh r + \sinh r \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix},$$

and integrating out the gauge fields, we obtain the following nonlinear-sigma-model action:

$$\frac{k}{\pi}\int d^2x (\partial r\, \bar{\partial} r + \tanh^2 r\, \partial\theta\, \bar{\partial}\theta) - \frac{ie}{2\pi}\int d^2x (\tanh^2 r\, \partial\theta\, \bar{\partial} X + \tanh^2 r\, \bar{\partial}\theta\, \partial X)$$
$$+ \frac{1}{2\pi}\int d^2x \left[1 + \frac{e^2}{2k}\frac{1}{\cosh^2 r} \right] \partial X \bar{\partial} X + \frac{1}{8\pi}\int d^2x\, R^{(2)}\phi, \tag{6}$$

where $\phi = \ln\cosh^2 r + \text{const}$. Comparing with Eq. (1), it is easy to see that this two-dimensional action describes a two-dimensional string theory in a nontrivial gravitational, gauge field, and dilaton background. There is also a scalar field associated with the vertex operator:

$$\partial X \bar{\partial} X e^{i(p_r r + p_\theta \theta)}. \tag{7}$$

The dilaton field arises from the integration measure and can be checked to one loop in sigma-model perturbation theory. It should be noted that the procedure described above is also applicable when we use fermions instead of bosons to incorporate the gauge fields or even when we have chiral fermions as would occur in the heterotic string. We note that $A_\mu = \tilde{A}_\mu$ in the case at hand. We can generalize our construction to the case $A_\mu \neq \tilde{A}_\mu$ by gauging the left and right U(1) independently in the WZW sector. For example, we can gauge only the symmetry generated by ∂X in the boson system together with a suitable modification of (2), which balances the left and right anomalies. In this case we would obtain a string theory with only an A_μ background and no extra scalar background. This results in either a naked singularity or a black hole with an imaginary gauge field.

To obtain a Lorentzian signature black hole we could gauge a noncompact U(1) subgroup. Here, we merely analytically continue to Lorentzian signature by $\theta \to it$. This yields the spacetime metric,

$$ds^2 = 2k\, dr^2 - 2k \tanh^2 r\, dt^2, \tag{8a}$$

dilaton,

$$\phi = \ln\cosh^2 r + a, \tag{8b}$$

and U(1) gauge field,

$$A_\mu = e \tanh^2 r\, \partial_\mu t, \tag{8c}$$

where a is a constant. In addition, we have a scalar background associated with the compact boson. This solution describes a charged black hole with a metric and dilaton which are identical to that of the uncharged black hole [1]. It should be observed that if we think of the solution as a three-dimensional metric, then shifting t by a multiple of X diagonalizes the metric and yields the product of the uncharged black hole and a circle. Hence, one might think that the charged black hole is equivalent to the uncharged black hole. However, this is not the case. Since X is compact, the new coordinates which one obtains by such a coordinate transformation are not good global coordinates on the cylinder. Moreover, it can be shown that the two underlying conformal field theories are inequivalent.

We now calculate the mass and charge of the black hole. The mass of the black hole can be obtained by the Arnowitt-Deser-Misner procedure. The low-energy effective action for the background fields is quadratic in the electromagnetic field strength and in gradients of the scalar field. Since both of these vanish asymptotically, the gauge field and the scalar field will not contribute to the mass. Thus, the mass is the same as that of the uncharged black hole [1]

$$M=\sqrt{2/k}\,\exp a\,.$$

We can also find the electric charge of the black hole. The equation of motion for the gauge field can be obtained by dimensionally reducing to two dimensions the low-energy three-dimensional effective action. Far from the singularity, the equation of motion takes the form

$$\nabla_\mu(e^\phi F^{\mu\nu})=0\,.$$

The electric charge is then given by

$$Q=\tilde{F}|_{r=\infty}\,, \tag{9}$$

where $\tilde{F}=e^\phi e_{\mu\nu}F^{\mu\nu}$. In four dimensions, $\tilde{F}$ would become a two-form, and the charge would be the integral of $\tilde{F}$ over a two-sphere. In two dimensions since $\tilde{F}$ is a scalar, we merely evaluate it at ∞. One should note that the equation of motion insures that Q is independent of r. Evaluating Eq. (9), we obtain

$$Q=2(e/k)\exp a\,.$$

We also note that one should include in the solution quantum corrections coming from the determinant which arise in the Gaussian integration over the gauge field. Asymptotically, the sole effect of this is to replace k by $k-2$. It should be noted that unlike the uncharged black hole, there will now be an infinite tower of propagating massive states. In addition, there will be an instability due to the tachyon. As in the case of the uncharged black hole, we expect that there exists a superconformal version of the charged black hole. In this case, there would be no tachyon, and the theory would be stable. Finally, we should point out that by constructions similar to that described in this paper it is also possible to obtain black string solutions [9].

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