Harmonic Gyroresonance of Electrons in Combined Helical Wiggler and Axial Guide Magnetic Fields

K. R. Chu
Department of Physics, National Tsing Hua University, Hsinchu, Taiwan, Republic of China

A. T. Lin
Department of Physics, University of California, Los Angeles, California 90024
(Received 28 August 1991)

A driven-oscillator model is developed to examine the orbital-velocity harmonics of off-axis electrons in combined helical wiggler and axial guide magnetic fields. The gyroresonance effect is found to occur at harmonics of the wiggler frequency in addition to the well-known fundamental-harmonic resonance. Resonance at the first negative harmonic is potentially important for providing an explanation of the unexpected observations of a recent free-electron-laser experiment. Exploration of the gyroresonance effect as a harmonic selection mechanism for the free-electron laser is discussed.

PACS numbers: 42.55.Tb

The free-electron laser (FEL) often employs an axial magnetic field to guide the electron beam through the wiggler. The guide field also plays an important role in the wiggle motion of the electrons and through which influences the gain, frequency, and efficiency. It is in fact an integral part of the physical process taking place in FEL's with a guide field. Electron dynamics in combined helical wiggler and axial guide fields have been studied in ideal (one-dimensional) [1-3] and realistic (three-dimensional) [3-5] wigglers. In the simplest case, the electron performs a steady-state axis-centered helical orbit with constant axial and transverse velocities. With the pitch distance equal to the wiggler period λ_w, the electron rotation can be synchronized with the wiggler field such that on its orbit there are *constant* azimuthal and axial magnetic fields $(B_\perp, B_\parallel)$ but no radial magnetic field. Also, the electron velocity has only azimuthal and axial components $(v_\perp, v_\parallel)$. Thus the magnetic force on the electron is radially directed which balances the centrifugal force to result in an equilibrium state given by

$$\gamma m_e v_\perp^2/r_b = (e/c)(v_\parallel B_\perp + v_\perp B_\parallel)\,, \tag{1}$$

where r_b is the electron orbital radius,

$$\gamma = (1 - v_\perp^2/c^2 - v_\parallel^2/c^2)^{-1/2} = \text{const}\,, \tag{2}$$

and $B_\parallel$ includes the uniform guide field $B_0\mathbf{e}_z$. Substituting the relation $v_\perp = k_w v_\parallel r_b$ into Eq. (1) to eliminate r_b gives the familiar equation derived by Friedland [1] and others [2-5],

$$v_\perp = \Omega_\perp v_\parallel/(k_w v_\parallel - \Omega_\parallel)\,, \tag{3}$$

where $\Omega_{\perp,\parallel} = eB_{\perp,\parallel}/\gamma m_e c$ and $k_w = 2\pi/\lambda_w$. Equations (2) and (3) together yield multiple solutions for $v_\perp$ and $v_\parallel$, corresponding to different classes of equilibrium orbits [1,4]. Equation (3) predicts a wiggler and guide field resonance (gyroresonance) at

$$k_w v_\parallel - \Omega_\parallel \simeq 0\,, \tag{4}$$

which separates the electron orbits into two groups, commonly referred to as group I $(k_\parallel v_\parallel < \Omega_\parallel)$ and group II $(k_\parallel v_\parallel > \Omega_\parallel)$.

The simple equilibrium model just discussed is only a very special case. Any departure from it, such as a perturbation of the electron velocity or a radial displacement of the electron guiding center, can result in complicated orbital behavior. This has led to detailed theories which shed light on the electron betatron oscillation [4,5], guiding-center drift [5,6], stability of [1-4] and accessibility to [1,3] the equilibrium orbit, etc. Numerical calculations generally show that the electrons execute non-steady-state helical orbits, with their $v_\perp$ and $v_\parallel$ oscillating about mean values $\bar{v}_\perp$ and $\bar{v}_\parallel$. For experimental studies of the FEL [7-11], it is of practical importance to have an accurate method of predicting $\bar{v}_\perp$ and $\bar{v}_\parallel$ in order to determine the FEL gain and frequency. Thus different schemes of approximating the $\Omega_\perp$ and $\Omega_\parallel$ terms in Eq. (3) (and hence $\bar{v}_\perp$) have been developed, as has been extensively discussed by Fajans, Kirkpatrick, and Bekefi [5].

Elaborate experimental investigations [8,10,11] of beam behavior near the gyroresonance have shown good agreement with theoretical predictions. A beam injected under the resonance condition (4) is found to be totally disrupted in the wiggler field [8,11]. In a recent *Ka*-band FEL amplifier experiment by Conde and Bekefi [11], the magnetic field is reversed from its usual (positive z) direction and this results in a dramatic increase in interaction efficiency (from 2% to 27%). When the reversed magnetic field is tuned to the value $|\Omega_\parallel| = k_w v_\parallel$, where no resonance is predicted, there is a large, unexpected dip in output power although most of the beam (> 90%) has passed through the wiggler. This peculiarity suggests the need for further examination of the electron behavior in the wiggler, especially in the reversed field configuration.

Motivated by these observations, the present paper develops an approximate model to examine the wiggle motion of the off-axis electrons. Off-axis electron motion is not amenable to exact analysis. It has been studied numerically [5,6,8] or by some averaging method [4,5]. In these treatments, the harmonic aspect of the electron motion has been either obscured or averaged out. Here, we shall focus on the harmonic content of the electron motion, but ignore the other complexities discussed earlier. In our model, the magnetic forces on an off-axis electron are evaluated on an ideal helical orbit, as is projected in the cross-sectional plane in Fig. 1 (validity discussed later). The wiggler plus guide field is expressed in cylindrical coordinates (r,θ,z) by [3]

$$\mathbf{B}=B_0\mathbf{e}_z+2B_w\{I_1'(k_wr)\cos(\theta-k_wz)\mathbf{e}_r-[I_1(k_wr)/k_wr]\sin(\theta-k_wz)\mathbf{e}_\theta+I_1(k_wr)\sin(\theta-k_wz)\mathbf{e}_z\}\,, \tag{5}$$

where I_n is the modified Bessel function of the first kind and I_n' its derivative. In Fig. 1, the electron phase angle is chosen so that in the limit $r_g\to 0$, the orbit reduces to the exact steady-state orbit.

The transverse force exerted on the electron by the transverse components of the wiggler field can be approximately written

$$\mathbf{F}_\perp=-(e/c)\bar{v}_\parallel\mathbf{e}_z\times\{I_1'(k_wr)\cos(\theta-k_wz)\mathbf{e}_r-[I_1(k_wr)/k_wr]\sin(\theta-k_wz)\mathbf{e}_\theta\}\,, \tag{6}$$

where $\bar{v}_\parallel$ is the average axial velocity of the electron.

Transverse orbital motion of an off-axis electron allows it to traverse the wiggler field over a radial distance of up to $2r_b$ (Fig. 1). Hence, the spatial gradient of the wiggler field results in a magnetic force on the electron composed of harmonics of the wiggler frequency $k_w\bar{v}_\parallel$. Applying the Bessel function addition theorem to eliminate the oscillatory variables r and θ in favor of r_b,r_g and phase angle k_wz, we rewrite Eq. (6) as

$$\mathbf{F}_\perp=\frac{eB_w\bar{v}_\parallel}{c}\sum_{q=-\infty}^{\infty}I_q(k_wr_g)\left[-\frac{2(q+1)}{k_wr_b}I_{q+1}(k_wr_b)\cos(qk_wz)\mathbf{e}_\rho+I_{q+2}(k_wr_b)\sin(qk_wz)\mathbf{e}_\varphi\right]\,, \tag{7}$$

where $\mathbf{e}_\rho$ and $\mathbf{e}_\varphi$ are unit vectors perpendicular and tangential to the circular orbit, respectively (see Fig. 1). Transforming to the Cartesian coordinates and rearranging terms, we obtain

$$\mathbf{F}_\perp=\frac{2eB_w\bar{v}_\parallel}{c}\sum_{n=-\infty}^{\infty}\frac{n}{k_wr_b}I_n(k_wr_b)I_{n-1}(k_wr_g)\mathbf{e}_n\,, \tag{8}$$

where

$$\mathbf{e}_n\equiv\sin(nk_wz)\mathbf{e}_x-\cos(nk_wz)\mathbf{e}_y$$
$$=\sin(nk_w\bar{v}_\parallel t)\mathbf{e}_x-\cos(nk_w\bar{v}_\parallel t)\mathbf{e}_y\,, \tag{9}$$

and $\mathbf{e}_n$ rotates in the same (opposite) sense as the wiggler field for positive (negative) n.

The total axial field on the helical orbit can be similarly expressed,

$$B_\parallel=B_0+2B_w\sum_{n=-\infty}^{\infty}I_{n+1}(k_wr_b)I_n(k_wr_g)\cos(nk_wz)\,, \tag{10}$$

and its average is simply

$$\bar{B}_\parallel=B_0+2B_wI_1(k_wr_b)I_0(k_wr_g)\,. \tag{11}$$

We may now write the equation of motion for the transverse electron velocity in the average axial magnetic field:

$$\frac{d\mathbf{v}_\perp}{dt}=-\bar{\Omega}_\parallel\mathbf{v}_\perp\times\mathbf{e}_z+2\Omega_w\bar{v}_\parallel\sum_{n=-\infty}^{\infty}\frac{n}{k_wr_b}I_n(k_wr_b)I_{n-1}(k_wr_g)\mathbf{e}_n\,, \tag{12}$$

where $\bar{\Omega}_\parallel\equiv e\bar{B}_\parallel/\gamma m_ec$, $\Omega_w\equiv eB_w/\gamma m_ec$, and the time variable t $(=z/\bar{v}_\parallel)$ is used. This is the equation of motion of a driven oscillator, with $\bar{\Omega}_\parallel$ being the natural frequency. The solution is

$$\mathbf{v}_\perp=v_{\perp0}[\cos(\bar{\Omega}_\parallel t+\varphi_0)\mathbf{e}_x+\sin(\bar{\Omega}_\parallel t+\varphi_0)\mathbf{e}_y]-\sum_{n=-\infty}^{\infty}v_{\perp n}\mathbf{e}_z\times\mathbf{e}_n\,, \tag{13}$$

where

$$v_{\perp n}\equiv\frac{2n\Omega_w\bar{v}_\parallel}{k_wr_b(nk_w\bar{v}_\parallel-\bar{\Omega}_\parallel)}I_n(k_wr_b)I_{n-1}(k_wr_g)\,. \tag{14}$$

The first term on the right-hand side of Eq. (13) is a general solution representing the cyclotron oscillation. Its amplitude $v_{\perp0}$ and phase φ_0 depend on the entrance con-

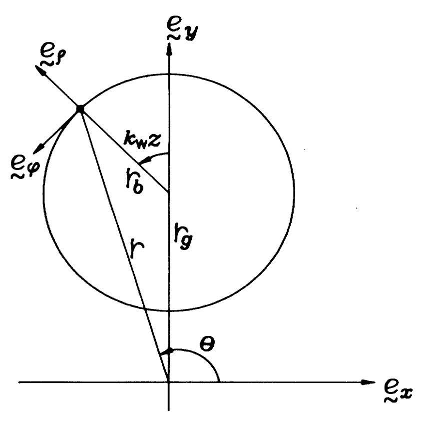

FIG. 1. Projection of the helical orbit on the transverse plane on which the magnetic forces on the electron are evaluated.

ditions of the electron. It can become significant under nonadiabatic entrance conditions. For the present purpose, it will not be discussed further. The second term comprises oscillations at the harmonics of $k_w \bar{v}_\parallel$ driven by the wiggler field. In the case of axis-centered motion ($r_g = 0$), all but the $n=1$ term vanish and we recover the steady-state solution discussed earlier. The $n=1$ term will henceforth be termed the fundamental harmonic and the $n \neq 1$ terms (including the $n=-1$ term) higher harmonics. For the off-axis electrons ($r_g > 0$), however, $\mathbf{v}_\perp$ is a rich composition of positive ($n>0$) and negative ($n<0$) harmonics. It is interesting to note that the fundamental-harmonic term

$$v_{\perp 1} = \frac{2\Omega_w \bar{v}_\parallel}{k_w r_b (k_w \bar{v}_\parallel - \Omega_\parallel)} I_1(k_w r_b) I_0(k_w r_g) \tag{15}$$

is precisely the semiempirical equation obtained by Fajans, Kirkpatrick, and Bekefi [Eq. (18) of Ref. [5]]. The averaging procedure of Ref. [5] has suppressed the higher-harmonic components of Eq. (13). These harmonic contributions, of the order of $(k_w r_g)^{|n-1|}$ when $k_w r_g < 1$, are negligible in the absence of harmonic resonances, since $k_w r_g \ll 1$ under realistic experimental conditions. Indeed, Eq. (15) is found to be in good agreement with computer simulations as well as experimental observations over the full range of r_g of practical interest ($k_w r_g < 1$) [5]. However, when there is a harmonic resonance

$$n k_w \bar{v}_\parallel - \bar{\Omega}_\parallel \simeq 0 , \tag{16}$$

the resonant component $v_{\perp n}$ is no longer a small term. The larger r_g is, the larger the $v_{\perp n}$ term. Electrons on the outer edge of the beam may thus obtain a sufficiently large $v_{\perp n}$ to be intercepted by the wall. This qualitatively explains the small but noticeable dip in beam current observed by Conde and Bekefi [11] in the reversed magnetic field ($\bar{\Omega}_\parallel < 0$) satisfying resonant condition (16) with $n=-1$. Under the same condition, the off-axis electrons, though mostly transported through the wiggler, have all acquired a harmonic ($n=-1$) quiver velocity of varying magnitude depending on their guiding-center positions, which amounts to a cross-sectional velocity spread and hence explains the large dip in the observed output power.

In our model, the evaluation of magnetic forces has been based on the assumption that the electrons stay near an ideal helical orbit. The validity of this assumption requires

$$\frac{1}{v_{\perp n}} \frac{\partial v_{\perp n}}{\partial r_{b,g}} \Delta r_{b,g} \ll 1 , \tag{17}$$

where Δr_b and Δr_g are deviations from the helical orbit. Substituting Eq. (14) into Eq. (17), we obtain

$$|n| \Delta r_b / r_b \ll 1, \quad |n-1| \Delta r_g / r_g \ll 1 , \tag{18}$$

where only the first term of the modified Bessel function has been retained because of its small argument. Condition (18) is generally lenient for low harmonics, but becomes progressively restrictive as the harmonic number increases.

From the analytical point of view, the present model depicts a much simplified picture of the electron behavior in the wiggler plus guide field. It nevertheless serves the purpose of a focused look at a new physical effect which appears to have clarified an unexpected aspect of the MIT experimental results [11]. Perhaps more significantly, the gyroresonance effect also suggests a potentially promising harmonic FEL scheme as discussed below.

Radiation generation at the harmonic frequencies has often been limited by problems of mode competition, insufficient gain, low efficiency, etc., which get worse with increased harmonic number. Thus, harmonic FEL operation generally requires a mechanism that selectively enhances the competitiveness of a desired harmonic. Methods for mode or harmonic selection include resonator tuning [12], magnetic-field tapering [13], signal injection [14], mode-orbit differential efficiency [15], periodic positioning interaction [16], harmonic wiggler field components [17], orbital harmonic enhancement by the focusing magnet [18], etc. In the present case, the axial guide field could conceivably be explored as a harmonic selection mechanism. With the guide field tuned near to the nth wiggler harmonic ($\bar{\Omega}_\parallel \simeq n k_w \bar{v}_\parallel$), Eq. (14) shows that the fundamental-harmonic component of the quiver velocity will be reduced by a factor approximately equal to n, whereas its nth-harmonic component is resonantly enhanced. The harmonic quiver velocity corresponds to an effective wiggler period of λ_w/n. Consider a 700-kV electron beam in a wiggler with $\lambda_w = 3$ cm as an example, a guide field of ~ 38 kG will be sufficient to reach the fifth wiggler harmonic. The appropriate beam source for such an application is preferably annular in shape with a reasonably large radius (r_g). So, upon resonant action in the wiggler region, the electrons will acquire a substantial harmonic quiver velocity with a minimum cross-sectional velocity spread. On the other hand, there are factors that are expected to smear out the harmonic resonance. Because of its finite orbital radius (r_b), an off-axis electron does not stay in a constant axial field [see Eq. (10)]. Further, there will be space-charge force and orbital oscillations of one kind or another, giving rise to an effective spread in real and velocity spaces. All these subtle details have not been incorporated in the present model. To further assess the feasibility and limitations of this scheme, a more detailed analysis needs to be conducted to address these issues.

The authors wish to thank G. Bekefi for informative discussions and a preprint of their experimental observations. This work is sponsored by the National Science Council of Taiwan, Republic of China, and the Air Force Office of Scientific Research.

[1] L. Friedland, Phys. Fluids **23**, 2376 (1980).

[2] H. P. Freund and A. T. Drobot, Phys. Fluids **25**, 736

(1982).
[3] P. Diament, Phys. Rev. A **23**, 2537 (1981).
[4] H. P. Freund and A. K. Ganguly, IEEE J. Quantum Electron. **21**, 1073 (1985).
[5] J. Fajans, D. A. Kirkpatrick, and G. Bekefi, Phys. Rev. A **32**, 3448 (1985).
[6] J. A. Pasour, F. Mako, and C. W. Roberson, J. Appl. Phys. **53**, 7174 (1982).
[7] T. C. Marshall, S. P. Schlesinger, and D. B. McDermott, in *Advances in Electronics and Electron Physics,* edited by L. Marton and C. Marton (Academic, New York, 1980), Vol. 53, p. 47.
[8] R. H. Jackson, S. H. Gold, R. K. Parker, H. P. Freund, P. C. Efthimion, V. L. Granatstein, M. Herndon, A. K. Kinkead, J. E. Kosakowski, and T. J. Kwan, IEEE J. Quantum Electron. **19**, 346 (1983).
[9] S. H. Gold, D. L. Hardesty, A. K. Kinkead, L. R. Barnett, and V. L. Granatstein, Phys. Rev. Lett. **52**, 1218 (1984).
[10] J. Fajans, G. Bekefi, Y. Z. Yin, and B. Lax, Phys. Rev. Lett. **53**, 246 (1984); Phys. Fluids **26**, 1995 (1985).
[11] M. E. Conde and G. Bekefi, MIT Report No. PFC/JA-91-19 (to be published); in Conference Record of the IEEE International Conference on Plasma Science, Williamsburg, 1991 (unpublished), p. 176.
[12] S. V. Benson and J. M. J. Madey, Phys. Rev. A **39**, 1579 (1989).
[13] H. P. Freund, Phys. Rev. A **37**, 3371 (1988).
[14] P. E. Latham, B. Levush, T. M. Antonsen, Jr., and N. Metzler, Phys. Rev. Lett. **66**, 1442 (1991).
[15] M. J. Schmitt and C. J. Elliott, Phys. Rev. A **34**, 4843 (1986).
[16] H. Bluem, R. H. Jackson, H. P. Freund, D. E. Pershing, and V. L. Granatstein, Phys. Rev. Lett. **67**, 824 (1991).
[17] M. J. Schmitt and C. J. Elliott, IEEE J. Quantum Electron. **23**, 1552 (1987).
[18] H. Bluem, H. P. Freund, and C. L. Chang, Nucl. Instrum. Methods Phys. Res., Sect. A **272**, 579 (1988).