Anomalous Noise Distribution of the Interface in Two-Phase Fluid Flow

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The fluctuations of the advancing interface observed in experiments on two-phase fluid flow in porous media are investigated using several evaluation techniques. We find that the amplitudes η of these fluctuations are distributed according to a power law of the form $P(\eta)\sim\eta^{-(\mu+1)}$, with $\mu\approx 2.7$. Our results together with a recent model of anomalous roughening suggest a plausible explanation for the nonuniversal values of the exponents observed in the experiment on two-phase flow.

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The growth of rough surfaces and interfaces under far-from-equilibrium conditions is a very common phenomenon in nature [1]. Examples include such processes as vapor deposition, crystallization, thin-film growth by atomic beams, sedimentation and settling of granular materials, and fluid flow in porous media. Small perturbations or noise play an essential role in the development of many of these surfaces. In particular, if the growth is such that the surface is marginally stable, i.e., perturbations of a smooth surface neither grow nor die out exponentially with time, the nature of the noise plays a fundamental role. It is therefore of great interest to investigate the actual noise distribution in laboratory experiments.

In this Letter we present the results of the first experimental study of the noise distribution in the evolution of a rough interface in an experiment on two-fluid displacement in porous media. When a fluid invades a porous medium and displaces a less viscous fluid, the interface is acted on by a combination of forces, such as the interfacial tension, which tend to smooth it out. These stabilizing forces are counteracted by the perturbations arising from the filling of various size pores in the random medium. The dynamic balance of these forces leads to the formation of a fluctuating interface that grows with time [2]. Our data suggest that in the case where the invading fluid wets the porous media [2], the distribution of the apparently random jumps of the interface have a power-law distribution without an apparent cutoff at small scales.

The investigation of rough surfaces and interfaces has gained considerable impetus in recent years [1], mainly due to the introduction of the dynamic scaling description [3] and the development of a nonlinear equation [4] for describing the spatial and temporal evolution of rough surfaces. The growth process typically starts with a smooth surface which roughens with time t. The development of the interface can be characterized by the width w, which obeys the dynamic scaling form [3] $w(L,t)\sim t^{\beta}f(t/L^{\alpha/\beta})$. Here, β is the growth exponent, L is the linear extension of the surface, and $f(x)$ is a scaling function which is constant for $x\ll 1$ and scales as $x^{-\beta}$ when $x\to\infty$.

The question of the form of noise and its effects on the values of α and β has been a topic of much discussion in some of the recent theoretical studies [5–12]. The main theoretical approach [4] for studying the dynamics of rough surfaces is based on a nonlinear Langevin-type equation, proposed by Kardar, Parisi, and Zhang [4] (KPZ equation), for the height fluctuations $h^*(\mathbf{x},t)$ of the growing interface in a frame moving with a velocity equal to that of the lateral growth,

$$\frac{\partial h^*}{\partial t}=\nu\nabla^2 h^*+\frac{\lambda}{2}(\nabla h^*)^2+\eta(\mathbf{x},t)\,, \tag{1}$$

where ν is the surface tension, λ is the correction to the velocity due to a local tilt, and $\eta(\mathbf{x},t)$ is the noise term. In the KPZ paper [4] the noise was assumed to be a δ-correlated white noise $\langle\eta(\mathbf{x},t)\eta(\mathbf{x}',t')\rangle=2D\delta^d(\mathbf{x}-\mathbf{x}')\times\delta(t-t')$, with a Gaussian distribution, which led to universal exponents $\alpha=\frac{1}{2}$ and $\beta=\frac{1}{3}$ in two dimensions, and to the general scaling relation [13] $\alpha+\alpha/\beta=2$ in all dimensions. However, the available experimental results [2,14–16] for α in two dimensions are in the range $0.73<\alpha<0.9$ and seem to disagree with the values of the above predictions for the exponents.

In addition to δ-correlated noise, several other kinds of noise distributions have been studied [5–12], which may be relevant to experiments. (i) Medina, Hwa, and Kardar [5] carried out calculations with spatially and temporally correlated Gaussian noise; (ii) Zhang [6] introduced a power-law noise with a non-Gaussian distribution of amplitudes $P(\eta) \sim \eta^{-(1+\mu)}$; and (iii) Kessler, Levine, and Tu [12] proposed a quenched, δ-correlated noise of the form $\langle \eta(\mathbf{x},h^*)\eta(\mathbf{x}',h'^*)\rangle = g^2\delta(\mathbf{x}-\mathbf{x}')\delta(h^*-h'^*)$. It is therefore of great interest to obtain data on the noise distribution in an experiment because of its importance in resolving some of the fundamental issues that have been raised.

We have investigated the growing interface in a two-phase viscous flow in porous media using various approaches to extract the noise spectrum from an extensive set of digitized images of the fluctuating interface. The conditions of the experiments were the same as reported in Ref. [2]. The experimental setup was a linear Hele-Shaw cell with randomly distributed glass beads between the plates. Glycerol with 4 vol.% water was injected at a fixed flow rate into the air between the plates along a line at one of the shorter sidewalls. The average velocity of the interace was 70–100 nm/sec and its roughness was generated by the almost complete filling of the random distribution of pores and voids between the glass beads. The stabilizing effect of the pressure distribution in the glycerol was negligible, because of the low applied flow rates used in the experiments.

The evolution of the two-fluid interface was recorded on a videotape and digitized with 768-pixel horizontal resolution, which defined the size of our system in pixels ($L=768$). A typical set of digitized interfaces is given in Fig. 1 of Ref. [2]. We analyzed the data from the moment the glycerol entered the plates along one of the shorter sides ($t=0$) until the average height of the interface reached $\frac{3}{4}$ of the cell size ($t=T$). Because of the very fast sampling rate ($\simeq 0.28$ sec), two successive digitized interfaces differed from one another only at a few points. In all our calculations the digitized interfaces were used and averages were taken over a number of independent experiments with the same system.

We used two different definitions to extract the noise distribution from the digitized data. In both cases we defined a reference interface $h(x,t_2)$ for all surfaces in the data set $h(x,t_1)_{t_1 \in [0,\ldots,T]}$. These reference interfaces were defined using the condition that the shift $\Delta h = \langle h(t_2)\rangle_{x\in[1,\ldots,L]} - \langle h(t_1)\rangle_{x\in[1,\ldots,L]}$ between the average heights of the surface pairs had to be a fixed value. Because of the constant injection rate this corresponded to $t_2 = t_1 + \tau$ with τ proportional to Δh.

Using the first definition we calculated the deviations from the average position of the interface $\tilde{h}(x,t) = h(x,t) - \langle h(t)\rangle_x$, for a set of t_1 and t_2 values, where averages were taken over $x \in [1,\ldots,L]$. Then the noise was defined as the difference of these deviations normalized by the shift Δh (see Fig. 1):

$$\eta(x,t_1) \equiv \frac{\tilde{h}(x,t_2) - \tilde{h}(x,t_1)}{\Delta h}. \tag{2}$$

Although the above definition of noise is very simple and natural, certain technical problems are associated with its calculation. Independently of whether the interfaces come from experiments or numerical simulations, there is a small probability of finding two interfaces separated exactly by Δh, because the values of the average heights are not integer. Consequently, the statistics in such a calculation is very poor. To avoid this problem in the evaluation of our data we introduced a parameter $\delta h \ll \Delta h$ and defined the reference interface as having an average height $\langle h(t_2)\rangle_x$ nearest to $\langle h(t_1)\rangle_x + \Delta h$ in the interval

$$[(\langle h(t_1)\rangle_x + \Delta h - \delta h), \ldots, (\langle h(t_1)\rangle_x + \Delta h + \delta h)]. \tag{3}$$

Then the actual shift $\Delta h' = \langle h(t_2)\rangle_x - \langle h(t_1)\rangle_x$ between the compared surfaces is close to Δh, but not exactly the same. The limit theoretically would be $\Delta h \to 0$ and $\delta h \to 0$ while $\Delta h/\delta h \to \infty$. In practice, obviously this is impossible. As the best possible choice we considered the parameters of the data sets and the limits described above and we chose $\Delta h = 1$ and $\delta h = 0.03$ in our calculations. These values are in the range where the results are not sensitive to the actual choice for Δh.

We determined $\eta(x,t)$ for different experimental runs. The open circles in Fig. 2 show the corresponding noise distribution for $\eta(x,t) > 0$ averaged over the experiments. Our experimental data points can be fitted by a straight line on a log-log plot indicating a power-law decay of the noise amplitudes distribution. The exponent of the corresponding power-law behavior $P(\eta) = c\eta^{-(\mu+1)}$ is $\mu = 2.67 \pm 0.19$.

Zhang [6] and others [8,11] have demonstrated that in a number of discrete surface growth models with power-law noise and in a mean-field approximation [7,9], the roughening exponent α depends on the value of μ. For $\mu = 2.67$ the numerical simulations of the Zhang model [6,8] and its variants [8,11] give estimates for α close to 0.8. These results are in remarkably good agreement with our earlier experimental value of $\alpha \simeq 0.81$ for the

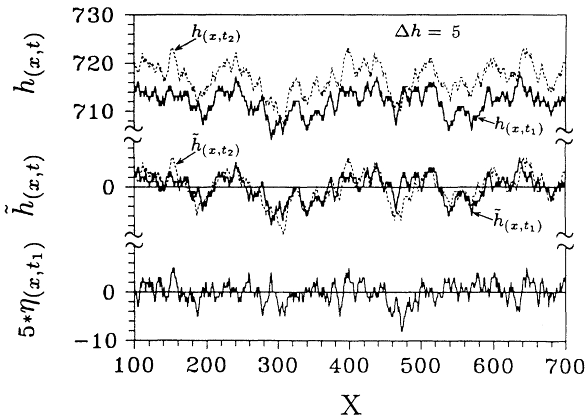

FIG. 1. The noise is defined as the random jumps between the surfaces $h(t_1)$ and $h(t_2)$ normalized by $\Delta h = \langle h(t_2)\rangle - \langle h(t_1)\rangle$ in the moving frame $\langle \tilde{h}(t)\rangle = 0$.

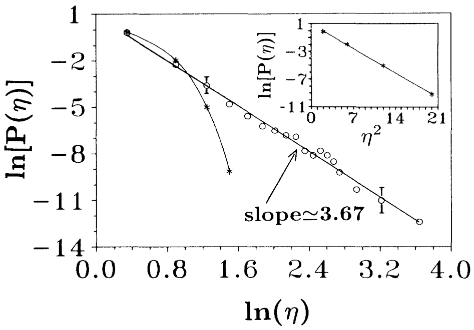

FIG. 2. The logarithm of the distribution $P(\eta)$ for $\Delta h=1$ and $\delta h=0.03$ plotted against the logarithm of the noise amplitude η. The data points plotted by circles and stars correspond to data from experiments and computer simulations, respectively. The uncertainty associated with the measured η values is illustrated by representative error bars. Inset: The logarithm of $P(\eta)$ plotted against η^2 for the RSOS model, demonstrating that the noise distribution in the computer simulations of this model is Gaussian.

present system [2]. Amar and Family [8] have simulated a model with a power-law noise distribution and found that the height fluctuations have the same distribution as the input noise. This demonstrates that the presence of a power-law distribution of height fluctuations is directly related to the presence of a power-law noise distribution. However, even though our results agree well with the recent results on growth anomaly, it is not obvious how the input noise arising from the presence of the porous media is transformed by the surface dynamics into the output noise measured experimentally from the set of digitized interfaces.

As a nontrivial test of our approach we determined the noise distribution in a computer model. We generated surfaces using the restricted solid-on-solid (RSOS) model of Kim and Kosterlitz [17] for a system of size $L=768$ and randomized sample rate as was used in our experiments. The measured noise distribution is denoted by stars in Fig. 2, which shows the expected Gaussian noise distribution in the computer model. This comparison underlines the highly nontrivial and unexpected nature of our experimental finding. To emphasize this point further, in the inset of Fig. 2, the logarithm of $P(\eta)$ is plotted against η^2 for the RSOS model, demonstrating that the noise distribution in the computer simulations of this model is Gaussian.

We also determined the noise distribution directly from the KPZ equation (1) by writing $\eta_{\rm KPZ}=\partial h^*/\partial t-(\lambda/2)(\nabla h^*)^2-\nu\nabla^2 h^*$. Using our data sets we numerically determined the derivatives in this expression. The only unknown parameters are the values of the constants λ and ν. We were able to calculate λ using the derivatives, because for large system sizes and/or periodic boundary conditions $\langle\nabla^2 h^*(x)\rangle_x=0$ and as in the case of the KPZ equation we assumed that $\langle\eta(x,t)\rangle=0$. Then from Eq. (1), λ is given by $\lambda=2\langle(\partial h^*/\partial t)\rangle_x/\langle(\nabla h^*)^2\rangle$. Unfortunately the parameter ν is still unknown. We calculated the noise for several values of ν in order to investigate the effect of ν on $\eta_{\rm KPZ}$. There is only a small difference between the noise distribution obtained from the KPZ interpretation and the earlier direct method. The data can be fitted well in both cases by a power-law function with a slight dependence on the value of ν in the range 0 to 5. The slope in the KPZ case is $\mu=2.5\pm0.3$. The larger error bar is due to the more noisy data. The reason for this difference in the error bars is that $\eta_{\rm KPZ}$ is much more sensitive to digitization errors. The details of this calculation will be given in a subsequent paper.

An alternative form of the noise correlation has been proposed by Medina, Hwa, and Kardar [5] who defined the noise through the spectrum $D(\mathbf{k},\omega)$ as

$$\langle\eta(\mathbf{k},\omega)\eta(\mathbf{k}',\omega')\rangle=2D(k,\omega)\delta^{d-1}(\mathbf{k}+\mathbf{k}')\delta(\omega,\omega')$$

with a power-law behavior of the form $D(k,\omega)\sim|\mathbf{k}|^{-2\rho}\omega^{-2\theta}$. For $\theta=0$ and for small ρ it has been found [5,10] that the exponent α is the same as in the uncorrelated case. For $\rho>\rho_c$ ($\rho_c=0.25$ in $d=1+1$) the exponent α was found to increase with ρ linearly up to $\alpha=1$ at $\rho=1$. According to these results [5,10] it is important to measure the correlations in the noise because long-range spatial correlations may also lead to higher values of the roughening exponents. Figure 3 shows the spatial correlation function $C(x)\equiv\langle\eta(x',t)\eta(x'+x,t)\rangle_{(x',t)}$ vs x (measured in pixels) for our experiments. These results indicate that the noise correlations in the experiment are short ranged and completely die out at about 10 pixels, similarly to the behavior we observed for the simulated surfaces. We also calculated the temporal noise correlation function $C(t)\equiv\langle\eta(x,t')\eta(x,t'+t)\rangle_{(x,t')}$ for the experimental and computer-generated surfaces. In both cases the correlations were found to vanish for $t>\tau$.

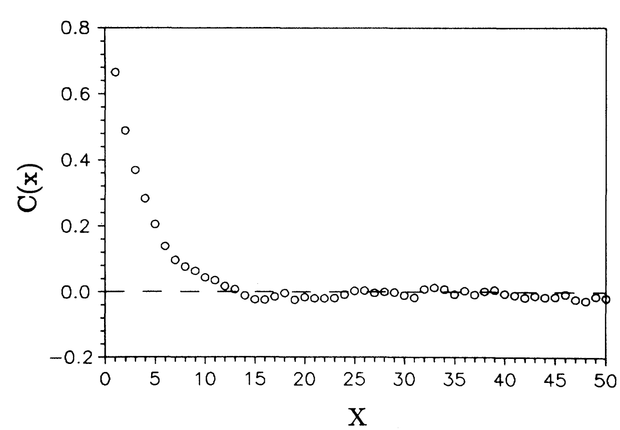

FIG. 3. The spatial correlation function $C(x)$ plotted vs x (measured in pixels) for the noise in the two-fluid displacement experiment. It demonstrates the short-ranged nature of the noise correlations which cannot give rise to anomalous growth.

Recently Martys, Cieplak, and Robbins [18] have simulated two-fluid flow in porous media using a quasistatic model. They simulated the medium by a triangular array of disks with random radii and they represented the interface by a sequence of arcs between pairs of disks with radii that were determined by the surface tension and the pressure. Their scaling studies [18] showed that in the case of a wetting fluid the interface is a self-affine fractal with a roughness exponent $\alpha = 0.81 \pm 0.04$ in close agreement with our experimental result [2]. In addition, they found that the distribution of invaded areas, when a single arc becomes unstable, has a power-law form. Since the distribution of invaded areas is related to the integral of the noise as defined by Eq. (2), this numerical finding is consistent with our experimental result.

To summarize, we have found that the noise distribution in experiments on two-phase flow in porous media has a power-law distribution. This result agrees with Zhang's suggestion that the anomalous values found in the experiments could be due to the presence of a power-law noise. However, there does not exist a first-principles explanation for this unusual result. In addition, it is not clear how the particular value of μ found in the experiments is selected. Because of the dynamic nature of the noise, it is quite possible that the value of μ could be nonuniversal. These questions clearly need to be explored both theoretically and by experiments on other systems.

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