Comment on "Second-Order Symmetric-Asymmetric Phase Transition of Randomly Connected Membranes"

In their Letter [1] Coulon, Roux, and Bellocq present static light-scattering results in systems of salted water, pentanol, and sodium dodecyl sulfate (SDS) as a function of SDS and pentanol concentrations. Close to a critical concentration ϕ_c (where ϕ is the total concentration of SDS and pentanol) they observe a strong divergence of the small-wave-vector limit of the density-correlation function,

$$\lim_{q\to 0} h(q) = \langle |\rho(q)|^2 \rangle \sim (\phi - \phi_c)^{-1/2}, \quad (1)$$

where $\rho = \phi - \langle \phi \rangle$. They interpret this effect as due to critical fluctuations at a symmetric-asymmetric second-order phase transition of randomly connected bilayers with order parameter η.

We show in this Comment that this interpretation is not adequate and that the strong divergence and the shape of $h(q)$ is due to *tricritical* point fluctuations.

We should follow the same theoretical analysis developed in a previous paper [2] by the authors. The system is represented by a free-energy density

$$\Delta f = H_0(\eta) + \tfrac{1}{2} a\rho^2 + \tfrac{1}{4} b\rho^4 + \tfrac{1}{2} c(\nabla\rho)^2 + \lambda\rho\eta^2, \quad (2)$$

with

$$H_0(\eta) = \tfrac{1}{2} A\eta^2 + \tfrac{1}{4} B\eta^4 + \tfrac{1}{2} C(\nabla\eta)^2.$$

The integration of the fluctuations of ρ leads to a shift of the parameters A and B in H_0 to A' and B' so that eventually B' can be very small leading to a tricritical point.

The scattering function $h(q)$ can be calculated in a Gaussian approximation by neglecting b:

$$h(q) = \frac{1}{a + cq^2} + \lambda^2 H(q), \quad (3)$$

where $H(q)$ is the Fourier transform of the "energy-energy" correlation function $G(r) = \langle \eta^2(0)\eta^2(r) \rangle - \langle \eta^2(0) \rangle^2$ estimated with the effective free-energy density

$$H_0'(\eta) = \tfrac{1}{2} A'\eta^2 + \tfrac{1}{4} B'\eta^4 + \tfrac{1}{2} C(\nabla\eta)^2. \quad (4)$$

In their Letter [1] and in a previous paper [2], the authors calculate $G(r)$ from a Gaussian approximation by neglecting B' in (4). This calculation gives a correct behavior only for *the tricritical point* ($B' = 0$) where it is well known from renormalization-group analysis [3] that in $d = 3$ dimensions the tricritical behavior is associated with a Gaussian fixed point (apart from the effect of marginal operators giving logarithmic corrections). On the contrary, for an ordinary critical point ($B' \neq 0$) the Gaussian approximation is *not* equivalent to a mean-field approximation [3] and the Gaussian estimates for $G(r)$ and $H(q)$ are incorrect. [For an ordinary critical point $H(q)$, $q \to 0$, presents only a very weak specific-heat-type singularity [4].]

In conclusion, the fact that the experiments agree well with the Gaussian calculations of the authors [1,2] now has to be interpreted as the signature of tricritical fluctuations with a scaling form for $H(q)$,

$$H(q) \sim \frac{1}{q^{2-\eta_t}} F(q\xi) \text{ with } \xi \sim (\phi - \phi_c)^{-\nu_t}, \quad (5)$$

and

$$\lim_{q\to 0} H(q) \sim \xi^{2-\eta_t} \sim (\phi - \phi_c)^{-\gamma_t},$$

with $\eta_t = 1$, $\nu_t = \frac{1}{2}$, $\gamma_t = (2 - \eta_t)\nu_t = \frac{1}{2}$.

This has to be compared with the tricritical behavior in ^{3}He-^{4}He mixtures [5] where the tricritical point is approached at constant concentration with renormalized tricritical exponents ($\nu_t' = \nu_t/\phi_t$, $\gamma_t' = \gamma_t/\phi_t$ with $\phi_t = \frac{1}{2}$).

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