Flux-Line Cutting in Superconductors

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The energy barrier for mutual cutting of a pair of twisted flux lines (prepared in a simple entangled configuration) is computed for isotropic and anisotropic superconductors by minimizing their total energy. The cutting barrier for the correctly *curved* flux lines is much smaller than the analytically calculated barrier for *rigid*, parallel flux lines and may even be *negative*, indicating instability of the entangled configuration. Vortex cutting is thus an effective mode of disentanglement of flux lines.

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The observation of a hexagonal flux-line lattice (FLL) in the high-T_c superconductor (HTSC) $YBa_2Cu_3O_7$ at 4.2 K and an *uncorrelated* FLL at 77 K [1] has renewed interest in the ground state and transport properties of extreme type-II superconductors in a magnetic field. A possible interpretation of these observations is in terms of FLL melting. In 3D, estimates exist showing that the FLL may melt well below the upper critical field $H_{c2}^0(T)$ [2–4]. Clarification of the *nature* of the proposed melting transition and the properties of a flux-line *liquid* requires more work.

Weak positional correlations between the flux lines imply that a *single* strong pinning center (if it existed) could not effectively pin *large parts* of the FL system. Flux-line *entanglement* [2] could produce an internal viscosity or shear stiffness which partly restores the lattice nature of the vortex system and enhances pinning of the FLL by a few pinning centers [5]. On the other hand, *collective* pinning [6] from dense randomly distributed weak pinning centers is *most effective* in a flux-line *liquid* [7], since in this case the flux lines can adjust to the pins in an optimal way. Entanglement would then *reduce* pinning.

It was shown in [8] that when one assumes the flux lines to be *rigid* and *parallel*, a large vortex-cutting barrier $\Delta U \approx 2lH_{c1}\Phi_0 = l\Phi_0^2\ln\kappa/2\pi\mu_0\lambda^2$ ($\sim 50kT_c$ in Ba-Sr-Ca-Cu-O) results. Here $\Phi_0 = 2.07\times10^{-7}$ G cm^2 is the flux quantum, λ the magnetic penetration depth, l a minimum characteristic length in the problem (e.g., coherence length ξ), and κ is the Ginzburg-Landau (GL) parameter. However, this estimate shows that a consideration of parallel, straight-vortex configurations is inappropriate for discussing vortex *cutting*, because the factor $\ln\kappa$ originates from the energy in the *far-field region*, which is *not* involved in cutting [9]. A more appropriate treatment of cutting assumes a finite angle α between the rigid flux lines. The result for the cutting energy $\Delta U_{\rm cut} \le \Phi_0^2\cot\alpha/2\mu_0\lambda$ [10] does not have the unphysical $\ln\kappa$ factor, although the assumption of rigid straight vortices still leads to an overestimate of the barriers for vortex cutting. In Ref. [8] essentially infinite relaxation times for vortex disentanglement were found by assuming that vortex cutting does not take place.

To get more insight into the cutting problem, we consider the static configuration of two curved flux lines by minimizing their energy with respect to arbitrary vortex shape subject to appropriate boundary conditions. These boundary conditions should simulate their interaction with the other vortices of the FLL in which they are embedded. We use anisotropic London theory, which applies when $b = B/B_{c2} < 0.2$ and $\kappa \gg 1$; B is the average induction and B_{c2} is the upper critical field.

Within anisotropic London theory, the total energy (interaction energy and self-energy) of a system of arbitrarily distorted vortices is given by [11]

$$U = \frac{\Phi_0^2}{2\mu_0}\sum_{i,j}\int\int dr_j^\alpha dr_i^\beta V_{\alpha\beta}(\mathbf{r}_i - \mathbf{r}_j)\,, \tag{1}$$

where the *tensorial* interaction between vortices is [12]

$$V_{\alpha\beta}(\mathbf{r}) = V_1(r)\delta_{\alpha\beta} + V_s^{\alpha\beta}(\mathbf{r});\quad (\alpha,\beta)\in(x,y,z)\,, \tag{2}$$

$$V_1(r) = \frac{1}{4\pi\lambda_{ab}^2 r}\exp(-\tilde{r})\,, \tag{3}$$

$$V_2^{\alpha\beta}(\mathbf{r}) = \frac{1}{4\pi\lambda_{ab}\rho^2}\left[G_1(\mathbf{r})\delta_{\alpha\beta} + G_2(\mathbf{r})\frac{x_\alpha x_\beta}{\rho^2}\right]. \tag{4}$$

In Eq. (4), we have $(\alpha,\beta)\in(x,y)$ and $V_2^{zz}(\mathbf{r}) = V_2^{xz}(\mathbf{r}) = V_2^{yz}(\mathbf{r}) = 0$ when the *average* vortex direction (z direction) is parallel to the uniaxial symmetry axis $\hat{\mathbf{c}}$. The functions $G_1(\mathbf{r})$ and $G_2(\mathbf{r})$ are given by

$$G_i(\mathbf{r}) = a_i\exp(-\tilde{r}) - b_i\exp(-\tilde{\rho})\,, \tag{5}$$

with $\tilde{r} = r/\lambda_{ab}$, $\tilde{\rho} = (\rho^2 + \Gamma^2 z^2)^{1/2}/\lambda_c$, $a_1 = 1 - a_2$, $b_1 = 1 - b_2$, $a_2 = 2 + \rho^2/\lambda_{ab}^2\tilde{r}$, $b_2 = 2 + \rho^2/\lambda_c^2\tilde{\rho}$, $r^2 = \rho^2 + z^2$, $\rho^2 = x^2 + y^2$, $\Gamma = \lambda_c/\lambda_{ab}$, and λ_{ab} and λ_c are the penetration depths for currents in the a-b plane and parallel to the $\hat{\mathbf{c}}$ axis, respectively.

When flux lines with finite core radii are in close contact, Eq. (1) should be supplemented by a *scalar* core attraction [10] obtainable from GL theory [13]. For simplicity we use only the potential (2), but with a *circular* inner cutoff which simulates this core attraction, achieved by replacing r by $(r^2 + \xi_{ab}^2)^{1/2}$ in (2) [14,15]; ξ_{ab} is the coherence length in the a-b plane, and will serve as our unit length throughout. For an isotropic superconductor, the cutoff *is* circular. As shown in Ref. [12], the cutoff for an anisotropic superconductor is really *elliptical*. For our numerical calculations, we use the simpler circular cutoff scheme. The elliptical cutoff will be used in forthcoming less transparent computations, where it will be shown that the results we obtain for the anisotropic case are changed only quantitatively, but not qualitatively.

To get an analytical estimate, we first consider two rigid, straight flux lines inclined an angle α with respect to each other and with a distance d at the point of their closest approach. We assume the symmetry axis between the vortices to be parallel to the $\hat{\mathbf{c}}$ axis of the uniaxial superconductor.

Rigid flux lines cannot curve locally to lower their interaction energy as they approach each other. The cutting barrier $\Delta U(\alpha,d)$ is then completely determined by their mutual *interaction* energy $U_{\rm int}(\alpha,d)$, $\Delta U = U_{\rm int}(\alpha,d) - U_{\rm int}(\alpha,0)$. For the magnetic interaction between straight vortices in a uniaxial superconductor we find in the present geometry,

$$U_{\rm int}(\alpha,d) = \frac{\Phi_0^2}{4\mu_0}\left[\frac{1}{\lambda_{ab}}\cot\frac{\alpha}{2}\exp\left(-\frac{d}{\lambda_{ab}}\right) - \frac{1}{\lambda_c}\tan\frac{\alpha}{2}\exp\left(-\frac{d}{\lambda_c}\right)\right]. \qquad (6)$$

Note that $U_{\rm int}(\alpha,d\to 0)$ is *finite.* Note also that even in this simple geometry, it is not possible to bring (6) into the form of the *isotropic* result [10] [see Eq. (8) below] simply by a scaling of lengths.

For $\Gamma = \lambda_c/\lambda_{ab} \to \infty$, we have from (6),

$$U_{\rm int}(\alpha,d) \approx \frac{\Phi_0^2}{4\mu_0\lambda_{ab}}\cot\frac{\alpha}{2}\exp\left(-\frac{d}{\lambda_{ab}}\right), \qquad (7)$$

which should be compared with the isotropic result [10] obtained from Eq. (6) when $\lambda_{ab} = \lambda_c$,

$$U_{\rm int}(\alpha,d) = \frac{\Phi_0^2}{2\mu_0\lambda_{ab}}\cot\alpha\exp\left(-\frac{d}{\lambda_{ab}}\right). \qquad (8)$$

For isotropic superconductors it is seen from $U_{\rm int}$ that the electromagnetic interaction between flux lines changes sign when $\alpha = \alpha_0 = \pi/2$. When $\Gamma \to \infty$, $U_{\rm int}$ *never changes sign*, but it reaches the value 0 at $\alpha_0 = \pi$. Thus, for rigid flux lines, we find for $\Gamma \geq 1$ that $\pi/2 \leq \alpha_0 < \pi$. This *increase* of the "neutral angle" α_0 reflects the tendency of supercurrents to flow predominantly in the basal plane in anisotropic superconductors.

From the above remarks, one might conclude that vortex cutting in general is *suppressed* in strongly anisotropic superconductors. However, this conclusion is premature: Vortices in anisotropic superconductors can *lower* their self-energy by increasing α; i.e., they prefer to lie almost in the *a-b* plane.

The *self-energy* per unit length of a straight flux line along $\hat{\mathbf{z}}$ and tilted an angle θ with respect to the $\hat{\mathbf{c}}$ axis follows from (1) [12]:

$$J(\theta) = \frac{\Phi_0^2}{2\mu_0}\int\frac{d^2k_\perp}{4\pi^2}\tilde{V}_{zz}(\mathbf{k}_\perp). \qquad (9)$$

Explicitly, one finds (an *elliptical* cutoff is used)

$$J(\theta) = \frac{\Phi_0^2}{4\pi\mu_0}\frac{\lambda_c}{\lambda_\theta}[g_1(\theta)l_1 + g_2(\theta)l_2], \qquad (10)$$

with $g_1(\theta) = A_1/\gamma_1$, $g_2(\theta) = A_2/\gamma_2$, $A_1 = (\gamma_3-\gamma_1)/(\gamma_2-\gamma_1)$, $A_2 = 1 - A_1$, $\gamma_1 = \lambda_{ab}^2(\lambda_c^2+\lambda_\theta^2)/2\lambda_\theta^2$, $\gamma_2 = \lambda_c^2$, $\gamma_3 = (\lambda_c^2+\lambda_\theta^2)/2$, $l_1 = \ln[\kappa(\lambda_c^2+\lambda_\theta^2)^{1/2}/2^{1/2}\lambda_\theta]$, $l_2 = \ln(\Gamma\kappa)$, and $\lambda_\theta = (\lambda_{ab}^2\sin^2\theta + \lambda_c^2\cos^2\theta)^{1/2}$. The normalized self-energy $J(\theta)$ and line tension $P(\theta) = (1+\partial^2/\partial\theta^2)J(\theta)$ are shown in Fig. 1 for $\kappa = 20$ and two values of Γ. $J(\theta)$ is a rapidly decreasing function of θ for $\Gamma \gg 1$, and it is symmetric around $\theta = 0$ and $\theta = \pi/2$. One has [12]

$$J(0) = \frac{\Phi_0^2\ln\kappa}{4\pi\mu_0\lambda_{ab}^2}; \quad J(\pi/2) = \frac{\Phi_0^2\ln(\Gamma\kappa)}{4\pi\mu_0\lambda_{ab}\lambda_c}. \qquad (11)$$

In large-κ, anisotropic superconductors with $\mathbf{B}\|\hat{\mathbf{c}}$, energy can be gained when vortices curve *locally* to achieve large angles α: It does not cost much self-energy, but reduces the interaction energy. This effect reduces the cutting barriers, illustrating the importance of allowing for local curving of the vortices in the vicinity of their closest approach in a calculation of cutting barriers. The enhanced tendency of FL curving in anisotropic superconductors can also be seen from the *anisotropic, nonlocal* tilt moduli of the FLL [3,12]. This nonlocal effect, contained in (1), implies that it costs little energy to bend a flux line sharply, in contrast to what one would expect from a local *string model* of the flux lines.

For our numerical computations based on (1)-(5), we write the two symmetric vortex shapes $\mathbf{r}_1(z) = [x(z), y(z), z]$ and $\mathbf{r}_2(z) = -\mathbf{r}_1(z)$ with symmetries $x(-z) = x(z)$, $y(-z) = -y(z)$. These shapes are quite general, describing, e.g., a double helix by $x(z) = \cos(kz)$, $y(z) = \sin(kz)$, or our starting configuration $x_{\rm in}(z) = x_\infty + (x_0 - x_\infty)\exp[-(z/z_x)^2]$, $y_{\rm in}(z) = y_\infty\tanh(z/z_y)$. Here, x_∞ is a parameter which describes the extent to which flux lines are twisted, while x_0 is a parameter which we will use to *constrain* the intervortex distance at $z=0$. When x_∞/y_∞ is reduced the vortices become more entangled.

In early computations, we used trial functions which

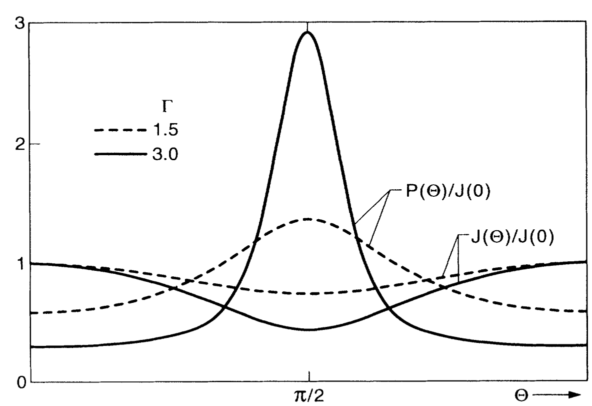

FIG. 1. Line energy $J(\theta)$ (10) and line tension $P(\theta) = J + J''$ of an isolated, straight flux line vs angle θ between the flux line and the $\hat{\mathbf{c}}$ axis for anisotropy ratios $\Gamma = \lambda_c/\lambda_{ab} = 1.5$ and 3. For $\Gamma = 1$ one has $P = J$. For $\Gamma \gg 1$ the maximum in $P(\theta)$ becomes very sharp and high.

had the appropriate boundary conditions built in and allowed for the variation of progressively more parameters. Later, a more accurate, more general, and much faster numerical method was developed which finds the minimum energy by a "viscous" relaxation of $x(z)$ and $y(z)$ by iterating the equations $x(z) \to x(z) - \eta \delta U/\delta x(z)$, $y(z) \to y(z) - \eta \delta U/\delta y(z)$. Here, U is our energy functional for the vortex pair, and η^{-1} a viscosity. Boundary conditions (asymptotically parallel FLs) are enforced by binding the vortices elastically to their initial configurations with a weak restoring force which vanishes in the cutting region $|z| < z_m \approx 20\xi_{ab}$.

The cutting barrier is obtained by prescribing the *closest distance of approach* $d = 2x_0$ and plotting the constrained equilibrium energy $U(d)$ with x_∞ and y_∞ as parameters. If U has a minimum at $2x_0 = d_{eq}$ (global equilibrium) and a maximum (a barrier) at $2x_0 = d_b < d_{eq}$, then a cutting barrier $\Delta U = U_{max} - U_{min}$ must be overcome. With decreasing x_∞, when $d_b = d_{eq}$, the vortices spontaneously collapse and cut. In Fig. 2 we plot U/U_0 when $\kappa = 20$ and $\Gamma = 1$ for various values of the parameter x_∞ as a function of the constraint parameter x_0, with energy unit $U_0 = \Phi_0^2 \xi_{ab}/8\pi\mu_0\lambda_{ab}^2$. Shown in the inset are cutting barriers as a function of the twisting parameter x_∞. We have chosen $\kappa = 20$; the *normalized* cutting barrier $\Delta U/U_0$ depends only weakly on κ. In Fig. 3 we show the corresponding vortex configurations for $x_\infty = -0.6$, $y_\infty = 4$ for various values of x_0 for the isotropic case $\Gamma = 1$. These states are obtained after 1000 relaxation steps from the initial state $[x_{in}(z), y_{in}(z)]$. Very similar ground-state configurations are found in the anisotropic case $\Gamma > 1$, but with vortices slightly more repulsive when held close together at $z = 0$. This is due to both a *harder* vortex interaction, and a *reduction* of line tension for $\Gamma > 1$. At slightly negative values $x_\infty \lesssim -1$, the maximum in F at d_b *disappears*: The vortex pair is unstable at rather small values of the prescribed twist. *Hence even moderate flux-line twisting will result in spontaneous flux-line cutting.*

Comparing our results with the parallel, rigid flux-line estimate, $\Delta U = 2lH_{c1}\Phi_0$ and using $l \approx 2\xi_{ab}$ and $z_m \approx 20\xi_{ab}$, we see that local curving of flux lines gives a reduction of ΔU per unit length by a factor of ~ 10 for $\kappa = 20$ even for a small twist. Hence, curving of the FLs in the vicinity of their closest approach substantially reduces the barriers compared to estimates using rigid flux lines.

In Fig. 4 we plot the normalized cutting barrier $\Delta U/U_0$ for $\kappa = 20$ and various values of Γ as a function of the parameter x_∞. The value of the energy barriers for vortex cutting and the stability of the entangled-vortex configuration *in this geometry* as the mass anisotropy is increased is determined by the balance between three effects: the *reduction* of the line tension for vortices $\sim \parallel \hat{c}$, the *increase* of line tension for vortices $\sim \perp \hat{c}$, Eq. (10) and Fig. 1, and the *hardening* of the interaction between vortices, Eq. (6). It turns out that the latter is the dominating effect, leading to cutting barriers that are slightly *increased* in anisotropic as compared to isotropic superconductors. This result is *not* obvious, as is clear from our discussion following Eq. (10).

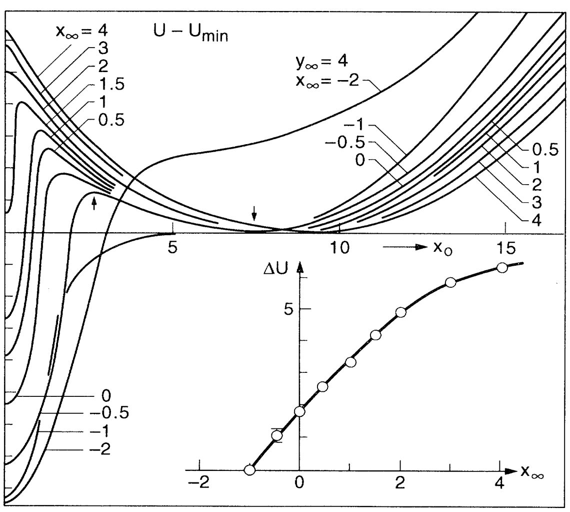

FIG. 2. Total energy $U(x_0)$ of the vortex pair for $y_\infty = 4$, $\kappa = 20$, $\Gamma = 1$, vs the prescribed distance $x_0 = d/2$ with x_∞ as a parameter. U is referred to its minimum U_{min} (equilibrium configuration), except for $x_0 = -2$ where no minimum occurs (unstable configuration). The maximum in $U(x_0)$ defines the energy barrier for vortex cutting, $\Delta U = U_{max} - U_{min}$, plotted vs x_∞ in the inset (error bar indicated with $x_\infty = -0.5$). Energy unit is $U_0 = \Phi_0^2 \xi_{ab}/8\pi\mu_0\lambda_{ab}^2$.

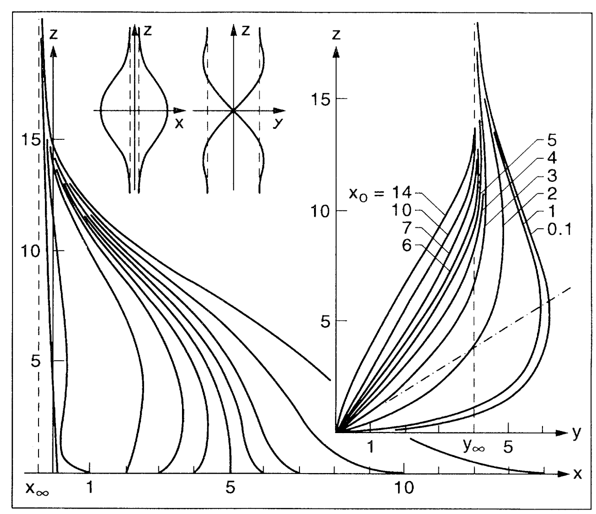

FIG. 3. Computed shapes $\mathbf{r}_1 = -\mathbf{r}_2 = (x,y,z)$ of a vortex pair for $y_\infty = 4$, $x_\infty = -0.6$, $\kappa = 20$, $\Gamma = 1$ (isotropic superconductor). Inset: $x(z)$ is even and $y(z)$ is odd. The closest distance $d = 2x_0 = 2x(0)$ is *prescribed* by a constraining force. Here, the equilibrium configuration [with $x'(0) = 0$] occurs for $x_0 \approx 5$. Length unit is ξ_{ab}.

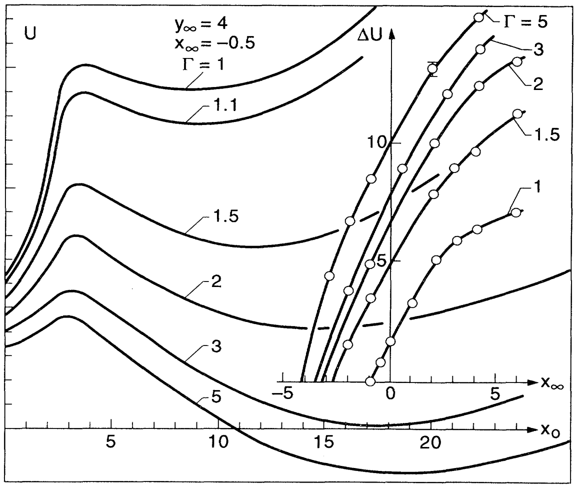

FIG. 4. Total energy $U(x_0)$ for various anisotropy ratios Γ for a case where the vortex pair for $\Gamma=1$ is on the verge of cutting ($x_\infty=-0.5$, $y_\infty=4$, $\kappa=20$). Inset: The energy barriers $\Delta U = U_{\max} - U_{\min}$ vs x_∞ (one error bar indicated). Energy unit is U_0. Note saturation of ΔU with increasing Γ.

In summary, we have studied the ground-state properties of twisted two-vortex configurations in isotropic and anisotropic superconductors. We have done this as a basic step towards a realistic description of the thermodynamics of entangled flux-line liquids. An essential input in our calculations is the exact 3D nonlocal, anisotropic London potential between vortex elements [11,12]. Exact analytical results for cutting of *straight* FLs and numerical results for cutting of *curved* FLs have been obtained. Exact ground states for symmetrically twisted two-vortex configurations have also been obtained by viscous relaxation of prescribed initial configurations. We have demonstrated that it is crucial to allow for curving of the FLs in the vicinity of their closest approach to get quantitatively and even qualitatively correct results for the cutting barriers. Including the self-energy and allowing for curving of the FLs leads to spontaneous vortex cutting, or equivalently, an *instability* of the model configurations we have studied, as soon as the vortices are slightly twisted. This feature will *not* be captured by calculations on rigid vortices. Increasing mass anisotropy slightly increases the cutting barriers and tends to stabilize the twisted-vortex configuration.

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