Synchronization of Parametrically Pumped Electron Oscillators with Phase Bistability

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Stochastic motions of parametrically pumped electron oscillators synchronize abruptly when the pump power exceeds a threshold and the oscillators are radiatively cooled in a cylindrical Penning trap. Self-organized collective behavior far from thermal equilibrium is manifested by the coherent, phase-bistable motion of the electrons' center of mass, which was observed. Synchronized electrons are an ideal probe of the radiation field of a trap cavity, opening the way to a new generation of electron magnetic-moment measurements and to sideband cooling of an electron motion to mK temperatures.

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Centuries ago, Huygens observed that pendula of two clocks on a wall tend to synchronize [1]. More recent efforts to characterize large dynamical systems have generated increased interest in larger systems of coupled oscillators. Concepts and techniques of phase-transition theory are applicable to lasers, for example, providing a fruitful analogy to critical phenomena in a ferromagnet [2]. Also, modest-sized arrays of Josephson-junction oscillators synchronize when they produce high-frequency microwaves, being coupled via a common load of passive circuit elements [3]. Large systems of well-characterized, coupled oscillators are difficult to realize under good control in the laboratory and no unified theoretical approach has yet emerged for collective behavior far from thermal equilibrium. Nonetheless, a few systems of coupled limit-cycle oscillators reveal recognizable cooperative phenomena such as oscillator synchronization, "clustering," and "attractor crowding" [4]. Examples include Van der Pol oscillators [5] and an "active rotator" model [6] which are studied using numerical solutions of coupled differential equations [5], using coupled iterative maps [4,7], as well as using generalized mean-field approaches [6].

In this Letter we report the discovery of self-organized [8], collective motion of a system of well-controlled, collisionally coupled electron oscillators suspended in a Penning trap. The oscillators are far from thermal equilibrium insofar as they are strongly pumped parametrically and they continuously dissipate energy. A unique feature is that the oscillators synchronize to produce an observable, coherent motion of their center of mass (CM) at half the frequency of the pump. Time translation symmetry requires that any such response be bistable in phase relative to a subharmonic of the parametric pump. The collective motion is self-organized insofar as the choice between the bistable phases depends upon the internal motions of the electrons (not upon the external pumping field) and characteristically requires sufficient energy dissipation [8]. Transitions between the bistable phase states depend upon the internal energy of the oscillators (relative to their CM), reminiscent of a two-state system coupled to a thermal bath. This energy is varied by tuning the radiative dissipation to the cold microwave cavity formed by electrodes of a specially designed, cylindrical Penning trap. Likely applications include a thousandfold reduction in the axial temperature of a trapped elementary particle, a new generation of measurements of the electron magnetic moment which are no longer limited by cavity frequency shifts or by damping linewidths, and radiative cooling of cryogenic electron plasmas and internal motions of molecular ions at adjustable rates.

The trap cavity (Fig. 1) has its vertical axis $\hat{z}$ along the axis of a 6-T magnetic field from a superconducting solenoid. Slits perpendicular to the magnetic field divide oxygen-free high-conductivity copper cavity walls into two end-cap electrodes (at z_0 above and below the trap center), a ring electrode (with radius ρ_0), and two compensation electrodes. The Penning trap formed by tuning the potentials of these electrodes produces an electric quadrupole potential near the center which is pure

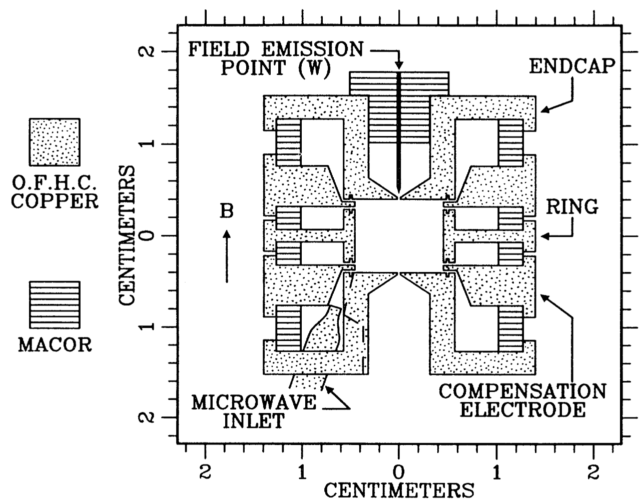

FIG. 1. Orthogonalized cylindrical trap cavity [to scale with $z_0=0.3844(4)$ cm and $\rho_0=0.4564(4)$ cm at 4 K]. A spatially uniform magnetic field ($\Delta B/B < 10^{-6}$ over $z_0/10$) is along the vertical axis.

enough to observe and study even a single electron [9] ($C_4 < 10^{-5}$ and $C_6 \approx -10^{-1}$ in common notation [10]). Familiar motions of a trapped electron [10] include a cyclotron orbit around magnetic-field lines (at frequency $\omega_c'/2\pi < 170$ GHz) and a harmonic axial oscillation along the magnetic-field direction $\hat{\mathbf{z}}$ (at frequency $\omega_z/2\pi = 63$ MHz). The latter motion is detected and damped (at a rate γ_z) by coupling to an *LCR* circuit nearly resonant at ω_z. To approximate an ideal, cylindrical microwave cavity, the trap was precisely constructed with small slits (0.015 cm) that incorporate choke flanges ($\lambda/4$ at 164 GHz). The circuit and trap are kept near 4.2 K by thermal contact to liquid helium.

An oscillatory, radio-frequency potential at $\omega_d \approx 2\omega_z$ (applied between one end cap and the ring) modulates the trapping potential and thus parametrically pumps the axial motion of N centered electrons at $\omega_d/2 \approx \omega_z$. Axial CM motion induces a potential (across a resonant *LCR* circuit connected between the opposite end cap and ring) which when squared is proportional to the energy in this motion. Power dissipated in the resistor damps this CM motion at a rate $N\gamma_z$, where the damping rate for a single electron γ_z is (30 ms)$^{-1}$ at maximum. This electrical setup is similar to that used for other experiments which will be mentioned [11-13]. Additional electronics, similar to that used for one electron [14], measures the response at $\omega_d/2$ that is phase coherent with the parametric pump. Because the pump is at twice the response frequency, any coherent (steady-state) response of the electron cloud (not previously observed) must have either of the bistable phases that differ by 180°.

The parametric pump can excite the axial CM motion of the cloud of N trapped electrons. We use a dimensionless CM coordinate $Z = \sum z_i/Nd$, where z_i/d is the axial position of one electron scaled by a suitable trap dimension. A rigid axial motion of N electrons near the trap center has the same differential equation as that for a single particle except with N times larger damping,

$$\ddot{Z} + (N\gamma_z)\dot{Z} + \omega_z^2[1 + h\cos(\omega_d t)]Z + \lambda_4 Z^3 + \lambda_6 Z^5 = 0\,, \tag{1}$$

where h is the pump strength. The nonlinear (anharmonic) terms, with strengths λ_4 and λ_6, arise from unavoidable or deliberate distortions of the pure electrostatic quadrupole potential of an ideal Penning trap [10]. Nontrivial steady-state solutions to Eq. (1) are limit-cycle oscillations [15] with an abrupt threshold at pump strength $h = h_T \equiv 2N\gamma_z/\omega_z$. For an isolated electron ($N=1$), this equation is an exact description of the unavoidable rigid motion which has been observed [14]. The generalization of Eq. (1) which describes the general, nonrigid motion of N electrons has anharmonic terms which depend upon the axial and radial coordinates of the individual electrons (z_i and ρ_i), rather than upon Z alone. For example, Z^3 becomes $\sum_i (z_i^3 - 3z_i\rho_i^2/2)/N$.

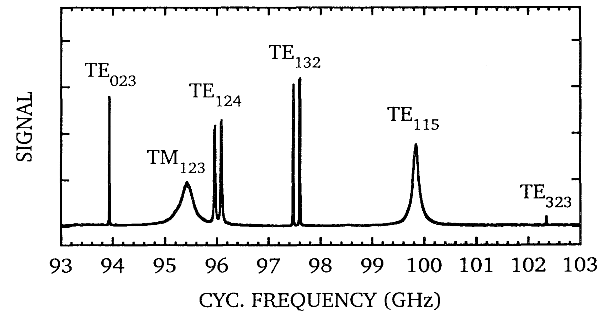

FIG. 2. Sample cavity mode spectrum (taken in 34 min) with a signal proportional to energy in the axial CM motion.

Rigid motion of many electrons is typically prevented by the large, stochastically changing coordinates (z_i,ρ_i) of the individual electrons. Below the threshold (i.e., $h < h_T$), the steady-state solution to Eq. (1) is $Z=0$ so only internal motions (relative to the CM) can possibly be excited by the parametric driving force. Such excitations are occurring, perhaps because the resonant frequencies of internal motions are broadly distributed by the Coulomb repulsion of the electrons, but the exact mechanism is not certain. Energy coupled into the CM motion by the anharmonic nonlinearities is observed in the form of incoherent transients which are damped by the *LCR* at a rate of order $N\gamma_z$, with a coherence time < 1 ms. This regime is well described by a "bolometric model" which treats the electron cloud as a gas which comes into thermal equilibrium via collisions between electrons [11]. However, no synchronized, coherent motion is anticipated or can be accounted for in this model.

If we cool the internal motions of the electrons, we enter a new regime where Eq. (1) approximates important observed features. Cooling by the *LCR* circuit does not suffice. However, electron-electron collisions transfer some internal energy to the cyclotron motions of the electrons. This energy is removed by coupling the cyclotron motion to a resonant mode of the cold, cylindrical cavity (e.g., Fig. 2). Cooled electrons switch abruptly from independent, stochastic motions to a highly synchronized motion [Fig. 3(a)] as the strength of the parametric pump at $\omega_d = 2\omega_z$ is increased by less than 0.5 dB across the threshold $h = h_T$. The amplitude of the CM motion increases by orders of magnitude and this motion is phase coherent with the subharmonic of the pump at $\omega_d/2$. We establish that $h_T = 2N\gamma_z/\omega_z$ by varying the electron number between $N=60$ and 18000 and by detuning ω_z from resonance with the *LCR* to change γ_z. (N is determined from the linewidth of a directly driven axial resonance.) Above the threshold, any small symmetry-breaking fluctuation in the CM location ($Z \neq 0$) begins to increase exponentially as the parametric pump overcomes the resistive damping. The anharmonicities shift the resonant fre-

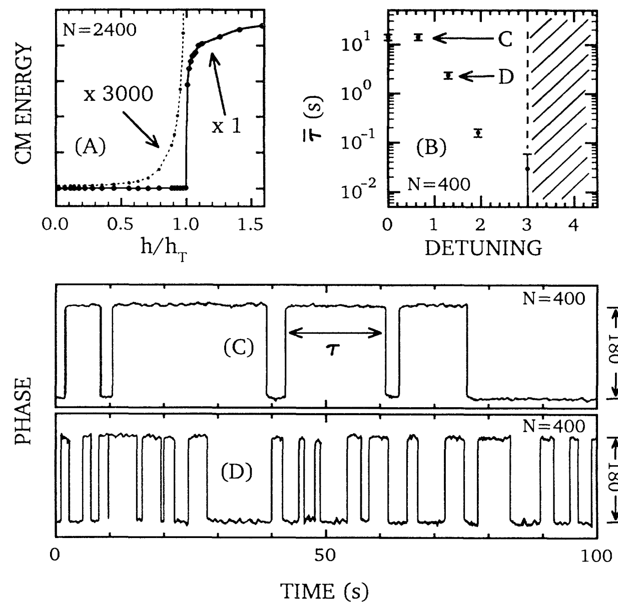

FIG. 3. (a) Abrupt threshold in axial CM energy vs pump strength for 2400 electrons resonant with the TE_{115} mode. (b) Mean time $\bar{\tau}$ between flips vs detuning, $2(\omega_c - \omega_{115})/\gamma_{115}$, for $N=400$ electrons. Transitions in CM phase in (c) and (d) at two detunings.

quency as the amplitude increases, arresting the rapid growth at a steady-state amplitude. Tuning the trap potentials to make either the λ_4 or the λ_6 term dominate yields different characteristic dependences of this amplitude upon pump frequency.

Figure 2 shows the large CM excitations which occur as the magnetic field B is swept to bring ω_c' into resonance with several of more than 100 resonant cavity modes observed below 170 GHz. The modes are identified as transverse electric TE_{mnp} or transverse magnetic TM_{mnp} using resonant frequencies which correspond well to those calculated for a perfect cylindrical cavity [16], typically to a percent or better. TE_{1np} and TM_{1np} modes with p odd give the strongest observed signals (i.e., largest areas) and have quality factors as high as $Q=10^4$. They couple most strongly to electron cyclotron motions owing to a nonvanishing transverse electric field at the trap center. Isolated modes fit well with Lorentzian line shapes, with exceptions in two interesting cases. First, for modes with standing-wave nodes at the trap center, the electron cloud sees a microwave field that is amplitude modulated by its driven motion at $\omega_d/2 \approx \omega_z$. Such modes (e.g., those mentioned above except with p even) thus produce two Lorentzians split by ω_d as illustrated in Fig. 2. One peak is seen again when we apply a dc offset potential to the electrodes to shift the electron cloud along the magnetic field by $\lambda/4$ to an antinode of the standing-wave field. (Such studies suggest a cloud size $< z_0/10$ for $N < 3000$.) The second exception is the broadening and splitting observed when an electron cloud and a cavity mode are strongly coupled, i.e., when N and Q are sufficiently large that the coupling time for the cloud and mode is shorter than the decay time for the cavity mode itself.

For electrons resonantly cooled by a cavity mode, the coherent CM motion above threshold is equally likely to have either of the two steady-state phases which differ by 180°. We observe abrupt transitions between the two phases (similar to those attributed to a much poorer vacuum in an early experiment with only one electron [14]). Thousands of flips observed over many hours show the flips to be random with an exponential distribution of transition times. Figure 3(b) shows the rapid decrease in mean time between flips $\bar{\tau}$ for a cloud of $N=400$ electrons as the frequency ω_c' is detuned from resonance with the TE_{115} cavity mode in Fig. 2. Transitions occur least rapidly very near to resonance with a cavity mode [Fig. 3(c)] where the internal motion is most strongly cooled. The transition rate $\bar{\tau}^{-1}$ increases [Fig. 3(d)] when a slight detuning of ω_c' from the mode resonance allows the internal energy to rise. Further off resonance [hatched region in Fig. 3(b)], the internal energy increases sufficiently so that the random, desynchronized motions of the electrons keep a detectable coherent CM motion from developing because of the nonlinear couplings. The transition rate increases rapidly with increasing internal energy. Consistent with this interpretation, an increase in the pump power or a stochastic modulation of ω_z (by applying a broadband noise potential to the ring) also increases the transition rate. Since the fluctuating motions of a larger number of electrons average to a smaller sized fluctuation of their CM, the transition rate decreases rapidly with increasing electron number. For $N > 2500$, no transition is observed over hours when ω_c' is resonant with the cavity mode.

As a practical application, synchronized electrons allow the identification of the radiation modes of a trap cavity for the first time, which should prompt a new generation of electron magnetic-moment measurements. These important measurements provide the most accurate comparison of theory (quantum electrodynamics) and experiment for an elementary particle and have alternatively provided the most precise value for the fine-structure constant. Past measurements [17] employed hyperbolic Penning trap cavities whose microwave properties were almost entirely unknown experimentally, and whose resonant modes have only been crudely calculated numerically [18]. A cavity-modified spontaneous emission rate for one electron in a hyperbolic trap cavity first focused attention upon these problems [19], and a "bolometric technique" later added evidence for some sort of modes in another hyperbolic cavity [13]. However, no mode was identified by its field symmetries and it remains unknown whether the observed modes of traps used for precision experiments even couple to a single electron at their center. Moreover, the Q values quoted are only estimates, given that Lorentzian line shapes were not established. The largest error assigned [17] was thus based on calculated frequency shifts for a cylindrical trap model [18] of unknown applicability. In the well-characterized radiation field of a cylindrical Penning trap, cyclotron damping and frequency shifts can be systematically studied as a function of detuning of the cyclotron frequency from the resonant frequency of identified modes. Comparisons with simple theoretical forms [18] (appropriately corrected for renormalization effects) should be possible because the field symmetries are well known. In addition, the intense microwave gradient field built up in a high-Q mode with p even and $m=1$ should allow sideband cooling [10,12] at $\omega_c' - \omega_z$ of undamped axial motion (uncoupled from the LCR circuit) to mK temperatures. This temperature would be 10^3 times lower than previously achieved with an elementary particle. These same modes have a transverse magnetic field at the cavity center that could directly flip an electron spin without exciting cyclotron motion.

In conclusion, we report the discovery of self-organized, collective behavior for collisionally coupled, parametrically pumped electron oscillators. A cylindrical Penning trap which is also a good microwave cavity controls the internal energy of the electron oscillators by controlling their radiative dissipation. For low enough internal energy, the electrons make an abrupt transition from stochastic motions to a highly synchronized motion as the pump strength increases through a threshold. The center of mass of the synchronized electrons makes random transitions between CM motions with bistable phases at a rate which increases with increasing internal energy. As an initial application, synchronized electrons are used to identify the radiation modes of a trap cavity for the first time. A thousandfold decrease in an electrons' axial temperature now seems feasible, as does a new generation of electron magnetic-moment measurements which avoid previous limitations from damping linewidth and cavity shifts of measured frequencies. The underlying simplicity suggested by our observations of Lorentzian line shapes, an exponential distribution of transition times, etc., will hopefully prompt a detailed theoretical analysis of parametrically pumped electron oscillators and perhaps even the energy-transfer processes within the cryogenic microplasma [20]. Such analysis, with the good experimental control that is possible in a cylindrical Penning trap, should allow fruitful quantitative studies of the onset of cooperative phenomena and related nonlinear dynamics far from thermal equilibrium.

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