Two-Frequency Wiggler for Better Control of Free-Electron-Laser Dynamics

D. Iracane and P. Bamas
Commissariat à l'Energie Atomique, BP 12, 91680 Bruyeres-Le-Chatel, France
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We study the physics of a free-electron laser based on a "two-frequency" undulator (TFU) which induces large nonlinear effects, especially on the spectral dynamics. These effects are analyzed in an extended formalism where the spontaneous emission, the low-gain regime, and the strong-field saturation regime are studied. Numerical simulations show that an optimized TFU generates a laser field having both a large extraction efficiency and a narrow spectrum.

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In the physics of the high-power Compton free-electron laser (FEL), linear mechanisms are presently well understood, but much remains to be done in the domain of nonlinear regimes. As in a large class of physical situations (traveling-wave tubes or Langmuir waves), complex behaviors are observed when an electron beam is strongly coupled with a multifrequency electromagnetic field. Such high-power systems present strong spectral broadening mechanisms. This has been experimentally observed in the FEL context where technical solutions have been implemented [1]. As additional technological devices may result in severe drawbacks like damage threshold or nontunability, it is worth trying to optimize the FEL dynamics in order to get both a strong efficiency and a narrow spectrum.

In view of this, we focus on the optimization of the envelope $\mathcal{B}(z)$ of the wiggler magnetic field. Classically, a wiggler is tapered with an adiabatic decreasing of the magnetic field to compensate the energy loss of the electrons. Such devices require a simple phase-space structure [2] which is not consistent with a large-spectrum FEL. As high-power FEL's are characterized by strong spectral broadenings [3], adiabatic taperings may then become inefficient. To investigate a larger class of tapering function $\mathcal{B}$, we consider a first generalization provided by the possibility to introduce a second frequency in the wiggler magnetic field. By deeply modifying the electron dynamics, a two-frequency undulator (TFU) alters the nature of the linear regime and of the asymptotic equilibrium.

Let us consider a plane wiggler magnetic field with a vector potential given by

$$A_w = \frac{mc}{e}\,\mathrm{Re}(a_1 e^{-ik_{w1}z} + a_2 e^{-i(k_{w2}z-\varphi)}),$$

where $\lambda_{w1}=2\pi/k_{w1}$ and $\lambda_{w2}=2\pi/k_{w2}$ are the two periods of the TFU. We assume that the beating wavelength

$$\lambda_b \equiv 2\pi/k_b = 2\pi/(k_{w1}-k_{w2}) = \lambda_{w1}/\varepsilon_f$$

is long compared to λ_{w1} and λ_{w2}, so that ε_f is a small parameter. As the second frequency induces a slow modulation of the first frequency taken as a carrier, a TFU can be designed by varying the magnet amplitudes of a one-period wiggler. Considering a wiggler parameter a_w and a small ε_a, the amplitudes a_1 and a_2 are given by $a_1 = a_w(1-\varepsilon_a) \simeq 1$, $a_2 = a_w\varepsilon_a/(1-\varepsilon_f) \simeq 0.1$. In this paper, we investigate the effects induced by variations of the small-value parameters ε_a and ε_f. The TFU phase displacement φ is equal to zero and its optimization is still open.

We assume a continuous electron beam characterized by a density per unit of length ρ_e. Each electron is specified by its energy $mc^2\gamma$ and its longitudinal position $\psi = (k_L + k_{w1})z - \omega_L t$, defined [2] as the phase displacement between the electron transverse oscillation and the light phase. We take advantage of the continuous-beam limit to expand the laser vector potential A_L as a discrete Fourier sequence

$$A_L(\tau = z - ct, z) = \frac{mc}{2e}\sum_{n\ll N}\frac{\mathcal{E}_n(z)}{k_n}e^{ik_n\tau}.$$

The laser frequencies $\omega_n = ck_n$ are periodically distributed around the central frequency $\omega_L = ck_L$; $\omega_n = (1+n/N)\omega_L$, where N is a large integer.

Following the classical technique, the equations of motion for a TFU are readily obtained:

$$\partial_z\psi = k_{w1} - \frac{k_L}{2\gamma^2}[1+\tfrac{1}{2}a_1^2+\tfrac{1}{2}a_2^2+a_1a_2\cos(k_bz)], \quad (1)$$

$$\partial_z\gamma = (4\gamma)^{-1}\,\mathrm{Im}\sum\mathcal{E}_n(G_1+G_2), \quad (2)$$

$$i\partial_z\mathcal{E}_n = (\mu_0e^2/m)\rho_e\langle(G_1^*+G_2^*)/2\gamma\rangle, \quad (3)$$

where $G_u = a_u\exp(i\psi_{kn}^u)$ is a bunching function.

The main new characteristics are the following.

First, Eq. (1) exhibits a slow modulation term $\cos(k_bz)$ which persists after the smoothing operations. To retrieve the usual notation, we introduce the wiggler parameter K as $2K^2 = a_1^2 + a_2^2$ and the detuning function:

$$\nu = k_{w1} - (k_L/2\gamma^2)(1+K^2).$$

The usual FEL physics rests on the idea that the detuning parameter is close to zero. The modulation term then becomes the driving term in Eq. (1).

Second, in Eqs. (2) and (3), the two terms G_1 and G_2 are associated with the wiggler periods λ_{w1} and λ_{w2}, respectively. Thus, each electron is characterized by the phase displacements

$$\psi_{kn}^u = (k_n + k_{wu})z - \omega_n t, \quad u = 1,2,$$

between the laser frequencies $\omega_n = k_n c$ and the motions induced by the wiggler period λ_{wu}. These variables are a trivial linear combination of ψ and z.

As a first investigation of the TFU physics, we analyze two weak-field regimes, the spontaneous emission and the linear regime.

In the weak-field limit, the electron energy γ is constant and the electron trajectory is deduced by a straightforward integration. Starting from the classical formula giving the radiation power emitted by an accelerated charged particle [4], we obtain after some smoothing procedure

$$\frac{d^2 I}{d\omega\, d\Omega} = \frac{e^2}{4\pi\varepsilon_0 c} \frac{K^2}{(1+K^2)^2} N_w^2 \gamma^2 |\mathcal{L}|^2 , \qquad (4)$$

where I is the radiated energy in the solid angle $d\Omega$ at the frequency ω and N_w is the number of wiggler periods. Equation (4) is very close to the formula obtained for standard wigglers [5]. Basic new features appear in the definition of the spectral factor $\mathcal{L}$:

$$\mathcal{L} = (a_1 I_v + a_2 I_{v-k_b})/K ,$$
$$I_v = \frac{1}{L_w} \int_0^{L_w} dz \exp\{i[vz + \alpha \sin(k_b z)]\} , \qquad (5)$$

with $\alpha = -\delta k_{w1}/k_b$ and $\delta = a_1 a_2/(1+K^2)$.

The function $\mathcal{L}$ is a generalization of the standard resonance shape $\sin(x)/x$, which is recovered when $\varepsilon_a \varepsilon_f = 0$. As a first difference, $\mathcal{L}$ is a sum of two terms $a_1 I_v$ and $a_2 I_{v-k_b}$, corresponding to the radiation associated with the two wiggler periods. However, as $a_2 \ll a_1$, this is not an essential feature. The second difference is more fundamental. In the usual case, $\alpha = 0$ and $|I_v|$ simply is a $\sin(x)/x$ function reducing to a Dirac signal for infinitely long wigglers. In this case, the maximal radiation is obtained for $v = 0$ with a width close to $2/L_w$. A TFU induces a modulation term proportional to α which becomes the driving term of Eq. (5). This term arises because a TFU modulates the longitudinal electron motion. When the beating-wave period is comparable to the wiggler length, the modulation clearly becomes a nontrivial term. First, it is a slow term which is not removed by smoothing procedures. Second, since we consider a full beating period, it is not an adiabatic change of the dynamics. Third, its intensity may be large, even for small ε_a. This modulation term may be analyzed by expanding the resonance function I_v with the help of integer-order Bessel functions. This yields a form factor

$$|I_v|^2 = \left| \sum J_n(\alpha) \frac{\sin(x)}{x} e^{ix} \right|^2 , \quad x = (v \pm n k_b) L_w/2 ,$$

which is a succession of equidistant peaks with a width equal to $2/L_w$. The radiated frequency associated with each peak is obtained by writing $v \pm n k_b = 0$:

$$\omega = c[2\gamma^2/(1+K^2)] k_{w1} (1 \mp n\varepsilon_f) . \qquad (6)$$

When $\alpha = 0$, only one peak $[J_{n\neq 0}(0) = 0]$ appears. As α increases, the $n = 0$ peak is rapidly overcome by the other peaks, $n = 1, 2, \ldots$.

Three situations are possible, depending on the relative values of the width $2/L_w$ and the distance k_b between peaks. When $L_w \ll \lambda_b$, we observe only one peak [Fig. 1(a)] which is characteristic of a classical wiggler. If $\lambda_b \ll L_w$, Fig. 1(c) shows three well-defined peaks with a usual shape because couplings are not possible between remote frequencies. When $L_w \simeq \lambda_b$, complex interferences occur between the different peaks [Fig. 1(b)]. From this, one may conjecture that a TFU basically modifies the FEL dynamics when the beating-wave period and the wiggler length are close.

By expanding Eqs. (1) and (2) using a time-dependent perturbative technique, we compute the weak-field gain for a TFU. The quantitative analysis of the gain formula is similar to the study of the spontaneous emission, since

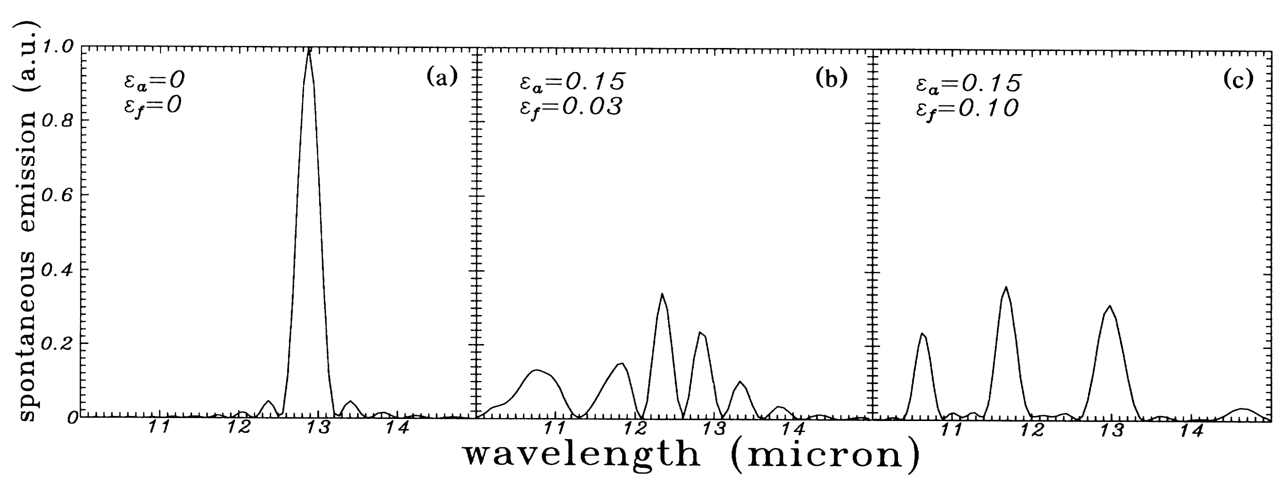

FIG. 1. Spectral form factor $|\mathcal{L}|^2$ of the spontaneous emission for three different magnetic modulations. When the beating-wave period is close to the wiggler length (b), we get a complex gain curve because of interference phenomena.

the expression depends on the same form factor $\mathcal{L}$ up to some derivative.

By a direct computation, we verify the Madey theorem: The mean-square first-order variation $\langle \gamma_1^2 \rangle$ (and the second order $\langle \gamma_2 \rangle$) of the electron energy is proportional to the spontaneous radiated energy (and the gain). In addition, the weak-field gain is proportional to the gamma derivative d_γ of the spontaneous radiated energy, i.e., $|\mathcal{L}|^2$. It is worth noticing that, in a TFU, the form factor $\mathcal{L}$ depends upon γ through the variables v and α, so that

$$d_\gamma = \partial_\gamma - \frac{2(v-k_{w1})}{\gamma}\left[\partial_v + \frac{\delta}{k_b}\partial_\alpha\right].$$

This shows that the gain curve for a TFU is a sequence of equidistant peaks governed by Bessel coefficients, with possible complex interferences when $L_w \simeq \lambda_b$. This multipeak behavior is an essential new feature, since different wavelengths may be above the oscillation threshold. By optimizing the TFU, it is possible to select the lasing peak. One can expect that peaks characterized by large detuning parameters lead to new FEL mechanisms.

As a second investigation of the TFU physics, we study the nonlinear behavior of the FEL. As the saturation is a very complex multifrequency equilibrium even for a standard wiggler, we propose (i) a qualitative interpretation of the electron dynamics in the presence of a constant monochromatic laser field and (ii) a quantitative analysis based upon a full simulation of the multifrequency laser dynamics.

Assuming a constant electric field $\mathcal{E}$, the saturation in a standard wiggler is due to the oscillation of bound electrons at the synchrotron frequency Ω:

$$\partial_z^2 \psi - \Omega^2 \sin\psi = 0, \quad \text{with } \Omega^2 = (k/2\gamma^4)(1+K^2)\mathcal{E}a_1 . \qquad (7)$$

For a TFU, the same derivation leads to a far more complex equation:

$$\partial_z^2 \psi - \Omega^2[1+\delta\cos(k_b z)][\sin\psi + (a_2/a_1)\sin(\psi - k_b z)] = (k/2\gamma^2)(1+K^2)\delta k_b \sin(k_b z) . \qquad (8)$$

The above equation is mathematically interpretable as a pulsed pendulum equation with nonconstant coefficients. When k_b is small compared to Ω, Eq. (8) leads to an adiabatic change of the electron trajectories. At saturation, the synchrotron period $2\pi/\Omega$ is roughly equal to the wiggler length. Hence, for a beating-wave period close to the wiggler length ($\Omega \simeq k_b$), the coefficients of Eq. (8) change at the same rate as the orbit period. This clearly shows that Eq. (8) is a nonadiabatic generalization of Eq. (7). It is no longer possible to provide a simple analysis of the phase-space structure since different electron trajectories cut each other.

The first noticeable distinction between Eqs. (7) and (8) is the forcing term of Eq. (8), which does not depend on the laser-field amplitude. This term arises from the longitudinal electron velocity modulation induced by a TFU. It may become the driving term of Eq. (8) since its amplitude, close to $a_2 a_1 k_{w1} k_b$, can be larger than Ω^2.

A second important difference appears when Eqs. (7) and (8) are linearized. In the standard case, this is the equation of an oscillator with frequency Ω. In a TFU, the linearized equation is a Mathieu equation which presents many resonant points in the phase space, either stable or unstable. Simulations of Eq. (8) show that the usual resonant electron [2] is no longer stable, so that localized trajectories disappear.

In a TFU, the electron trajectories present a stochastic behavior in phase space, even with a monofrequency laser. This means that the synchrotron motion is dislocated because the TFU physics obey nonconstant-coefficient differential equations which behave like Mathieu equations. A TFU modifies the sideband generation, since it destroys the collective synchrotron motion which amplifies the sidebands $\omega_L \pm \Omega$. A direct computation of this effect is not possible due to the complexity of Eq. (8). So, we have developed a full numerical simulation, taking into account both multifrequency laser dynamics and magnetic modulation. The code written for this purpose works in the continuous-beam limit from noise up to saturation. For a standard wiggler, simulations of that type have shown that a high-power FEL presents a large spectrum equilibrium with a relative width roughly equal to the extracted efficiency [3].

A TFU can lead to large efficiencies despite complex electron trajectories which induce nontrivial modification in the sideband generation and, then, in the spectral evolution. To investigate this point, we have performed a systematic mapping in the $(\varepsilon_a, \varepsilon_f)$ plane for ε_f ranging from 0 to 4% and ε_a from 0 to 30%. The main FEL parameters are close to those of the LANL experiment [1].

By optimizing the laser brightness (ratio of the efficiency to the spectral width), we get $\varepsilon_a = 17.5\%$ and $\varepsilon_f = 3.5\%$. This means that the beating-wave period (0.78 m) has the same order of magnitude as the synchrotron period which is close to the wiggler length (1 m). The comparison between the behavior of this TFU and a standard wiggler is shown in Fig. 2. The TFU simulation exhibits a sharp spectrum with a mean width in the range of 0.2%. The extracted efficiency is larger than 4% and is remarkably close to the efficiency obtained with a monofrequency laser simulation.

These simulations prove the possibility, for a high-efficiency FEL, to inhibit the sideband generation and then the spectral broadening for a large number of round trips (typically 1000 or 2000). However, some fundamental issues remain open since we cannot present a definitive interpretation of this sideband suppression that goes beyond the above qualitative arguments. A nonnumerical study of the asymptotic FEL behavior would be required to check if sidebands are suppressed or only delayed.

In summary, facing the complexity of nonlinear equi-

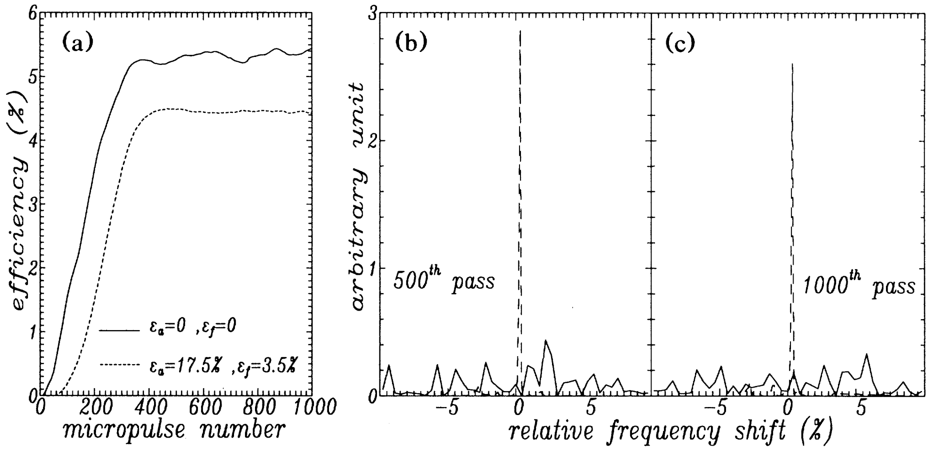

FIG. 2. Comparison between a standard wiggler and a TFU. The efficiencies (a) are comparable, but the spectral behaviors measured at the 500th round trip (b) and at the 1000th round trip (c) are quite different. The sideband generation is clearly delayed.

libriums in high-power FEL physics, we have tried to basically modify the running conditions by optimizing a two-frequency modulation of the magnetic field. Both the evaluation of the weak-field regime and the qualitative analysis of the saturation regime show that the magnetic modulation modifies the equation coefficients that move at the same rate as the fundamental synchrotron oscillation. Full simulations demonstrate the possibility (i) to reach high efficiency and (ii) to strongly inhibit the sideband generation and then the spectral broadening. The optimal beating wave is found around $\varepsilon_a \simeq 0.175$ and $\varepsilon_f \simeq 0.035$, which correspond to a period close to the synchrotron period.

A qualitative interpretation of this result has been given from the analysis of electron trajectories with a monofrequency laser field. A strong destabilization of the synchrotron motion occurs, whose mechanism is similar to the one occurring in the Mathieu equation. A full description of the collective motion of the electrons in a TFU is a complex problem that would benefit from new ideas. For example, a complementary insight could be gained from recent work [6] suggesting that weak periodic forcing can be used to control chaotic behaviors of driven pendulums.

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