Submarine Measurement of the Newtonian Gravitational Constant

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We have measured the Newtonian gravitational constant using the ocean as an attracting mass and a research submersible as a platform for gravity measurements. Gravitational acceleration was measured along four continuous profiles to depths of 5000 m with a resolution of 0.1 mGal. These data, combined with satellite altimetry, sea surface and seafloor gravity measurements, and seafloor bathymetry, yield an estimate of $G=(6.677\pm 0.013)\times 10^{-11}$ $m^3 s^{-2} kg^{-1}$; the fractional uncertainty is 2 parts in 1000. Within this accuracy, the submarine value for G is consistent with laboratory determinations.

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Several recent geophysical experiments have sought deviations from Newton's inverse square law by testing for range dependence of the gravitational constant G. For example, gravitational accelerations observed along tall towers are compared with the predictions of the inverse square law [1]. These experiments place limits on the variation of G with distance but provide no direct estimate of the gravitational constant. In another type of investigation, the variation of gravity with depth is observed through material of known density, either in mines and boreholes [2] or above fluctuating lakes [3]. Such "Airy" experiments yield G directly and assess the inverse square law by comparing the result with the value of G obtained at laboratory scales. Collectively, these studies offer no compelling evidence for deviations from Newtonian gravity in the scale range 10 to 1200 m.

We have determined the gravitational constant in the ocean over a length scale of 5000 m. A variety of gravity observations were needed: along vertical profiles through the water column using the submersible Sea Cliff, on submerged horizontal planes in the research submarine Dolphin, on the ocean bottom with a remotely operated gravity meter, and along the ocean surface by means of a shipboard gravity meter. Additionally, multiple samples of sea water, a precise map of the seafloor topography, seismic profiles, and satellite radar altimetry showing undulations in the sea surface shape were analyzed. Besides the large scale, an important aspect of our experiment is that gravity measurements were made over multiple vertical profiles and along several horizontal planes [4].

In an Airy experiment, the relative gravitational acceleration g is measured with depth z inside a material of known density ρ. Gravitational effects of masses throughout the Earth (excluding the material traversed by the experiment) must be modeled to predict the gravity gradient γ; G is determined from $g(z)=g_0+\gamma(z)z-4\pi G\rho(z)z$. To put the various gradients in perspective, note that the nominal gradient γ in air above the Earth's surface is approximately 309 $mGal\,km^{-1}$ ($1\ mGal=10^{-3}\ cm\,s^{-2}$); the gradient underwater is less by $4\pi G\rho\approx 87$ $mGal\,km^{-1}$ [5]. Perturbations to γ from seafloor topography typically amount to 0.2 $mGal\,km^{-1}$; from seafloor density variation the contribution is no more than 0.16 $mGal\,km^{-1}$. To achieve a fractional uncertainty of 0.1% in G, we must account for anomalous gradients of order 0.087 $mGal\,km^{-1}$. Before describing the gravity measurements made along the vertical profiles, we discuss the measurements used to predict the local gravity gradient and its weak dependence on depth.

Our experimental site in the northeast Pacific ocean (35°13′N, 132°00′W) was chosen to minimize gravity perturbations from the ocean-continent boundary (1000 km away), from oceanic fracture zones, and from oceanic currents and fronts [6]. A 7000-km^2 area surrounding the site was mapped by multibeam echo sounding with a resolution of about 5 m in depth and a 200-m footprint. The site has minimal relief: There are north-south abyssal hills with a mean depth of 5104 m and a standard deviation of 85 m. Seismic reflection profiling revealed a mean sediment thickness of 36 m having a standard deviation of 12 m.

To calculate the attraction of local terrain we constructed a digital representation of the multibeam echo-sounding map. The average seafloor density in the region, 2690 $kg\,m^{-3}$, is derived from the on-bottom gravity survey [4]. The attraction of the seafloor topography in a 60-km-square region was subtracted from the observed gravity. The standard deviation of these terrain corrections is 1.02 mGal, but, more importantly, the vertical gravity gradient associated with them is typically less than 0.2 $mGal\,km^{-1}$. The sediment layer produces a negligible vertical gravity gradient.

An important consideration is the possibility of variations in regional density; such inhomogeneity can create vertical gravity gradients that mimic possible non-Newtonian effects. To look for regional density structure we examined the on-bottom gravity survey and the horizontal submarine gravity survey for short-wavelength (1–10 km) anomalies, sea-surface gravity tracks for intermediate-wavelength (10–100 km) anomalies, and satellite altimetry for long-wavelength (100–1000 km) anomalies.

The ocean-bottom gravity survey consisted of 32 measurement stations distributed somewhat randomly in a 15-km-square region around the dive sites. A terrain-corrected gravity anomaly map from these data shows a standard deviation of 0.55 mGal in the residual gravity values and reveals no systematic pattern. Moving platform gravity surveys were performed with LaCoste-Romberg shipboard gravity meters [7] (S-110 and S-38). On the research submarine Dolphin, a survey was conducted at depths ranging from 150 to 950 m along the perimeter of a 15-km-square path surrounding the study site; the research vessel Thomas Washington performed a survey over a square region along 50-km-long tracks spaced 2.5 km apart.

The terrain-corrected gravity from the Dolphin submarine survey is flat to the 1-mGal resolution of the survey. Similarly, the surface survey shows a flat gravity field (the rms variation is 0.52 mGal). The observed lateral variation in gravity along the sea surface is much less than the computed attraction of the discernible submarine topography. The small variation in sea surface gravity is caused by the widely observed phenomenon of isostatic compensation, in which topographic loads on the crust are buoyantly supported by similar but inverted undulations in the shape of the crust-mantle boundary. The attraction of this interface largely cancels the varying attraction of the seafloor topography. An idealized Airy model [8] consisting of an image of the terrain buried at a depth of 7 km below the seafloor with the same density contrast yields a terrain-corrected gravity field at the sea surface which is consistent with the gravity data observed there. Because the surface data demand compensation, the model was also employed for the terrain correction along the vertical profiles. The difference between the compensated terrain correction and the simple terrain correction (from the topography alone) along the vertical profiles is typically 0.2 $\mathrm{mGal\,km^{-1}}$.

The local vertical free-air gravity gradient is related by Bruns' equation [9] to the surface Laplacian of the geoid height. Geoid heights are measured directly by radar altimetry from artificial satellites. We obtained ascending and descending tracks from the Exact Repeat Mission of Geosat [10]. Each sea surface height value, spaced about 3 km along track and 100 km between tracks for our area, represents the average of 44 passes over the ocean. After correction for small orbit errors, the rms crossover error is 2 cm. The deviation from the standard ellipsoid was modeled by a local quadratic function; the remaining residuals were analyzed in terms of a statistical model based on the assumption of local stationarity and isotropy [11]. Estimates were made of the vertical gradient, low-pass filtered to a scale of 100 km; the calculated uncertainty rises very rapidly if averaging is carried out on a shorter scale (the local ship survey covers the smaller scales appropriately). We found that the contribution to dg/dz from gravity fields of wavelength 100 km or more is -0.04 ± 0.15 $\mathrm{mGal\,km^{-1}}$. Similarly, the limit on dg/dz at shorter wavelengths based on spectral analyses of the surface and bottom gravity surveys is 0.062 $\mathrm{mGal\,km^{-1}}$. Undoubtedly these uncertainties will be reduced following analysis of higher density Geosat data which have recently become available for our site.

The sea water density must be known with a precision equal to the hoped-for uncertainty in G. We characterized the sea water with conductivity and temperature profiles (checked with direct sampling and laboratory salinity analysis) which give density to better than 1 part in 10^4 using the sea water equation of state [12]. The density varied from 1023.6 near the surface to 1050.5 $\mathrm{kg\,m^{-3}}$ at 5000 m depth with no significant lateral changes observed across the site. The slight seasonal changes in density were also negligible.

The density profile is important not only for an estimate of the gravitational effects of the sea water, but also because the depths of the gravity observations were determined from measured pressure. Pressure in the oceanic water column was monitored by two quartz pressure gauges (Paroscientific model 410KT). Over the pressure and temperature ranges encountered in our experiment, the difference between gauge-determined pressure and a dead-weight calibration pressure was less than 7 parts in 10^5. This translates to a depth uncertainty of 0.35 m. The depth uncertainty associated with water density uncertainty is 0.5 m.

The measurements central to the determination of G are the vertical gravity profiles obtained in a submersible with a Bell Aerospace BGM-3 gravity meter [13]. The gyrostabilized sensor can resolve less than 0.1 mGal. We calibrated the BGM-3 outside San Diego along a 405-mGal absolute gravity calibration line [14] and found that the disagreement between the manufacturer's calibration (obtained through rotations of the sensor) and our own was less than 10^{-4} of the gravity interval.

The gravimeter was placed in the U.S. Navy submersible Sea Cliff. A pilot and an equipment operator accompanied the gravity meter to the seafloor in four separate dives over a period of five days. The durations of the 5000-m-deep dives averaged 11 h, during which time gravity and pressure were recorded every 12 sec. At the beginning of each dive the submersible descended to the same central position on the ocean bottom (the vehicle's depression in the mud provided confirming evidence that the site was occupied repeatably). Once there, on-bottom data were collected for a 20–30-min motionless period to check for instrumental drift. No drift was observed in the pressure sensors; the standard deviation of the four average depths recorded during the reoccupations of the central site is 4 cm. After correction for tides and for a historical instrumental drift of 0.033 $\mathrm{mGal\,day^{-1}}$, the standard deviation of the four gravity records from the

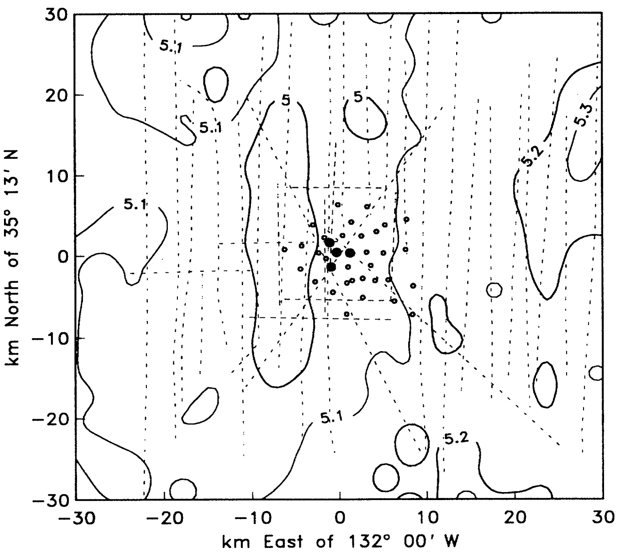

FIG. 1. A map showing the location of the submarine measurement of G. The contour lines indicate water depth in km; the larger points show the locations of continuous vertical gravity profiles obtained in a submersible, and the smaller points mark locations of bottom gravity measurements. Dashed lines indicate the tracks along which gravity was recorded on a ship and in a submarine.

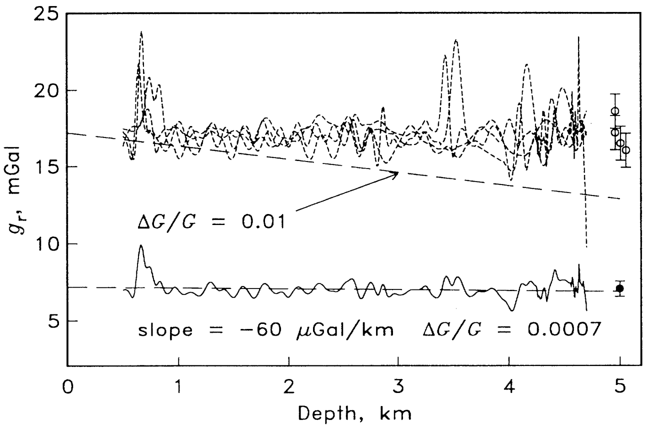

FIG. 2. The dashed curves are the corrected (using the laboratory value of G) gravity profiles along four vertical tracks. The solid curve is their average, displaced downward for display purposes. The consistency of the means of the four dashed curves (which have not been displaced relative to one another) provides further evidence that the local gravity field is uniform. The peaks are the results of incompletely corrected vertical accelerations that occurred during weight drops. The open circles are the individual values obtained on the bottom in the submersible. The solid circle (the average of the four open circles) differs from the average of the corrected gravity values measured in the water column by less than 0.05 mGal. The dashed line shows the slope that would result from a 1% non-Newtonian effect.

bottom reoccupations is 0.105 mGal. During three of the dives, the submersible was driven along the bottom approximately 2 km and an additional on-bottom gravity record was obtained before ascent to the surface. Figure 1 is a map showing the resulting pattern of dive-ascent locations.

To ascend at a uniform rate (between 10 and 25 $\mathrm{m\,min^{-1}}$), the submersible's buoyancy was adjusted by dropping weights and by pumping sea water ballast. The vertical accelerations produced by these events are almost completely correctable using the second derivative of the depth.

The submersible's lateral position was obtained with an accuracy of 20 m using five acoustic transponders tethered 100 m above the seafloor. The vehicle drifted at most 445 m laterally during the 5-h-long ascents. From this navigation we calculated Coriolis corrections (these "Eötvös" corrections range from −0.406 to 0.015 mGal). Likewise, the gravity data were corrected for Earth and ocean tides.

After correcting the gravity measurements for all known effects, including horizontal velocity, vertical acceleration, tides, latitude, meter drift, isostatically compensated topography, and corrected regional gradient, the data were reduced using our sea water density model and the laboratory value [15] of G, 6.6726×10^{-11} $\mathrm{m^3s^{-2}kg^{-1}}$. A nonzero slope in reduced gravity g_r would indicate a deviation from the laboratory value given by $\Delta G/G = -(dg_r/dz)/4\pi G\rho$, where $\Delta G = G_{\rm obs} - G_{\rm lab}$ and g_r and z are both positive downward. Figure 2 is a plot of the four separate vertical profiles (dashed curves) and the average profile. Between the depths of 500 and 4800 m, the fit to the average of all the profiles is $-0.060\ \mathrm{mGal\,km^{-1}}$. Slopes in the individual profiles vary from −0.213 to 0.053 $\mathrm{mGal\,km^{-1}}$. The average profile yields an estimate for G of 6.677×10^{-11} $\mathrm{m^3s^{-2}kg^{-1}}$; the fractional difference between this and the laboratory value is only 7 parts in 10^4, which is about

TABLE I. Sources of uncertainty are listed in terms of both the particular components and their effects on the gravity gradient assuming each is distributed systematically along the 5-km length of the experiment. The root-sum-square (RSS) result determines the overall uncertainty in the determination of G.

Source	Uncertainty	Gradient uncertainty ($\mathrm{mGal\,km^{-1}}$)
Gravity measurements	0.11 mGal	0.022
Depth	0.61 m	0.027
Water density	0.1 $\mathrm{kg\,m^{-3}}$	0.009
Terrain correction	0.23 mGal	0.046
Eötvös correction	0.013 mGal	0.003
Local gradient		0.062
Regional gradient		0.150
Total RSS uncertainty		0.172

one-third the uncertainty of our measurement. The rms residual to the fit is 0.537 mGal. For comparison, a dashed line indicates the slope that would occur for a fractional difference of 0.01 between the laboratory and oceanic values.

Table I lists the various sources of uncertainty in the experiment and the root-sum-square (RSS) total. The dominating factor is the limit in our knowledge of the regional gradient; 0.172 $\mathrm{mGal\,km^{-1}}$ corresponds to a fractional uncertainty in G of $0.172/87=0.002$.

To eliminate the possibility that unsurveyed density anomalies exactly cancel a non-Newtonian effect, we applied a two-dimensional ideal-body inversion of our gravity survey geometry [16]. We found that an unacceptably high-density contrast (greater than 300 $\mathrm{kg\,m^{-3}}$) would be required to mask a non-Newtonian signal larger than our uncertainty in G [4].

In conclusion, we have estimated G from measurements of gravity through a 5-km-thick slab of sea water and found that it agrees with the laboratory value to within less than 1 part in 1000, with a resolution of 2 parts in 1000. Roughly speaking, this result constrains the magnitude of the coupling constant of a single Yukawa modification to Newtonian gravity to be less than 0.002 for scale lengths in the range from 1 m to a few km. The uncertainty is limited by the extent to which a geologically caused anomalous gravity gradient would not be recognized in the satellite geoid measurements. There is little doubt that the uncertainty will improve when our analysis of the densely sampled satellite geoid is completed.

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[1] D. H. Eckhardt *et al.*, Phys. Rev. Lett. **60**, 2567 (1988); D. F. Bartlett and W. L. Tew, Phys. Rev. Lett. **63**, 1531 (1989); J. Thomas *et al.*, Phys. Rev. Lett. **63**, 1902 (1989); C. Jekeli *et al.*, Phys. Rev. Lett. **64**, 1204 (1990); C. Speake *et al.*, Phys. Rev. Lett. **65**, 1967 (1990).

[2] A. T. Hsui, Science **237**, 881 (1987); F. D. Stacey *et al.*, Rev. Mod. Phys. **59**, 157 (1987); M. Ander *et al.*, Phys. Rev. Lett. **62**, 985 (1989); M. Zumberge *et al.*, J. Geophys. Res. **95**, 15483 (1990).

[3] G. I. Moore *et al.*, Phys. Rev. D **38**, 1023 (1988); G. Müller *et al.*, Phys. Rev. Lett. **63**, 2621 (1989); G. Müller *et al.*, Geophys. J. Int. **101**, 329 (1990).

[4] Details will be described elsewhere.

[5] These are approximate; we actually used an ellipsoidally layered model described by F. D. Stacey *et al.*, Phys. Rev. D **23**, 1683 (1981); A. Dahlen, Phys. Rev. D **25**, 1735 (1982).

[6] J. A. Hildebrand *et al.*, Eos Trans. Am. Geophys. Union **69**, 769, (1988).

[7] L. J. B. LaCoste, Rev. Geophys. **5**, 477 (1967).

[8] D. L. Turcotte and G. Schubert, *Geodynamics* (Wiley, New York, 1982).

[9] W. A. Heiskanen and H. Moritz, *Physical Geodesy* (Freeman, San Francisco, 1967).

[10] K. Marks, D. McAdoo, and D. Sandwell, Eos Trans. Am. Geophys. Union **73**, 145 (1991).

[11] Appendix *B* in the latter of Ref. [5].

[12] F. J. Millero *et al.*, Deep-Sea Research **27A**, 255 (1980).

[13] R. E. Bell and A. B. Watts, Geophysics **51**, 1480 (1986).

[14] G. Sasagawa *et al.*, J. Geophys. Res. **94**, 7661 (1989).

[15] G. G. Luther and W. R. Towler, Phys. Rev. Lett. **48**, 121 (1982).

[16] R. L. Parker and M. A. Zumberge, Nature (London) **342**, 6245 (1990).