Vortex Dynamics in Superconducting Fractal Networks

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Superfluid and vortex dynamics in a fractal system were investigated by measuring the complex sheet impedance of a wire network of interconnected Sierpinski gaskets exposed to a weak magnetic field. By probing the network response over a range of length scales covering four stages of hierarchy in the gaskets, we find strong evidence for unusual scaling of the vortex energy U_h with the size r_h of the loops, $U_h \propto r_h^{-\zeta}$ with $\zeta = \ln\frac{5}{3}/\ln 2$, predicted by theory.

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Disorder in superconductors can assume a variety of forms and has been modeled in a number of ways. An elegant geometrical approach revealing some of the basic aspects of superconductivity in percolating materials is due to Alexander and Halevi (AH) [1]. Near the percolation threshold, such systems exhibit a natural self-similar structure with geometrical inhomogeneities occurring over a broad range of length scales. They can therefore be described by a family of scale-invariant fractal lattices, such as the Sierpinski gasket (SG) originally proposed by Gefen *et al.* [2] to mimic the topological features of the percolating cluster's backbone.

So far, studies of SG networks have focused on their superconducting-to-normal phase boundary $T_c(H)$ [3,4], which was found to agree with calculations [4,5] based on the Ginzburg-Landau (GL) theory. In this Letter we report the first study of superfluid and vortex dynamics in a SG network. A fundamental prediction emerging from the AH theory and its extensions [6] is that the energy required to create a vortex excitation in a loop of size r scales as $r^{-\zeta}$, where $\zeta = \ln\frac{5}{3}/\ln 2$. It follows that the pinning potential experienced by a vortex moving in a gasket has a marked hierarchical character which should become uniquely manifest in experiments probing the dynamics of vortices at different length (or time) scales. To this end, we have measured the complex sheet impedance Z of a network of interconnected gaskets exposed to a weak perpendicular magnetic field H over a wide range of driving angular frequencies ω. Close to $T_c(H)$, we observe richly structured Z-vs-H curves reflecting flux quantization in loops with a hierarchical distribution of sizes. Relying on thermally activated vortex motion to explain the evolution of the quantum structure with frequency, we find strong evidence for the scaling properties of vortices over four stages of hierarchy in the gasket. At high frequencies, where pinning and fluctuation effects are weak, the fine structure predominantly reflects the effect of frustration on the superfluid background.

The SG wire network was photolithographically patterned from a 1000-Å-thick granular aluminum film with a normal-state electrical resistivity of 14 $\mu\Omega$ cm. The sample consisted of sixth-order gaskets sitting on the sites of a 32×32 triangular lattice and connected to each other at the vertices. The length and width of the elementary links were, respectively, $a = 4.9$ μm and $w \approx 1$ μm. Real and imaginary parts of $Z = R + i\omega L$ were extracted from measurements of the screening properties of the sample performed with an inductive drive-receive coil technique [7]. Excitation levels were chosen to ensure a linear response. Data were taken over 5 orders of magnitude in frequency, from 100 Hz to 10 MHz, using a lock-in technique above 1 kHz and a SQUID detector at lower frequencies.

Measurements of L^{-1} and R performed at the upper and lower limits of our spectral range are shown in Fig. 1 as a function of the frustration parameter f expressing the number of flux quanta in an elementary triangular

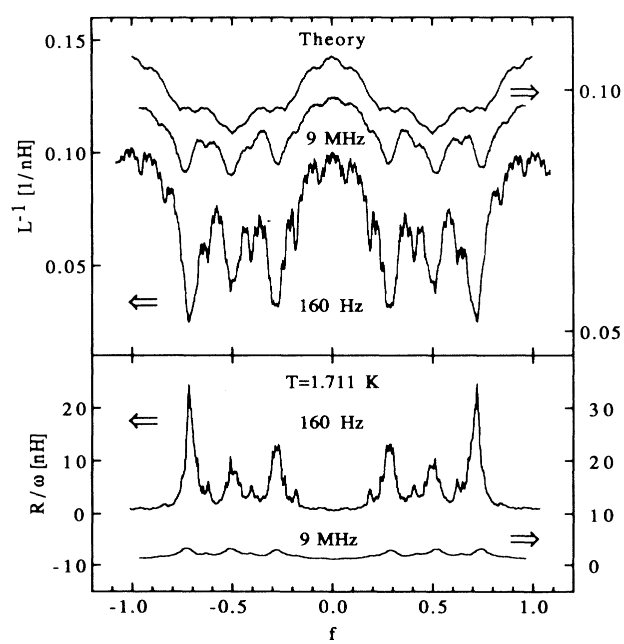

FIG. 1. Inverse sheet inductance and sheet resistance of the SG network at two different frequencies as a function of frustration. The theoretical prediction for a third-order gasket (shifted upward for clarity) should be compared with the 9-MHz $L^{-1}(f)$ data.

cell of the gaskets. The data, periodic in f with period 1, exhibit a complex fine structure arising from flux-quantization phenomena occurring in the loops of various size which generate the self-similar pattern of the gaskets. As expected, "dips" in $L^{-1}(f)$, reflecting a lower degree of superconducting phase coherence in the fractal lattice, correspond to "peaks" in the dissipative component $R(f)$. Quantum oscillations were observed only in a very narrow temperature range (~ 15 mK) just below the transition temperature $T_c(0) = 1.718$ K of the network. This dramatic temperature dependence as well as the observation that the fine structure becomes richer and sharper with decreasing frequency suggest that thermal fluctuations play a major role in the response of the system near the transition.

A theoretical description of superfluid and vortex dynamics in a fractal system including fluctuation effects does not exist so far. To get some insight into this complex problem, it is useful to consider the response at very high frequencies (VHF), where vortex motion is governed by viscous forces only. Since pinning effects, vortex-vortex interactions, and thermal fluctuations are negligible in this regime, Z can be written as [8] $Z = i\omega L_k + R_F$, where L_k is the bare kinetic inductance associated with nondissipative motion of the accelerated superfluid and R_F the flux-flow resistance due to viscous damping of otherwise freely moving vortices. Since R_F is a structureless (linear) function of f, in the VHF limit, quantum structures merely arise from the nonmonotonic effect of frustration on the superfluid background. To find the functional dependence of L_k on f, we recall that L_k^{-1} is proportional to the second derivative of the network ground-state energy with respect to the vector potential [9] and assume, for the sake of simplicity, that the network behaves as a Josephson-junction array (JJA) with a sinusoidal current-phase relation. Then, relying on the quadratic approximation of the cosine in the expression of L_k and using the AH recursive relation [1] for the gasket ground-state magnetic energy, we can express the frustration-induced change $\Delta L_k^{-1}(f) \equiv L_k^{-1}(0) - L_k^{-1}(f)$ of the inverse kinetic inductance as

$$\Delta L_k^{-1}(T,f) = L_k^{-1}(T,0)\frac{4\pi^2}{18} \times \left[f^2 + \frac{1}{2}\sum_{h=0}^{n-1} \frac{(2^{2h+1}f - p_h)^2}{5^h} \right], \quad (1)$$

where n is the gasket order and the sum runs over the n families of loops in the gasket (h is the hierarchical index labeling a family of identical loops). The integers $\{p_h\}$, chosen to minimize the gasket energy, denote the total number of flux quanta threading a loop of the species h and one of the three gaskets (of order h) which surround it. As shown in Fig. 2(a) for $n=2$, $\Delta L_k^{-1}(f)$ is a fractal curve [even in f and symmetric with respect to $f = 0.5(\mathrm{mod}\,1)$], whose parabolic segments correspond to

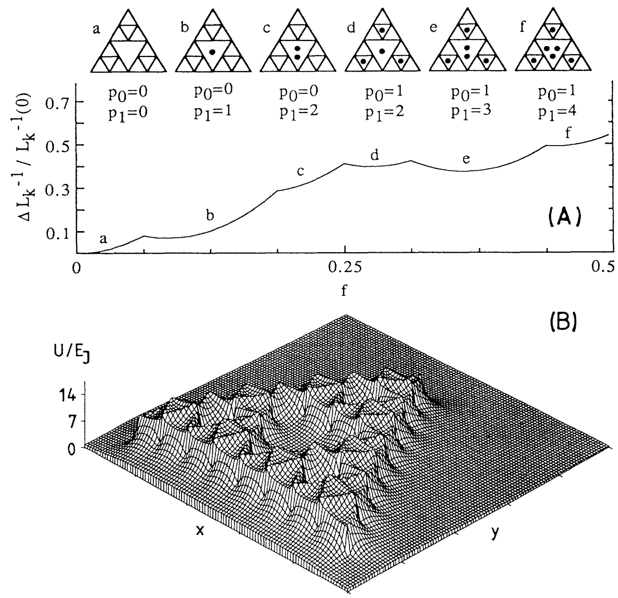

FIG. 2. (a) Relative change of the inverse kinetic inductance and corresponding vortex configurations for a second-order gasket as a function of frustration; (b) hierarchical vortex pinning potential for a third-order gasket.

vortex configurations specified by a well-defined set of $\{p_h\}$. Inspection [1] of Eq. (1) reveals that, in the interval $f_{c1} < f < 0.5$ ($f_{c1} = 4^{-n}$ is the frustration corresponding to the nucleation of the first vortex in the central loop of the gasket), $\Delta L_k^{-1}(f)$ scales as f^β with the anomalous exponent $\beta = \ln 5/\ln 4$. Deviations from a sinusoidal current-phase relation depending on the relative size of the GL coherence length $\xi(T)$ to the link dimensions tend to depress the magnitude of $\Delta L_k^{-1}(f)$, which vanishes in the limit of a linear current-phase relation [$a \gg \xi(T)$] [10].

In Fig. 1 we compare the 9-MHz $L^{-1}(f)$ curve with a calculation based on Eq. (1). To agree with the data at $T = 1.711$ K (where $a/\xi \approx 5$), the amplitude of the superfluid depression at $f = 0.5$ predicted by Eq. (1) was reduced by a factor of ~ 3. In order to reproduce the richness of the fine structure, it was sufficient to compute $L_k^{-1}(f)$ for $n=3$; features reflecting flux quantization in the higher hierarchical stages of the gaskets were beyond experimental resolution. Although the overall agreement tends to support the VHF interpretation, closer inspection of the data reveals a marked dip near $f = 0.25$ not predicted by theory. This suggests that, even at the highest frequencies studied in this work, vortex fluctuations were sufficiently important to weaken phase coherence in the system at some selected values of f. Additional evidence that the response at 9 MHz only partially fulfills the VHF requirement is provided by the presence of weak structures in $R(f)$, showing that pinning effects are still affecting vortex dynamics at 9 MHz.

A more crucial test of the VHF model is presented in

Fig. 3. To emphasize the low-field scaling properties of the response resulting from the self-similar nature of the gaskets, we show the frustration-induced relative variation $\Delta L^{-1}(f)/L^{-1}(0)$ of the network's inverse inductance at three different frequencies on a $\log_5$-$\log_4$ plot. As shown in the upper part of Fig. 3, the 9-MHz data exhibit two well-resolved stages of dilational invariance in magnetic flux, in good agreement with the third-order calculation described above, and scale as $\Delta L^{-1}(f) \propto f^{\beta}$ with $\beta = 1.19 \pm 0.05$, a value consistent with that predicted by theory ($\beta = 1.16$).

With decreasing frequency, thermal fluctuations make the fine structure sharper and richer; the growing richness is demonstrated by the increasing number of self-similar stages (at least four at 160 Hz) emerging from the $\log_5$-$\log_4$ plots of Fig. 3. This interpretation is consistent with similar behavior observed in renormalization-group calculations of the phase boundary [11]. Thermal fluctuations are believed to be responsible also for the deviations of the low-frequency data of Fig. 3 from the power-law scaling prediction of the AH theory.

While the inclusion of fluctuation-induced renormalization effects in a dynamical context appears to be an extremely difficult problem, the evolution of the fine-structure richness with frequency finds a natural explanation in terms of thermal activation of the vortices in the

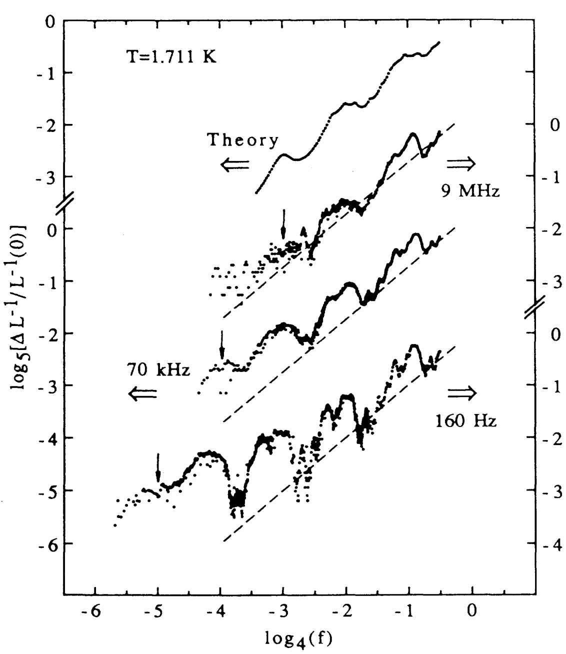

FIG. 3. $\log_5$-$\log_4$ plot of the relative change of the inverse sheet inductance of the SG network at three different frequencies as a function of frustration. The theoretical prediction for a third-order gasket should be compared with the 9-MHz data. The arrows denote the critical frustration. The power-law scaling prediction of the Alexander-Halevi theory is indicated by the dashed lines.

potential-energy landscape $U(x,y)$ created by the gaskets. Continuing with the JJA analog, we have computed $U(x,y)$ using the arctan approximation to the vortex-phase configuration [12]. The result, shown in Fig. 2(b) for a third-order gasket, reveals a pronounced hierarchical structure; the energy U_h of the local minima of $U(x,y)$ located near the centers of the various loops scales with the loop size r_h as

$$U_h(T) \approx \pi^2 E_J(T)(a/r_h)^{\zeta}, \qquad (2)$$

where $E_J(T)$ is the coupling energy of adjacent nodes [13] and $\zeta = \ln\frac{5}{3}/\ln 2$. This relation follows from the AH theory by calculating the energy of the first excited state (one single vortex in the central loop) of a gasket in zero field. Since the recursive procedure underlying the AH approach only applies to symmetric vortex configurations, Eq. (2) slightly underestimates U_h for vortex nucleation in off-center loops [6]. Notice that ζ is the ratio of the conductivity and correlation-length exponents in two-dimensional percolating clusters [2].

To proceed further in the description of thermally activated vortex motion in the fractal lattice, we need an estimate of the energy barrier Δ_h experienced by a vortex of species h. In this connection, the quantity of interest is the energy U_s of the saddle points of $U(x,y)$ located on the interconnecting links. In contrast with the strong hierarchical dependence of U_h, U_s turns out to be roughly independent of the link position in the gasket, a result allowing great simplification. Writing $U_s = sE_J$, we find that s varies from ~ 10 to ~ 12 as the form of the current-phase relation changes from sinusoidal to linear.

Assuming that vortices of species h behave as independent Brownian particles with an activation energy $\Delta_h = U_s - U_h$, we now argue that there will be detectable contributions to the fine structure arising from loops of species h only if the vortices have an appreciable probability to diffuse out of the corresponding potential wells on the characteristic time scale set by the measurement, i.e., only if the thermal escape rate $\Omega_h = \Omega_J \exp(-\Delta_h/kT)$ is larger than ω. Since the loops which dominate the response at any level of frustration in the low-field scaling regime are those enclosing one flux quantum [1], i.e., having a size $r_f \approx a/\sqrt{f}$, we can express the depinning condition $\Omega_h > \omega$ in the form $f > f_c(T,\omega)$ by setting $r_h = r_f$ in Ω_h. The critical frustration $f_c(T,\omega)$, the analog of the irreversibility line in high-temperature superconductors, is given by

$$[f_c(T,\omega)]^{\zeta/2} = A(T) + [kT/c_2E_J(T)]\ln\omega , \qquad (3)$$

where $A(T) = c_1 - [kT/c_2E_J(T)]\ln\Omega_J(T)$ and c_1 and c_2 are constants of order 1 and 10, respectively. In the JJA approach, one expects $\Omega_J(T) \approx (2e/\hbar)^2 E_J(T) R_J$, where R_J is the normal-state resistance of an elementary link [8].

To test the logarithmic frequency dependence predicted by Eq. (3), we take advantage of the self-similar proper-

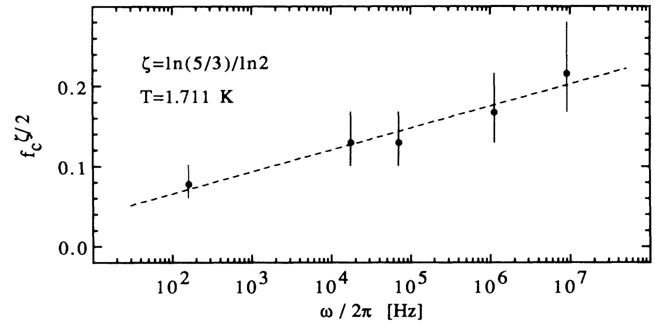

FIG. 4. Dependence of $f_c^{\zeta/2}$, as calculated from the f_c values defined in Fig. 3, on the logarithm of the driving frequency. The dashed line is a fit based on Eq. (3). See text for the definition of the error bars.

ties of the data of Fig. 3 to define $f_c(\omega)$ as the lowest value of frustration at which we are still able to resolve traces of a hierarchical stage. Adopting this procedure, which allows one to locate $f_c(\omega)$ with an accuracy corresponding roughly to one hierarchical stage, we find the f_c values denoted by arrows in Fig. 3. A more elaborate method, based on a Fourier analysis of the data, led to similar results. Using $\zeta = \ln\frac{5}{3}/\ln 2$, in Fig. 4 we have plotted $f_c^{\zeta/2}$ against $\log_{10}\omega$ as a check of Eq. (3). Taking $R_J = 7\ \Omega$ to compute $E_J(T)$ [13] at $T = 1.711$ K, we find a good fit for a quite reasonable set of the adjustable parameters c_1 and c_2 ($c_1 = 0.3$ and $c_2 = 9.5$). It should be noticed, however, that the logarithmic frequency dependence of the data is rather insensitive to the value of ζ, although drastic deviations from the theoretical value would imply unreasonable values of the fitting parameters. Also notice that the attempt frequency $\Omega_J(T)/2\pi$ at $T = 1.711$ K is roughly an order of magnitude higher than the highest frequency (9 MHz) studied in this work, an observation lending further support to the conclusion reached above in connection with the VHF regime.

In conclusion, ac impedance measurements performed on a network of interconnected Sierpinski gaskets have provided novel insight into the dynamics of superfluids and vortices in a fractal system. In particular, by probing the network response over a range of length scales covering four levels of hierarchy in the gaskets, we have found strong evidence for unusual scaling properties of vortices in a fractal lattice emerging from the Alexander-Halevi theory.

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