Cosmological Consequences of High-Frequency Oscillations of Newton's Constant

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We show that high-frequency, small-amplitude oscillations of Newton's constant G can dramatically alter cosmology even if the frequency is very high compared to the expansion rate. For example, it is possible to have a spatially flat universe in which dynamical tests of $\Omega \equiv$(matter density)/(critical density)—tests which attempt to directly measure the mass density of the universe—obtain values less than unity ($\Omega = 0.1$–0.3, say). The cosmological effects can be obtained in a frequency-amplitude range allowed by all known constraints on G and its time derivative.

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The notion of a time-varying Newton's constant G has received considerable theoretical and experimental attention in recent decades, inspired by Dirac's large-number hypothesis and the advent of Brans-Dicke cosmology [1]. Strong experimental constraints have been obtained which rule out significant *monotonic* variation of G on time scales comparable to the age of the Universe (e.g., Viking experiments indicate $\dot{G}/GH \leq 0.3$, where H is the Hubble parameter). Recently, *low-frequency* ($\nu \approx 160H$) oscillations in G have been proposed as an explanation for the apparent spatial periodicity ($2\pi\nu^{-1} \approx 128$ Mpc) observed in pencil-beam galaxy redshift surveys [2]. However, oscillations in this frequency regime are highly constrained by classical tests of general relativity.

In this paper, we turn attention to oscillations of G at *very high frequency* compared to the Hubble expansion rate, $\nu > 10^{12}$ Hz. (The lower bound has been chosen to ensure that the oscillations cannot be detected by classical tests of general relativity [3].) We find that oscillations in G can surprisingly affect cosmological measurements even though the frequency is fantastically greater than the present expansion rate. For example, oscillating G can make $\Omega \equiv$(matter density)/(critical density) appear to be less than unity even though the Universe is spatially flat. Hence, it is conceivable that oscillations in G can lead to a discrepancy between measurements of Ω (e.g., $\Omega \approx 0.1$–0.3 is reported in some observations) and the prediction that $\Omega = 1$. Cosmologies are also possible with the oscillating-G effect combined with a dark-matter or cosmological-constant component; however, the constraint that the hidden-mass component make up the entire difference between measured Ω and unity is removed.

Before proceeding to discuss the effects on cosmological parameters, it seems best to first address concerns which undoubtedly spring to the reader's mind when presented with the proposal of rapidly oscillating G: *Does the proposal violate existing astrophysical or cosmological limits on $\dot{G}$?* The answer is no; the best limits (e.g., Viking measurements or binary-pulsar limits) [1] are measured over time scales much greater than the oscillation period ($\leq 10^{-12}$ sec) so that $\dot{G}$ averages to zero. *Does the change in G affect stellar evolution, nucleosynthesis, or terrestrial measurements of G?* No; the amplitude of the oscillation in G is likely to be extraordinarily small, e.g., $\Delta G/G \leq 10^{-30}$ today. The amplitude is many orders of magnitude below limits set by stellar evolution, and $\Delta G/G \ll 1$ extrapolating back to nucleosynthesis as well. Terrestrial measures of G are insensitive to such small amplitudes. We note that frequencies above a few kilohertz lie beyond the range of proposed laser-interferometry tests for gravity waves. Although the amplitude is incredibly small, we will show that the cosmological effect of oscillations is proportional to $\dot{G}/GH \propto \nu H^{-1}(\Delta G/G)$; hence, the effect can be non-negligible because the frequency is large and because the Hubble parameter is small. *Is there any microphysical reason for high-frequency, low-amplitude oscillations?* Yes. The key microphysics requirement is a massive scalar field that is nonminimally coupled [i.e., an interaction $f(\phi)\mathcal{R}$, where $\mathcal{R}$ is the scalar curvature]. Then, the effective value of G obtains an oscillatory contribution as the nonminimally scalar field oscillates about its ground-state value with a frequency ν determined by its mass, $\nu \sim m^{-1}$. We note that even if nonminimal couplings do not appear in the classical theory describing particle fields coupled to gravity, they are generated by quantum corrections since no symmetry forbids them, in general. While there is no accepted, renormalizable unified theory in hand, it is reasonable to suppose that finite nonminimal couplings are present in the effective, low-energy Lagrangian that describes our Universe. In fact, virtually every approach attempting to unify particle physics and gravity, including superstring, Kaluza-Klein, induced gravity, and supergravity theories, also predicts weakly interacting, massive scalar fields which are nonminimally coupled. It is necessary that the initial oscillation amplitude be small or else the oscillation energy dominates the energy at an early epoch destroying the successful predictions of nucleosynthesis. (In more recent work [3], we have shown that the requisite initial oscillation amplitude can be produced naturally in an inflationary epoch.) Since the oscillation amplitude decays with time due to redshift, a consequence is that the present amplitude would be unobservably small, which is why nonminimal couplings have been ignored by many cosmologists. Here, however, we show how oscillations can affect cosmology even though the present amplitude is small. *Does the proposal violate solar-system or fifth-force tests for intermediate-range forces?* No. Nonminimal couplings result in a short-range Yukawa interaction that modifies Newtonian gravity on scales smaller than m^{-1}. Masses $m > 10^{-12}$ GeV or frequencies $\nu > 10^{12}$ Hz correspond to length scales less than 0.03 cm, which is well below the range of tests for intermediate-range forces [4] or solar-system tests of general relativity [3]. Particle-physics models would typically suggest greater masses (and, hence, modifications on shorter length scales).

The remarkable fact is that, even though high-frequency, low-amplitude oscillations in G easily evade all known tests on Einstein gravity, they can, nevertheless, profoundly affect cosmology. One's natural intuition is that the high-frequency effect must wash out on long time scales, and here time scales 10^{29} or more times the oscillation period are being considered. The intuition fails because of the nonlinearities in the Einstein equations modified for an oscillating G:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\dot{G}}{G}H + \frac{8\pi G}{3}(\rho_m + \rho_G), \tag{1}$$

where $a(t)$ is the Robertson-Walker scale factor, ρ_m is the ordinary matter density (including radiation), and ρ_G is the energy density associated with the oscillating field (say, a scalar field ϕ) responsible for the oscillations in G. [The derivation of Eq. (1) is a simple generalization of the derivation for Brans-Dicke theories [5].]

The universe is assumed to be flat owing to an inflationary phase early in the history of the universe. For simplicity, it is further assumed that ρ_G is negligibly small compared to ρ_m, i.e., ordinary matter dominates the energy density. The dominant oscillation effect is due to the term $(\dot{G}/G)H$ on the right-hand side.

Solving Eq. (1) for H illustrates why there is a nontrivial effect even at high frequency:

$$H = \frac{1}{2}\frac{\dot{G}}{G} + \left[\frac{8\pi G\rho_m}{3} + \frac{1}{4}\left(\frac{\dot{G}}{G}\right)^2\right]^{1/2}. \tag{2}$$

If $\dot{G}/G \propto \cos(mt)$, where $m \gg H$ (frequency much greater than the averaging time), the first term averages to zero, but $(\dot{G}/G)^2$ in the second term retains a nonzero contribution even at ultrahigh frequencies.

How the frequency-independent contribution affects measurements of the expansion, Ω, and the age of the universe depends upon how the frequency and amplitude vary with time, which depends on the nonminimal couplings to the scalar curvature, $G^{-1}\mathcal{R} \to f(\phi)\mathcal{R}$, and other interactions of the ϕ field. As a simple example, we shall analyze the case where $f(\phi) = \bar{G}^{-1}[1 + \alpha\phi/M_P]$ and $\rho_G = \frac{1}{2}(\dot{\phi}^2 + m^2\phi^2)$, where $\bar{G}$ is the mean Newton's constant and $M_P \equiv \bar{G}^{-1/2}$ is the mean Planck mass. Assuming a scenario in which an early inflationary epoch smooths out the spatial gradients in ϕ [3], we shall illustrate the constraints on the model and show that, in this example, measurements of H are unchanged while dynamical measurements of Ω yield a value less than unity. Discussion of other models with different effects on cosmological measures will be presented elsewhere [6].

Since the potential for ϕ is harmonic, ρ_G redshifts just as nonrelativistic matter ($\propto a^{-3}$): $\phi \approx [\phi_b(a/a_b)^{-3/2}] \times \cos(mt)$, where ϕ_b is the amplitude and a_b is the scale factor at the beginning of the oscillations (when $H = H_b \approx m$). For the desired frequency range, oscillations begin when the universe is less than a picosecond old during the radiation-dominated epoch. It is possible that ρ_G grows to dominate the radiation energy density before the usual onset of the matter-dominated epoch. Since we do not require a significant ρ_G in this scenario, we will apply a conservative constraint on the initial amplitude ϕ_b to ensure that ρ_G is always less than the radiation density and remains less than ρ_m even through the present epoch. Since ρ_G and ρ_m scale the same way as a function of the scale factor, it is sufficient that $\rho_G = \xi^2\rho_m$ with $\xi \lesssim 1$ at the present epoch. If $a_0 \equiv 1$ and H_0 are the scale factor and Hubble parameter today, respectively, this means $m^2\phi_{\text{today}}^2 \approx \xi^2(3/8\pi)H_0^2M_P^2$, or

$$\phi_{\text{today}} \approx \xi(3/8\pi)^{1/2}(H_0/m)M_P. \tag{3}$$

Oscillations in ϕ induce oscillations in $G \propto f^{-1}(\phi) = \bar{G}[1 + \alpha\phi/M_P]^{-1}$. The oscillation amplitude for G today,

$$\alpha\phi_{\text{today}}/M_P \approx \alpha\xi(3/8\pi)^{1/2}(H_0/m), \tag{4}$$

is negligibly small for the high frequencies that we consider ($\nu > 10^{12}$ GeV or $m > 10^{-12}$ GeV): $\Delta G/G \le 10^{-30}$. On the other hand, because of the high frequency m, the amplitude of $\dot{G}/G$ today,

$$\mathcal{A} \equiv m(\alpha\phi_{\text{today}}/M_P) \approx \alpha\xi(3/8\pi)^{1/2}H_0, \tag{5}$$

is not negligible; rather, it is comparable to the expansion rate H_0 and, hence, can significantly affect the cosmological expansion according to Eq. (1). In general, the amplitude of $\dot{G}/G$, which is proportional to the amplitude of ϕ, scales as $\mathcal{A}/a^{3/2}$.

Since the amplitude of $(\dot{G}/G)^2$ and the matter density ρ_m both scale as $a(t)^{-3}$, Eq. (2) is separable in terms of $a(t)$ and t, and an exact solution for $a(t)$ can be found in terms of elliptic integrals. If $\gamma \equiv \mathcal{A}/2H_{\text{MD}} = \alpha\xi/2$ and if $\chi \equiv \gamma^2/(1+\gamma^2)$, then

$$a(t) \approx \{(\tfrac{3}{2}h_m t)[(1+\gamma^2)^{1/2}E(\chi)]\}^{2/3}, \tag{6}$$

where $h_m \equiv (8\pi G\rho_m/3)^{1/2}$ evaluated today and $E(\chi)$ is the complete elliptic integral of the second kind. The factor in square brackets ($\approx 1 + \frac{1}{4}\gamma^2$) is the leading correction.

The scale factor $a(t)$ is changed by a time-independent factor: $a \propto t^{2/3}$ and $H \equiv \dot{a}/a$, just as in a flat universe without oscillating G. We have also assumed that the spatial curvature is negligible due to an earlier inflationary epoch. Consequently, all kinematic tests of flatness (e.g., measurements of the deceleration parameter or galaxy number counts) are predicted to find $\Omega_{\rm kin}=1$ and the age of the universe is unaltered.

What oscillating G changes is the relationship between H_0 and the matter density, as can be seen directly from Eq. (2). A consequence is that dynamical techniques for measuring Ω (e.g., via a virialized mass distribution or peculiar velocities) are predicted to find $\Omega<1$, perhaps as small as $\Omega=0.1$–0.3. Attempts to measure ρ_m directly (assuming Kepler's third law for motion about a galaxy or assuming a virialized mass distribution for tests on larger scales) should find

$$\Omega_{\rm virial} \equiv 8\pi G\rho_m/3H_0^2=(h_m/H_0)^2$$
$$=[(1+\gamma^2)^{1/2}E(\beta)]^{-2} \approx (1+\tfrac{1}{2}\gamma^2)^{-1}\,; \qquad (7)$$

for $\gamma \approx 3$, $\Omega_{\rm virial}\approx 0.2$.

The value of Ω inferred from peculiar-velocity measurements is slightly smaller [7]. The peculiar velocity on a given scale is related in linear theory to the growth of perturbations on that scale, $\delta \equiv \delta\rho/\rho$. The perturbation amplitude for nonrelativistic species on subhorizon scales during the matter-dominated epoch obeys an equation of the form [5]

$$\ddot{\delta}+2H\dot{\delta} \approx 4\pi G\rho_{\rm tot}(\xi_m\delta+\xi_G\delta_G)\,, \qquad (8)$$

where $\rho_{\rm tot}=\rho_m+\rho_G$, $\xi_m=\rho_m/\rho_{\rm tot}$, $\xi_G=\rho_G/\rho_{\rm tot}$, $\delta=\delta\rho_m/\rho_m$, and $\delta_G=\delta\rho_G/\rho_G$. If $\rho_G \ll \rho_m$, the right-hand side reduces to $4\pi G\rho_m\delta$. The peculiar velocity is proportional to $\kappa \equiv d(\ln\delta)/d(\ln a)$. In an open universe without oscillations in G, the solution to Eq. (8) is $\Omega=\kappa^{5/3}$ [8]. In a flat universe with oscillating G, the corrections to Eq. (8) are negligibly small; any new terms are either suppressed by a factor of $\Delta G/G \ll 1$ or average to zero over many oscillations. The solution to Eq. (8) is $\kappa=\frac{1}{4}[-1+(1+24\Omega_{\rm virial})^{1/2}] \le 1$, where we have used $\Omega_{\rm virial}=8\pi G\rho_{\rm tot}/3H_0^2$. Given peculiar-velocity measurements alone, an oscillating G with $\Omega=\Omega_{\rm virial}$ could be misconstrued for an open universe with $\Omega_{\rm peculiar}=\kappa^{5/3}$. For $\Omega_{\rm virial}=0.2$, $\kappa=0.35$, and $\Omega_{\rm peculiar}=0.18$. The discrepancy between $\Omega_{\rm virial}$ and $\Omega_{\rm peculiar}$ is small for the interesting range $0.1<\Omega<1$.

Note that one consequence of this last analysis is that $\delta \propto a^{\kappa}$ with $\kappa<1$. The growth of perturbations is significantly suppressed. Admittedly, this simplistic model is not complete since some new physics is required to explain structure formation.

In this discussion of Eq. (8), we have assumed that the oscillations in ϕ are spatially uniform. In a later paper [3], we will present a generic scenario in which ϕ is displaced from its equilibrium position during an early inflationary epoch, setting up the conditions to initiate oscillations. In this scenario, inflation naturally leads to an acceptable amplitude today and smooths ϕ such that the perturbations in ϕ remain negligible today. It is also possible to construct scenarios in which the perturbations in ϕ are non-negligible and Eq. (8) must be modified. These scenarios will also be discussed in Ref. [3].

We have also assumed that ϕ should not have decayed through the present epoch. Even if all direct ϕ couplings are extremely weak, ϕ can decay due to its nonminimal coupling to the scalar curvature $\mathcal{R}$. In general, if ϕ has mass m, the decay rate is [9]

$$\Gamma_g \lesssim m m_d^2/M_P^2\,, \qquad (9)$$

where we have assumed a dominant two-body mode to fermions of mass $m_d<m$. For scenarios where the ϕ oscillations are supposed to continue through the present epoch, we require $\Gamma_g \lesssim H$. For the most pessimistic case, $m_d \approx m$, we find $m \lesssim [HM_P^2\alpha^{-2}]^{1/3} \sim \alpha^{-2/3}(100\ \text{MeV})$. Recall that the lower bound to evade classical relativity tests on laboratory length scales is $m>10^{-12}$ GeV, so a significant allowed mass range remains.

It is possible to consider models in which the decay of ϕ to gravitons is suppressed by imposing a $\phi \to -\phi$ symmetry [9], e.g., $f(\phi)=\bar{G}^{-1}[1+\xi\phi^2/M_P^2]$ and $\langle\phi\rangle=0$. The upper bound on m is lifted when the decay is suppressed. However, it should be noted that the mean value of G changes with time as the oscillation amplitude decreases due to redshift. The nucleosynthesis bound, $\Delta\bar{G}/G_{\rm today} \lesssim 0.4\pi$, must then be considered [10]; typically, the new constraint modestly raises the lower bound on m but there remains no upper bound.

Other cosmological effects of oscillating G can be obtained by altering the dependence of G on ϕ and/or introducing additional nonminimally coupled fields. The $(\dot{G}/G)^2$ in Eq. (2), instead of scaling as nonrelativistic matter ($\propto a^{-3}$), can be made to scale as relativistic matter ($\propto a^{-\beta}$), where the exponent β is model dependent [3]. An important effect is that suppression of perturbation growth is reduced to a negligible level if β is small. Also, we have shown how these more general models can alter not only $\Omega_{\rm virial}$, but also galaxy count versus redshift or the cosmological age [6].

At present, a number of potentially embarrassing conflicts are developing between measured cosmological parameters and the predictions of inflationary or big-bang models. For example, inflationary models predict $\Omega=1$, while some dynamical measurements suggest that Ω may be 0.3 or less. There may also be problems reconciling the age of the Universe, the growth of large-scale structure, and galaxy number counts. The conflicts have led to the appeal to dark matter, a small, but nonzero cosmological constant, and other exotic possibilities. The notion of rapidly oscillating G is, in this context, perhaps no more exotic and perhaps even fits with current attempts to unify gravity with the strong, weak, and electromagnetic forces. The frequency and amplitude ranges required to affect cosmological measures seem safely within all known experimental constraints at present.

With so much at stake, it may become imperative to develop novel tests for high-frequency, low-amplitude oscillations in G.

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