Top-Quark and Higgs-Boson Mass Bounds from a Numerical Study of Supersymmetric Grand Unified Theories

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We run all the couplings of the minimal supersymmetric (SUSY) extension of the standard model, taking account of the Yukawa sector. After identifying the scale at which the gauge couplings unify, we place bounds on the top-quark mass by requiring equality of the bottom-quark and τ Yukawa couplings at that scale. For $M_{\rm SUSY}=1$ TeV, $M_b=4.6$ GeV, we find $139 \le M_t \le 194$ GeV, which remarkably satisfy the ρ-parameter bound. Furthermore, using the minimal SUSY boundary condition on the scalar quartic coupling, we obtain bounds for the mass of the Higgs boson, $44 \le M_{\rm Higgs} \le 120$ GeV.

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In this Letter we present bounds on the mass of the top quark in a minimal supersymmetric extension of the standard model (MSSM) with minimal Higgs structure in the context of a grand unified theory (GUT) by numerically evolving the couplings using their renormalization-group equations. This analysis improves on previous endeavors by taking full account of the Yukawa sector. A more detailed account of the method will be presented in a subsequent paper [1]. Here we present our results and give a brief description.

The modified-minimal-subtraction ($\overline{\rm MS}$) renormalization-group equations for the standard model and the MSSM [2] are numerically integrated and used to evolve the parameters of the model to Planck scale. Although it is not possible to analytically express certain parameters (e.g., Cabibbo-Kobayashi-Maskawa angles) in terms of the Yukawa couplings, equations for the running of the quantities themselves can be arrived at by making some approximations. Typically one assumes that the contribution of the Yukawa couplings is essentially given by the top-quark one, y_t, since it is larger than the others. Otherwise one can retain the effects of the other Yukawa coupling by keeping only the diagonal entries, but since our approach is numerical we opt to run the quantities by diagonalizing the Yukawa matrices at every step of the Runge-Kutta method.

In the expectation that the standard model is only the low-energy manifestation of some yet unknown GUT or of a possible supersymmetric (SUSY) extension thereof, the three couplings g_3, g_2, and g_1 corresponding to the standard model gauge groups, $SU(3)^c \times SU(2)^W \times U(1)^Y$, should meet at some large grand unification scale. Our study begins here. Using the accepted values and associated errors of these couplings, we observe unification in the SUSY GUT case but not in the pure GUT case, as noted by several groups [3,4] (see Fig. 1). However, this should not be viewed as proof of supersymmetry since, given the values of $\alpha_1,\alpha_2,\alpha_3$ at some scale, and three unknowns (the value of α at the unification scale, the unification scale, and an extra scale such as the SUSY scale), there is always a solution. The exciting aspect of the analysis of Ref. [3] is the numerical output, namely, a low SUSY scale, $M_{\rm SUSY}$, and a perturbative solution below the Planck scale which does not violate proton decay bounds [5].

Furthermore, in the context of a minimal GUT [6] there are constraints on the Yukawa couplings at the scale of unification. In this Letter we first restrict ourselves to an SU(5) SUSY GUT [7] where y_b and y_τ, the bottom and τ Yukawa couplings, are equal at unification. The crossing of these renormalization-group flow lines is sensitive to the physical top-quark mass M_t. This can be seen in the down-type Yukawa renormalization-group equation (above $M_{\rm SUSY}$, for example), from which we extract the evolution of y_b, since the top contribution is large and appears already at one loop through the up-type Yukawa dependence:

$$\frac{d\mathbf{Y}_d}{dt}=\frac{1}{16\pi^2}\mathbf{Y}_d[3\mathbf{Y}_d^\dagger\mathbf{Y}_d+\mathbf{Y}_u^\dagger\mathbf{Y}_u+\mathrm{Tr}(3\mathbf{Y}_d^\dagger\mathbf{Y}_d+\mathbf{Y}_e^\dagger\mathbf{Y}_e) -(\tfrac{7}{15}g_1^2+3g_2^2+\tfrac{16}{3}g_3^2)]\,, \qquad (1)$$

where $\mathbf{Y}_{u,d,e}$ are matrices of Yukawa couplings. Demanding that their crossing point be within the unification region determined by the gauge couplings allows one to constrain M_t. This yields an upper and a lower bound for M_t which nevertheless is fairly restrictive.

Let us now briefly describe our method [1]. We work in a mass-independent renormalization scheme where the running couplings are unphysical. From the decoupling theorem [8] we expect the physics at energies below a given mass scale to be independent of the particles with masses higher than this threshold. Therefore for a correct interpretation of these running couplings we must take into account the thresholds [9–11]. For the electroweak threshold we use one-loop matching functions [11] with the two-loop beta functions valid in the standard model regime below the SUSY scale. These matching functions are obtained in $\overline{\rm MS}$ renormalization by integrating out the heavy gauge fields in such a way that the remaining effective action is invariant under the residual gauge group [10]. At the electroweak threshold, near M_W, we integrate out the heavy gauge fields and the top quark. Below this threshold there is an effective $SU(3)^c \times U(1)^{EM}$ theory. Thresholds in this region are obtained by integrating out each quark to one-loop level at a scale equal to its physical mass. At these scales the one-loop matching functions in the gauge couplings vanish and the threshold dependence appears through steps in the number of quark flavors [12] as the renormalization-group scale passes each physical quark mass. There is also a threshold at M_{SUSY}. Here the matching condition is the naive one of simple continuity due to the lack of knowledge about the superparticle spectrum. We take this scale to be variable to account for this ignorance.

We consider the simplest implementation of supersymmetry and run the couplings above M_{SUSY} to one loop. The superpotential for the supersymmetric theory is

$$W=\hat{\Phi}_u \hat{Q} \mathbf{Y}_u \hat{u}^c+\hat{\Phi}_d \hat{Q} \mathbf{Y}_d \hat{d}^c+\hat{\Phi}_d \hat{l} \mathbf{Y}_e \hat{e}^c+\mu \hat{\Phi}_d \hat{\Phi}_u \,, \tag{2}$$

where the caret denotes a chiral supermultiplet. We assume the MSSM above M_{SUSY}, and a model with a single light Higgs scalar below it. This is done by integrating out one linear combination of the two doublets at M_{SUSY}, thereby leaving the orthogonal combination in the standard model regime as the "Higgs doublet":

$$\Phi_{(SM)}=\Phi_d \cos\beta+\tilde{\Phi}_u \sin\beta \,, \tag{3}$$

where $\tilde{\Phi}=i\tau_2\Phi^*$, and where $\tan\beta$ is also the ratio of the two vacuum expectation values (v_u/v_d) in the limit under consideration. This sets boundary conditions on the Yukawa couplings at M_{SUSY}. Furthermore, in this approximation the quartic self-coupling of the surviving Higgs boson at the SUSY scale is given by

$$\lambda(M_{SUSY})=\tfrac{1}{4}(g_1^2+g_2^2)\cos^2(2\beta) \,. \tag{4}$$

This correlates the mixing angle with the quartic coupling and thereby gives a value for the physical Higgs-boson mass M_{Higgs}. Using the experimental limits on the M_{Higgs} further constrains some of the results. By using the renormalization group we take into account radiative corrections to the light Higgs-boson mass [13] and hence relax the tree level upper bound, $M_{Higgs} \sim M_Z$ [14].

We determine the bounds on M_t and M_{Higgs} by probing their dependence on β. In SUSY SU(5), $\tan\beta$ is constrained to be larger than 1 in the one light Higgs limit. It seems natural to us to require that $y_t \gtrsim y_b$ up to the unification scale [15], thereby yielding an upper bound on $\tan\beta$. The initial values at M_Z for the gauge couplings are taken to be [3,16]

$$\begin{aligned} \alpha_1 &= 0.016\,887 \pm 0.000\,040 \,, \\ \alpha_2 &= 0.033\,22 \pm 0.000\,25 \,, \\ \alpha_3 &= 0.109^{+0.004}_{-0.005} \,, \end{aligned} \tag{5}$$

where GUT normalization for α_1 is used. We use the set

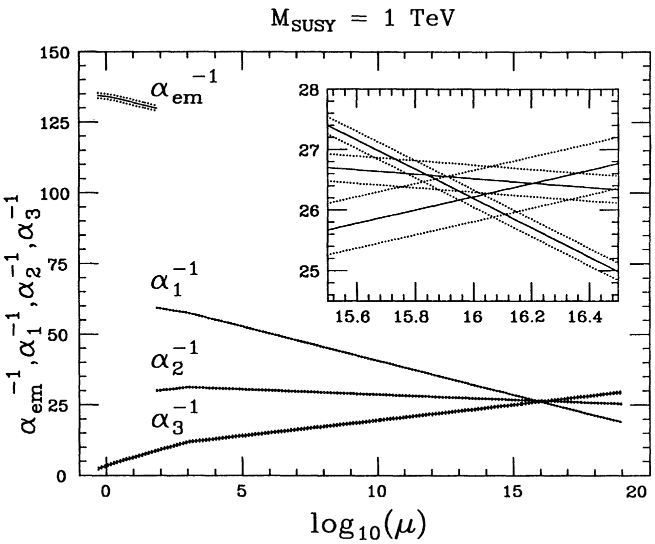

FIG. 1. Plot of the running of the inverse couplings. The dotted lines above and below the solid lines represent the experimental error for each coupling. Inset: Blowup of the area around the unification point. (Note the small region where all three couplings intersect. We found that this region reduced to a point when M_{SUSY}=8.9 TeV and was nonexistent above that scale.)

of four quark running masses defined at 1 GeV by the Particle Data compilation [17]: m_u=5.6 MeV, m_d=9.9 MeV, m_s=199 MeV, and m_c=1.35 GeV. For the bottom-quark mass we use the Gasser and Leutwyler bottom-quark mass value of 5.3 GeV at 1 GeV, which translates into a physical mass of M_b=4.6 GeV [18]. To probe the dependence of our results on M_b we also study the case of M_b=5 GeV, the typical value obtained from potential model fits for bottom-quark bound states [19]. We also investigate the effect of varying M_{SUSY}. Given the values of the gauge couplings, we find unification up to a SUSY scale of 8.9 TeV, and as low as M_W. For empirical reasons we did not investigate solutions below that scale.

From Fig. 1 we determine that the lower end scale, M_{GUT}^L, of the unification region corresponds to an α_3 value of 0.104 at M_Z, while the higher end scale, M_{GUT}^H, corresponds to a value of 0.108 at M_Z for α_3. We find that the unification region is insensitive to the range of top-quark, bottom-quark, and Higgs-boson masses considered. In our analysis of the bounds for M_t, the values for α_1 and α_2 are chosen to be the central values since their associated experimental uncertainties are less significant than for α_3. Demanding that y_b and y_τ cross at M_{GUT}^L and taking α_3=0.104 then sets a lower bound on M_t. Correspondingly, demanding that y_b and y_τ cross at M_{GUT}^H and taking α_3=0.108 yields an upper bound on M_t. These bounds are found for each possible value of β.

Figure 2 shows the upper and lower bound curves for both M_t and M_{Higgs} as a function of β and for M_{SUSY}=1

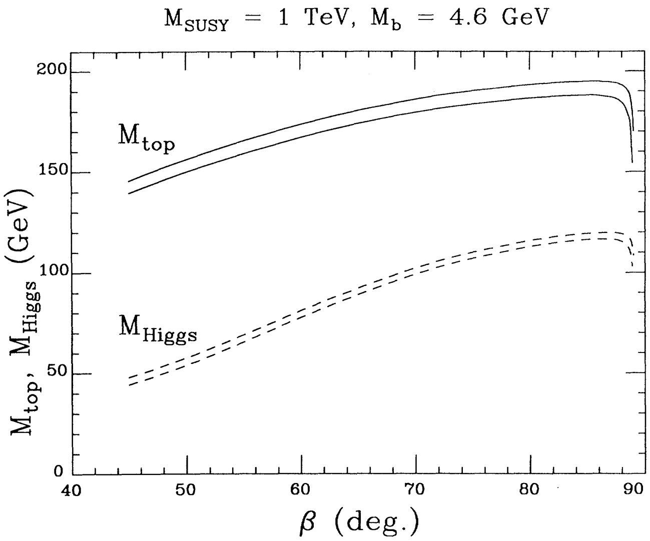

FIG. 2. Plot of the top-quark mass M_t and of the Higgs-boson mass M_{Higgs}, as a function of the mixing angle β for the highest value of α_3 (high curves) and the lowest value of α_3 (low curves) consistent with unification as *per* Fig. 1.

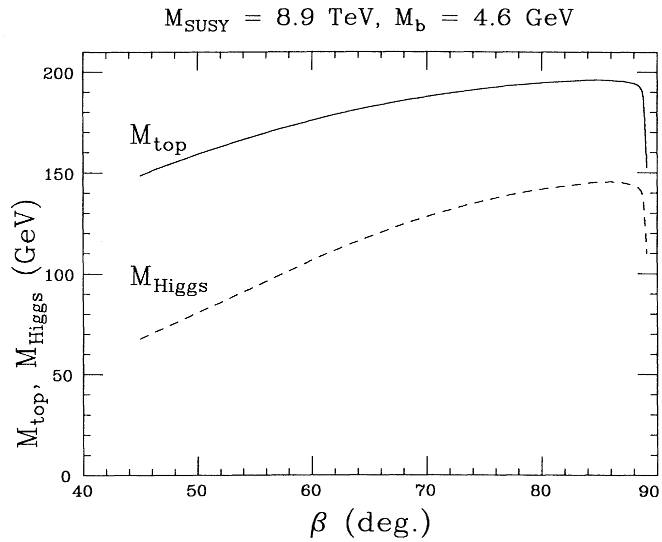

FIG. 3. Same as Fig. 2 for $M_{\text{SUSY}}=8.9$ TeV and $M_b=4.6$ GeV.

TeV and $M_b=4.6$ GeV. When applicable, we use the current experimental limit of 38 GeV on the light supersymmetric neutral Higgs-boson mass [20] to determine the lowest possible M_t value consistent with the model. We find $139 \leq M_t \leq 194$ GeV and $44 \leq M_{\text{Higgs}} \leq 120$ GeV. We investigated the sensitivity of these results on M_{SUSY} in the range 1.0 ± 0.5 TeV. We find that the bounds on M_t are not modified, but the upper bound on the Higgs boson is changed to 125 GeV, and the lower bound drops below the experimental lower bound.

For $M_b=5.0$ GeV, we see an overall decrease in the top-quark and Higgs-boson mass bounds: $116 \leq M_t \leq 181$ GeV, $M_{\text{Higgs}} \leq 111$ GeV. Varying M_{SUSY} as above modifies the respective bounds. The top-mass lower and upper bounds become 113 and 119 GeV, respectively. The upper bound on M_{Higgs} changes to 115 GeV. We display the results of our analysis for the extreme case, $M_{\text{SUSY}}=8.9$ TeV, in Fig. 3, with $M_b=4.6$ GeV. This only significantly changes the upper bound on M_{Higgs} to 144 GeV compared to the $M_{\text{SUSY}}=1$ TeV case.

We have also run y_t up to the unification region and compared it with y_b and y_τ to see what the angle β must be for these three couplings to meet [21], as in an SO(10) or E_6 model [22,23] with a minimal Higgs structure. It is clear that this angle is precisely our upper bound on β as described earlier. In Fig. 4 we display y_t/y_b at the GUT scale as a function of $\tan\beta$ for $M_{\text{SUSY}}=1$ TeV and for the two bottom masses we have considered. If we demand that the ratio be 1, we can determine the mixing angles for the low and high ends of the unification region. Then, going back to Fig. 2, we find as expected a much tighter bound on the masses of the top quark and the Higgs boson. Indeed, for $M_b=4.6$ GeV, we have $49.40 \leq \tan\beta \leq 54.98$, which yields $162 \leq M_t \leq 176$ GeV and $106 \leq M_{\text{Higgs}} \leq 111$ GeV. When $M_b=5.0$ GeV, we obtain $31.23 \leq \tan\beta \leq 41.18$, which gives $116 \leq M_t \leq 147$ GeV and $93 \leq M_{\text{Higgs}} \leq 101$ GeV.

Several issues have been left untouched. We have not implemented the supersymmetric two-loop beta functions and the corresponding thresholds. The effects of soft SUSY breaking terms were not investigated nor was the possible role of a large top mass on this breaking. Also, we have integrated out all the supersymmetric particles at

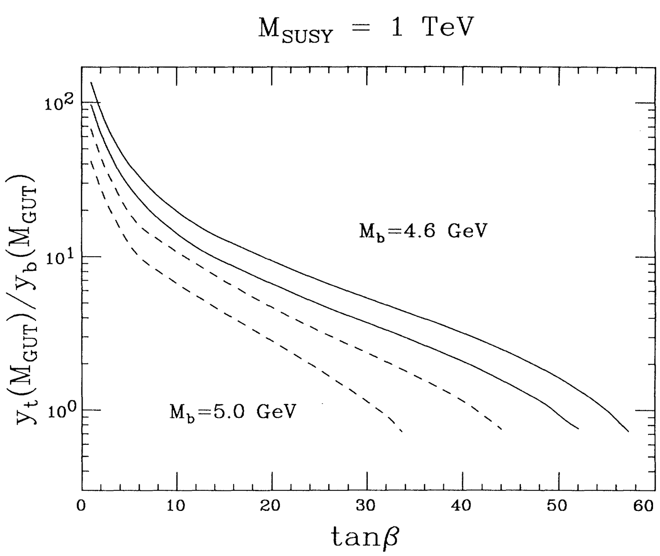

FIG. 4. Plot of the ratio of the top- and bottom-quark Yukawa couplings, y_t/y_b, for two different bottom masses (solid and dashed curves) as a function of $\tan\beta$ for the highest value of α_3 (high curves) and the lowest value of α_3 (low curves) consistent with unification as *per* Fig. 1.

the same scale. It would be interesting to study the effect of lifting this restriction. We should also note that our bounds on the top mass are very similar to those of Ref. [15], although the physics is very different. We plan to return to these issues in a forthcoming paper [1]. However, given the relative crudeness of the approximations in this paper, it is remarkable that the experimental bounds on the ρ parameter were satisfied, which in our mind gives credence to our program.

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