Exact Black Five-Branes in Critical Superstring Theory

Steven B. Giddings [(a)] and Andrew Strominger [(b)]
Department of Physics, University of California, Santa Barbara, California 93106
(Received 23 July 1991)

Exact solutions of ten-dimensional superstring theory corresponding to black five-branes [(5+1)-dimensional extended objects surrounded by event horizons] are constructed. The solutions are represented as products of conformal field theories.

PACS numbers: 11.17.+y, 04.20.Jb

The desire to unravel the mystery of singularities in general relativity is a primary motivation for developing a tractable framework for quantum gravity. String theory, whether or not it is the theory of our Universe, appears to provide a quantum mechanically consistent extension of classical gravity. For this reason we should explore in string theory the analogs of singular solutions of classical general relativity.

Recently, there has been some progress in understanding various aspects of black holes and their higher-dimensional generalizations, black p-branes, in string theory. In particular, in [1], Callan, Harvey, and one of the present authors found an exact solution of ten-dimensional superstring theory corresponding to an extremal black five-brane. This solution was labeled by one discrete parameter (the charge) and one continuous parameter (the asymptotic value of the dilaton) and had spacetime and world-sheet supersymmetry. Horowitz and one of the present authors [2] then found, using low-energy string theory, a larger family of approximate solutions with the black-hole mass as a second continuous parameter. The extremal member of this family with equal mass and charge coincides with the exact solution of [1]. Both the event horizon and the singularity disappear in this extremal limit, leaving in their place an infinitely long semiwormhole. While this is interesting in its own right, the issues associated with event horizons and singularities are sidestepped by this exact solution. This situation was markedly improved by Witten's interpretation [3] of an SU(1,1)/U(1) conformal field theory [4,5] (i.e., exact solution of the string equations of motion) as a black-hole geometry. Related work has appeared in [6–12]. However, this solution is the somewhat unphysical case of a black hole in two spacetime dimensions and does not have the usual asymptotically flat behavior at infinity. This latter point complicates investigations of phenomena such as Hawking radiation.

For these reasons it is important to search for exact black-hole-like solutions in higher-dimensional spacetimes. Various attempts have subsequently been made in this direction, principally through generalizations of the noncompact coset construction that yields the two-dimensional solution. However, such higher-dimensional examples have not previously been found.

In this paper we show that the tensor product of the supersymmetric version of the exact two-dimensional black-hole solution of [3] with the super Wess-Zumino-Witten (WZW) model used in [1] together with free field theories gives an exact construction of some of the approximate black five-brane solutions of [2]. The exact solutions are labeled by one continuous and one discrete parameter, and coincide in an appropriate limit with the solution in [1]. (We further suggest that the extra continuous parameter, present in [2] but missing from our exact solution, can be identified with a marginal operator of the conformal field theory.) Our construction therefore does provide an example of an exact conformal field theory corresponding to a solution of critical, ten-dimensional superstring theory with an event horizon.

We begin with a description of the field theory limit, which clarifies the geometry involved, and then give the conformal field theory construction.

In [2] approximate solutions of ten-dimensional superstring theory were found corresponding to extended black holes by solving the low-energy field theory equations of motion. The charged black five-brane solution was given by

$$\begin{aligned} ds^2 &= -(1-\Delta^2/y^2)dt^2 + (y^2+M-\tfrac{1}{2}\Delta^2) \\ &\quad \times \left(\frac{dy^2}{y^2-\Delta^2} + d\Omega_3^2 \right) + dx^i dx_i\,, \\ e^{2(\phi-\phi_0)} &= 1+(M-\tfrac{1}{2}\Delta^2)/y^2\,, \\ H &= Q\epsilon_3\,, \end{aligned} \tag{1}$$

where $i=6,\ldots,9$, $d\Omega_3^2$ and ϵ_3 are the line element and volume form on the unit three-sphere, and M is proportional to the mass per unit five-volume. The axion charge Q of the black five-brane is

$$Q=(M^2-\tfrac{1}{4}\Delta^4)^{1/2} \tag{2}$$

and is quantized. There is a singularity at $y=0$, and an event horizon at $y=\Delta$. This ten-dimensional solution may be viewed as the product of a five-dimensional black hole with five-dimensional flat space.

It is useful to transform to the coordinates

$$r=\operatorname{arccosh}(y/\Delta)\,, \quad \tau=t/\sqrt{Q}\,. \tag{3}$$

The solution is then

$$
\begin{aligned}
& ds^2 = -Q\tanh^2 r\, d\tau^2 + [M+\Delta^2(\cosh^2 r - \tfrac{1}{2})] \\
& \qquad \times (dr^2 + d\Omega_3^2) + dx^i dx_i\,, \\
& e^{2(\phi-\phi_0)} = [M+\Delta^2(\cosh^2 r - \tfrac{1}{2})]/\Delta^2 \cosh^2 r\,, \\
& H = Q\epsilon_3\,.
\end{aligned}
\tag{4}
$$

An interesting limit of these solutions can now be found by letting

$$e^{\phi_0} \to 0, \quad \Delta \to 0\,, \tag{5}$$

with

$$e^{-\phi_0}\Delta = \text{const} \equiv e^{-\hat{\phi}_0} \tag{6}$$

fixed. This gives

$$
\begin{aligned}
& ds^2 = -Q\tanh^2 r\, d\tau^2 + Q\,dr^2 + Q\,d\Omega_3^2 + dx^i dx_i\,, \\
& e^{2(\phi-\hat{\phi}_0)} = Q\cosh^{-2} r\,, \\
& H = Q\epsilon_3\,.
\end{aligned}
\tag{7}
$$

The first two terms in the metric, together with the expression for the dilaton, agree with the leading-order α' approximation to the $(1+1)$-dimensional black-hole solution of [3]. The third term in ds^2, together with the expression for H, comprise an SU(2) level-Q WZW model. We see that to leading order the level of the 2D black hole matches that of the SU(2) model. The last term is, of course, a free field theory. In this context we see that the $(1+1)$-dimensional black-hole solutions of [3-8] are a special case of the higher-dimensional solutions of [2].

It is now straightforward to give the exact construction of the approximate ten-dimensional solution of Eq. (7). In [3] Witten interprets an SU(1,1)/U(1) coset model [4,5] as a $(1+1)$-dimensional black hole. This has central charge $c = 2+6/(k-2)$ at level k. The supersymmetric version of this [13], which contains two additional world-sheet fermions, has the central charge

$$c_{1+1} = 3+6/(k-2)\,. \tag{8}$$

The level-Q super WZW model on SU(2) has, as explained in [1], the central charge

$$c_{\mathrm{WZW}} = \tfrac{9}{2} - 6/Q\,. \tag{9}$$

Finally, five free bosons together with their world-sheet superpartners have the central charge

$$c_5 = \tfrac{15}{2}\,. \tag{10}$$

Fixing the SU(1,1) level k by

$$k = Q+2\,, \tag{11}$$

which includes the quantum corrections to the leading-order relation between the levels, we find that the tensor product of these three theories has the central charge

$$c_{1+1} + c_{\mathrm{WZW}} + c_5 = 15 \tag{12}$$

for any level Q. This $c=15$ superconformal field theory is the exact solution of string theory corresponding to the approximate solution of Eq. (7).

We note that these exact solutions correspond to limiting cases of the general approximate black five-brane solutions of Eq. (1), which are not asymptotically flat. Rather, they are asymptotic to $S^3 \times M^7$. While we believe that many interesting questions about black holes (such as Hawking radiation) can be fruitfully addressed within this limit, it would certainly be of interest to construct the more general asymptotically flat case as an exact conformal field theory. (The following observations were made in collaboration with C. Callan and J. Harvey.) To this end, we note that the operator which takes the form

$$\mathcal{O} = (\cosh^2 r - \tfrac{1}{2})[\partial r\,\bar{\partial} r + \mathrm{Tr}(g^{-1}\partial g\, g^{-1}\bar{\partial} g)] + O(\alpha') \tag{13}$$

to leading order in sigma-model perturbation theory is in the limit of (5) and (6) a (1,1) superconformal primary field. We may therefore consider the new theory obtained by perturbing the $c=15$ conformal field theory with $\mathcal{O}$. Deforming (7) in the direction defined by $\mathcal{O}$ should give an action corresponding to the general asymptotically flat five-brane.

It should be noted, however, that this perturbation is singular because $\mathcal{O}$ is divergent as $r \to \infty$, and further work is required to define the perturbed theory. (This is reminiscent of adding a cosmological constant to the free Liouville theory.) Of course, a singular perturbation is obviously required to open up the S^3 throat of (7) to an asymptotically flat region.

This $c=15$ superconformal field theory provides a solution for both type-II and heterotic superstrings. In the former case, the spacetime background has nontrivial metric, dilaton, and axion fields. In the latter case, the gauge field must, in a manner described in [1], be equal to the spin connection with torsion. The black five-brane will then carry, in addition to axion charge, a nonzero Pontryagin charge given by

$$\nu = \frac{1}{8\pi^2}\int F \wedge F = 1 \tag{14}$$

independent of Q.

Finally, we note that the construction used in this paper should generalize to yield other interesting solutions corresponding to black holes and p-branes. For example, in a limit similar to that for the five-brane, the four-dimensional black-hole solution of Refs. [14-16] becomes a product of conformal field theories, one of which corresponds to the solution of Refs. [3-5]. The investigation of such constructions is in progress.

We would like to acknowledge the hospitality of the Aspen Center for Physics, where this work was carried out. We thank I. Bars, C. Callan, and J. Harvey for useful conversations, and G. Horowitz for valuable comments on an earlier draft. The work of S.B.G. was supported in part by DOE Outstanding Junior Investigator Grant No. DE-FG03-91ER40168. The work of A.S. was supported in part by DOE Grant No. DE FG03-91ER40168.

[a]Electronic address: giddings@denali.physics.ucsb.edu, steve@voodoo (bitnet).

[b]Electronic address: andy@denali.physics.ucsb.edu, andy @voodoo (bitnet).

[1] C. Callan, J. Harvey, and A. Strominger, Nucl. Phys. **B359**, 611 (1991).

[2] G. Horowitz and A. Strominger, Nucl. Phys. **B360**, 197 (1991).

[3] E. Witten, Phys. Rev. D **44**, 314 (1991).

[4] L. J. Dixon, J. Lykken, and M. E. Peskin, Nucl. Phys. **B325**, 329 (1989).

[5] I. Bars and D. Nemeschansky, Nucl. Phys. **B348**, 89 (1991).

[6] S. Elitzur, A. Forge, and E. Rabinovici, Hebrew University Report No. RI-143-90, 1990 (to be published).

[7] G. Mandal, A. Sengupta, and S. Wadia, IAS Report No. IASSNS-HEP-91/10, 1991 (to be published).

[8] M. Roček, K. Schoutens, and A. Sevrin, IAS Report No. IASSNS-HEP-91-14, 1991 (to be published).

[9] E. J. Martinec and S. L. Shatashvili, University of Chicago Report No. EFI-91-22, 1991 (to be published).

[10] R. Dijkgraaf, H. Verlinde, and E. Verlinde, Princeton University–IAS Report No. PUPT-1252–IASSNS-HEP-91/22, 1991 (to be published).

[11] I. Bars, USC Reports No. USC-91/HEP-B3, 1991 (to be published) and No. USC-91/HEP-B4, 1991 (to be published).

[12] N. Ishibashi, M. Li, and A. R. Steif, UCSB Report No. UCSB-91-28, 1991 (to be published).

[13] I. Bars, Nucl. Phys. **B334**, 125 (1990).

[14] G. W. Gibbons, Nucl. Phys. **B207**, 337 (1982).

[15] G. W. Gibbons and K. Maeda, Nucl. Phys. **B298**, 741 (1988).

[16] D. Garfinkle, G. Horowitz, and A. Strominger, Phys. Rev. D **43**, 3140 (1991).