Dark Matter and the Equivalence Principle

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If the dark matter in galaxies and clusters is nonbaryonic, it can interact with additional long-range fields that are invisible to experimental tests of the equivalence principle. We discuss the astrophysical and cosmological implications of a long-range force coupled only to the dark matter and find rather tight constraints on its strength. If the force is repulsive (attractive), the masses of galaxy groups and clusters (and the mean density of the Universe inferred from them) have been systematically underestimated (overestimated). Such an interaction also has unusual implications for the growth of large-scale structure.

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The notion that there may be hidden matter in the Universe has a long history. In the last century, anomalies in the orbit of Uranus were ascribed to the gravitational pull of an unseen planet, leading to the discovery of Neptune. More recently, the observed flat rotation curves of galaxies and the application of the virial theorem to clusters of galaxies have revealed the presence of large amounts of dark matter, constituting perhaps 90% of the total mass in these systems [1]. Several lines of argument suggest that much of the dark matter in galaxies and clusters is not baryonic, while particle physics models provide a gallery of exotic elementary particles as dark matter candidates [2,3].

In keeping with the principle of equivalence, it is generally assumed that the dark matter gravitates like the visible baryons and leptons, i.e., that it is subject only to gravitational forces. However, since the existence of dark matter is inferred solely from its gravitational effects, and its nature is otherwise unknown, this assumption is open to question. Although a new long-range force of gravitational strength coupled to *ordinary* matter is experimentally ruled out by the spate of recent "fifth-force" experiments [4], there may be an additional long-range interaction which couples to a quantum number carried exclusively by *nonbaryonic* matter. Such an additional force clearly evades laboratory tests of the equivalence principle. Its effects would be manifest in systems where the dark matter is dynamically important, that is, in the outer regions of galaxies and in clusters. In this Letter, we investigate the implications of additional long-range forces acting between nonbaryonic dark matter particles.

Long-range interactions have been proposed in the context of a variety of particle theories. For example, in extended supergravity models [5], a vector field coupling to particles of mass m and effective charge $\sim m/m_{\rm Pl}$ (where $m_{\rm Pl}=G_N^{-1/2}=1.2\times10^{19}$ GeV is the Planck mass) gives rise to a repulsive force of gravitational strength. Alternatively, ultralight pseudo Nambu-Goldstone bosons with scalar couplings, called schizons, can arise naturally in extensions of the standard electroweak model [6]. As a concrete example, consider the phenomenological schizon model with Lagrangian

$$L=\bar\psi i\gamma_\mu\partial^\mu\psi+m_\psi\bar\psi\psi+\tfrac{1}{2}\partial_\mu\phi\,\partial^\mu\phi+(\epsilon/f)\phi\bar\psi\psi-\tfrac{1}{2}m_\phi^2\phi^2\,, \tag{1}$$

where the fermion ψ of mass $m_\psi\sim1$–10 eV constitutes the dark matter ($\Omega_\psi\simeq1$), ϕ is the schizon, and f is a global symmetry-breaking scale. In the nonrelativistic limit, the static potential between two separated fermion sources with masses M_1 and M_2 is given by

$$V_\phi=-\frac{\epsilon^2}{4\pi m_\psi^2f^2}\frac{M_1M_2}{r}e^{-m_\phi r}\,. \tag{2}$$

Thus, on scales $r\ll m_\phi^{-1}$, the relative magnitude of the scalar force is $\alpha=G_\phi/G_N=\epsilon^2m_{\rm Pl}^2/m_\psi^2f^2$; for $\epsilon\sim m_\psi$ and $f\sim m_{\rm Pl}$ it has roughly gravitational strength. In these models the scalar mass is of order [6] $m_\phi\sim m_\psi^2/f$, so the range of the attractive force is astronomical, $\lambda=m_\phi^{-1}\sim1$–100 kpc or more.

In general, the potential energy of two nonbaryonic masses M_1 and M_2 at separation r may be parametrized by

$$V(r)=-G_N\frac{M_1M_2}{r}(1+\alpha e^{-r/\lambda})\,, \tag{3}$$

where the range of the additional interaction is fixed by the Compton wavelength of the exchanged vector ($\alpha<0$) or scalar ($\alpha>0$) particle, $\lambda=1/m_{v,s}$. The resulting force is

$$F(r)=-(G_NM_1M_2/r^2)[1+\alpha(1+r/\lambda)\exp(-r/\lambda)]\,.$$

We study constraints on the relative strength α for a range of wavelengths λ. Since there are gravitationally bound systems of dark matter, we infer that $\alpha>-1$ for $\lambda\gtrsim10$ kpc.

The dark matter density is often expressed in terms of the mass-to-light ratio, $\Upsilon=\langle M/L\rangle_V$, in the V band. From the observed mean luminosity density [7], $j_V\simeq1.7\times10^8hL_\odot$ Mpc^{-3}, the density parameter can be expressed as $\Omega=6\times10^{-4}h^{-1}\Upsilon/\Upsilon_\odot$, where the Hubble constant $H_0=100h$ km/sec Mpc. Thus, the critical mass-to-light ratio for an $\Omega=1$ universe is $\Upsilon_c=1600h\Upsilon_\odot$. The observation of stars in the solar neighborhood (presumably bound to the Galaxy) with velocities of 500 km/sec implies [7] that the total mass-to-light ratio for the Milky Way is at least $\Upsilon_{\rm MW}\gtrsim 30\Upsilon_\odot$. This is consistent with mass-to-light ratios inferred from flat rotation curves in other spiral galaxies. Dynamical measurements of the mass within galaxies rely on baryonic tracers (stars or gas) of the gravitational potential. Since we assume that the new interaction does not couple to baryons (the experimental bound [4] on a long-range force *between* baryons [8] is $\alpha_b\lesssim 10^{-4}$), the masses inferred for individual galaxies, $M_{\rm inf}(r)\sim v^2r/G_N$, are the true masses, i.e., they are independent of α.

In systems of galaxies, however, such as binaries, groups, and clusters, galaxies themselves are used as test particles. If the galaxy mass is dominated by nonbaryonic dark matter, one must take into account the additional force on the dark mass. Consider two galaxies in a binary system with separation $r\ll\lambda$, radially approaching each other with speed v_r; from Kepler's law

$$rv_r^2\propto G_N M_{\rm inf}=G_N[1+\alpha(1-F)]M_{\rm true}\,, \qquad (4)$$

where $M_{\rm true}$ is the true (luminous plus dark) mass of the binary system, $M_{\rm inf}$ is the mass one erroneously infers without knowledge of the additional force, and $F=M_b/M_{\rm true}$ is the baryon mass fraction of the system. Thus,

$$\frac{\Omega_{\rm true}}{\Omega_{\rm inf}}=\frac{\Upsilon_{\rm true}}{\Upsilon_{\rm inf}}=\frac{1}{1+\alpha(1-F)}\,. \qquad (5)$$

A repulsive force ($\alpha<0$) would delude us into believing that the dark matter density is smaller than it actually is. For example, a flat universe with $\Omega_{\rm true}=1$ could masquerade as an open universe with $\Omega_{\rm inf}<1$, perhaps reconciling the theoretical prejudice for a flat universe with the observational indications on scales of groups and clusters that $\Omega_{\rm inf}\lesssim 0.2-0.3$. Although an intriguing possibility, we show below that the value of α required, $\alpha\simeq -0.75$, is inconsistent with cluster observations.

To first approximation, the local group (LG) of galaxies can be thought of as a binary system, dominated by our Galaxy and Andromeda (M31), with a separation $r=700$ kpc. The two galaxies are approaching each other at relative speed $v_r=119$ km/sec. Assuming the orbit is radial, Eq. (4) implies [7] $\Upsilon_{\rm inf}({\rm LG})=(76\text{–}130)\Upsilon_\odot$. (The range arises from uncertainty in the age of the Universe.) Since M31 is expected to have a ratio of dark to luminous matter and a stellar population similar to those of the Milky Way, the true mass-to-light ratio of the local group should at least equal that of our Galaxy, $\Upsilon_{\rm true}({\rm LG})\gtrsim 30\Upsilon_\odot$. From Eq. (5) this implies the upper bound $\alpha\lesssim 4$ for $\lambda\gtrsim 1$ Mpc.

So far we have assumed that the luminous baryons are gravitationally enslaved to their dark halos. However, if $\alpha\neq 0$ a *nontidal*, bulk stripping force arises from the fact that the orbital speeds of the baryonic core and the dark halo of a galaxy (each treated as test particles) *at the same point* in the field of a central mass M (another galaxy or a cluster) do not coincide. For example, requiring that a typical galactic disk outside the core of the Coma cluster is bound to its halo leads to the (relatively weak) constraint $|\alpha|\lesssim 100$. (In the core of a typical rich cluster, the outer halos of most galaxies are thought to be tidally stripped off by the cluster potential [9], while the halos of galaxies at distances $R\gtrsim 1h^{-1}$ Mpc from the cluster center appear to be intact.) If this bound is saturated, a cluster spiral would be displaced from the center of its halo, and its rotation curve would be noticeably asymmetric.

A stronger constraint on α arises by considering the intracluster gas distribution [10]. From x-ray surface brightness observations, most clusters are well fitted by the density profile $\rho_{\rm gas}(r)=\rho_0[1+(r/r_c)^2]^{-3\beta_f/2}$, with [11] $\beta_f=0.6\text{–}0.8$. For isothermal gas in hydrostatic equilibrium in the potential well of a spherical cluster with spatially constant, isotropic, one-dimensional velocity dispersion σ, the gas density satisfies [12] $\rho_{\rm gas}(r)\propto\rho_{\rm clus}(r)^{\beta_s}$, where

$$\beta_s\equiv\sigma^2\mu m_p/kT_{\rm gas}\,. \qquad (6)$$

Here, $\rho_{\rm clus}(r)$ is the (luminous plus dark) cluster density, μ is the gas mean molecular weight in amu, and m_p is the proton mass. The observed surface density profile for galaxies in a cluster is generally well fitted by a King model, with $\rho_{\rm gal}\propto[1+(r/r_c)^2]^{-3/2}$. We thus expect $\beta_s=\beta_f$ if galaxies trace the cluster mass. However, observations of the x-ray spectral temperatures and the galaxy velocity dispersion of a number of clusters imply [13] $\beta_s\simeq 1.2\simeq 2\beta_f$; this difference is known as the β discrepancy. Since most cluster galaxies retain their dark halos (and therefore orbit the cluster as dark matter particles), if $\alpha\neq 0$, then $\beta_s=\beta_{s(\alpha=0)}[1+\alpha(1-F)]^{-1}$. Thus, the discrepancy would be resolved if $\alpha\simeq 1$. However, the "β problem" is most likely a reflection of the simplified assumptions used above [10] (e.g., the gas may not be isothermal, galaxies may not faithfully trace the mass, the galaxy velocity dispersion may be anisotropic and/or a function of radius, and the King model may be a poor fit at large radii) rather than a signal of new physics. Instead, we can use the factor of 2 agreement between the two determinations of β to place constraints on α; given the observational and theoretical uncertainties, we estimate the conservative bound $-0.5\lesssim\alpha\lesssim 2$. An independent bound comes from the giant luminous arcs, high redshift galaxies gravitationally lensed by foreground clusters [14]. From the observed lensing geometry, one can estimate the true cluster mass interior to the radius of the arc (since photons do not couple to the new force) [15]. Comparison with the inferred virial mass in the case of Abell 370 yields a similar bound to that above.

Cosmology is the final arena where the effects of an additional dark matter interaction would be played out. Despite the form of Eq. (3), the gravitational constant G_N is *not* replaced by a function of α in the Einstein equations for a homogeneous and isotropic universe. This is most easily seen by considering the scalar field example of Eq. (1). The homogeneous field $\phi=\phi(t)$ leads to only two effects: a cosmological density of coherent scalar particles, $\rho_\phi(t)$, which behaves like nonrelativistic matter, and a time-dependent mass for the dark fermions. For $f \gtrsim m_{\rm Pl}$ and temperatures $T \lesssim m_\psi$ both effects have negligible impact on the density of the Universe [16]. Consequently, we can assume that the standard cosmology is unaltered by the additional interaction.

The new force will, however, dramatically affect the growth of inhomogeneities. Consider the fractional density perturbations in the nonbaryonic component, $\Delta(\mathbf{x}) = [\rho_{\rm nb}(\mathbf{x}) - \langle\rho_{\rm nb}\rangle]/\langle\rho_{\rm nb}\rangle$. We focus on small-amplitude fluctuations inside the horizon, so we can apply linear perturbation theory in the Newtonian approximation. The dark matter obeys the usual perturbed fluid equation [17], except that it is augmented by the additional potential ϕ, which satisfies

$$\nabla_x^2\phi - m_{v,s}^2\phi a^2 = 4\pi\alpha G_N\langle\rho_{\rm nb}\rangle\Delta a^2\,, \qquad (7)$$

where the gradient is taken with respect to the comoving coordinate $\mathbf{x}$ and a is the cosmic scale factor. For a spatially flat, matter-dominated universe ($\Omega=\Omega_{\rm b}+\Omega_{\rm nb}=1$), the Fourier transform of the perturbation amplitude satisfies

$$\ddot{\Delta}_k + \frac{4}{3t}\dot{\Delta}_k - \frac{2}{3t^2}\left[\Omega_{\rm b}\delta_k + \left(1+\frac{\alpha}{1+(m_{v,s}/k_p)^2}\right)\Omega_{\rm nb}\Delta_k\right] = 0\,, \qquad (8)$$

where $k_p \propto a^{-1}$ is the physical wave number of the perturbation. The fractional perturbation in the baryonic component, δ, obeys

$$\ddot{\delta}_k + \frac{4}{3t}\dot{\delta}_k - \frac{2}{3t^2}(\Omega_{\rm b}\delta_k + \Omega_{\rm nb}\Delta_k) = 0\,. \qquad (9)$$

Since $\Omega_{\rm nb} \gg \Omega_{\rm b}$, the approximate solution for short-wavelength modes ($k_p \gg m_{v,s}$) is $\Delta_k \propto t^p$, where $p = -\frac{1}{6}[1 \pm (25+24\alpha)^{1/2}]$. As expected, the perturbation growth rate is enhanced for an attractive interaction ($\alpha>0$) and retarded by a repulsive force relative to the usual $\Delta \sim t^{2/3}$ behavior. For standard gravity ($\alpha=0$) the asymptotic growing mode satisfies $\delta=\Delta$, so the baryonic fluctuations track the dark matter. However, for $\alpha \neq 0$, the asymptotic ratio δ/Δ is a function $b(\alpha,\Omega_{\rm b})$, which is larger (smaller) than unity for a repulsive (attractive) interaction. Thus the additional force automatically generates a scale-dependent bias (or antibias) between the dark matter and the light.

On large scales, $k_p \ll m_{v,s}$, we retrieve the standard result $\Delta \sim t^{2/3}$ for the growth rate. This introduces a feature in the transfer function $T(k) = \Delta_k(t_0)/\Delta_k(t_i)$, which relates the perturbation amplitude today (t_0) to the amplitude when it was generated (t_i). There is standard growth on *large* scales, but enhanced (retarded) growth on *small* scales for positive (negative) α. If the present spectrum is normalized in the usual way (e.g., by the second moment, J_3, of the mass-correlation function at $r=10h^{-1}$ Mpc), this leads to less (more) relative power on large scales for an attractive (repulsive) force. The reduced power on large scales for $\alpha>0$ effectively excludes a scenario based on cold dark matter with initial adiabatic Harrison-Zel'dovich fluctuations and an attractive long-range interaction with $\lambda \gtrsim 100$ kpc. A detailed discussion of the implications for large-scale structure formation, peculiar velocity flows, and the microwave background anisotropy will be given elsewhere [18].

A final bound on α results from considering the response of dark halos to dissipational baryonic infall in the early stages of galaxy formation. For a collisionless dark matter particle in a circular orbit in a protogalaxy, conservation of angular momentum implies a relation between its initial radius r_i (before baryonic infall) and its final radius r, after the baryons have cooled and collapsed into a disk [19], $r_i/r = f(r/l)F + 1 - F$. The initial fraction of dissipational baryonic mass in the protogalaxy is given by $F = M_{\rm b}/M$; the function $f(r/l)$ is determined completely by the disk scale length l and the ratio of the core radius to the outer radius of the initial mass distribution (assumed to be a truncated isothermal sphere). The resulting rotation curve is relatively flat and continuous from the baryon-dominated core to the dark-matter-dominated halo only if F lies in the range [19] $0.05 < F < 0.2$. If $\alpha \neq 0$, this constraint is replaced by an identical bound on the parameter $\tilde{F} = F/[1+\alpha(1-F)]$. Now, if all the initial baryons are dissipational, then $F = b\Omega_{\rm b}/[1+\Omega_{\rm b}(b-1)]$, where $b(\alpha,\Omega_{\rm b})$ is the linear bias parameter, mentioned above. Using the big-bang nucleosynthesis constraint, $0.01 < \Omega_{\rm b} < 0.2$, and the rotation curve bound on $\tilde{F}$, we find the constraint $-0.8 < \alpha < 1.3$. If α is outside this range, spiral rotation curves would be sharply rising or steeply falling or nearly flat.

We have touched upon only a few of the many interesting phenomena which arise if the dark matter violates the principle of equivalence in the sense of interacting with "hidden" long-range forces. From the mass-to-light ratios in galaxies and binaries, the gas distribution in clusters, and the relaxation of dark halos due to baryonic infall, we conclude that the relative strength of such an interaction must satisfy $-0.5 \lesssim \alpha \lesssim 1.3$ if its range $\lambda \gtrsim$ a few hundred kpc. Even so, such an interaction can alter the apparent density of dark matter and profoundly change the spectrum and amplitude of large-scale density fluctuations.

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Note added.—After this work was completed, a report by Kawasaki *et al.* [20] appeared, proposing that such a force would allow 10-eV neutrinos to cluster in dwarf galaxies, contrary to the usual phase-space constraints; however, their required value of $\alpha \sim 10^4$ is strongly ruled out by our results.

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