Resonant Tunneling and Hopping through a Series of Localized States in a Two-Dimensional Electron Gas

Dragana Popović, A. B. Fowler, and S. Washburn[a]
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
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We report measurements of the temperature and length dependence of the conductance of a wide two-dimensional electron gas in a strongly localized regime. Our results indicate that the conduction occurs via resonant tunneling below about 0.1 K and, at higher temperatures, via hopping through channels that contain more than one localized state. A dramatic *increase* of the average logarithm of conductance $\langle \ln G \rangle$ with the increasing sample length is observed in the tunneling regime, for which there is no satisfactory theoretical explanation yet. This effect persists even up to $\gtrsim 4.2$ K.

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Transport in small strongly localized structures has attracted a lot of attention [1] in recent years. The three types of conduction processes that may occur in these systems at low temperatures are thermally assisted tunneling from one localized state to another [called variable-range hopping (VRH)], resonant tunneling through localized states with energies equal, within the energy-level broadening, to the Fermi energy in the contacts, and direct tunneling between the contacts. Most studies have concentrated on the behavior of quasi-one-dimensional (1D) devices, where the geometrical width W of the sample is smaller than the most probable hopping length R_0. The issue of special interest was the lack of self-averaging in these systems. Studies of resonant tunneling through a single site [2] required the use of devices that were also very short, with the sample length L of the order of R_0. The limit of relatively wide ($W \gg R_0$) but short devices ($L \sim R_0$) has been hardly studied at all, especially in experiments.

The conductance of a macroscopic system in a strongly localized regime ($L, W \gg R_0$ for a 2D case) obeys Mott's law for VRH and can be describe successfully using percolation theory [3]. Obviously, this picture breaks down when L is reduced so that it becomes comparable to the correlation radius of the critical network. It has been proposed [4,5] that, in that case, the conduction proceeds via VRH along exponentially rare, almost rectilinear and equidistant, isolated chains of hops. This model seems to be in agreement with recent experiments [6,7]. If the sample length is reduced further, chains with decreasing numbers of hops become dominant [8]. Finally, when $L \sim R_0$, inelastic processes are no longer important and the conduction is expected [9] to occur via resonant tunneling through channels with an almost periodic distribution of localized states. The number of states N in a channel will depend on L and the density of states. It should be pointed out once again that the probability of formation of these chainlike paths is exponentially small, which means that the physics of these systems is determined by very rare events. Obviously, the chances for their observation in quasi-1D structures, for example, are then also exponentially small. Novel effects may also be expected to arise as a result of self-averaging made possible by the special (wide and short) sample geometry. So far, resonant tunneling through a single state ($N = 1$) and direct tunneling have been studied experimentally in thin a-Si barriers [10,11]. Evidence for hopping through channels with a few localized states has also been found [12] on the same system at higher bias voltages.

In this Letter, we report the observation of the weak, power-law dependence of conductance G on temperature, characteristic [8] of hopping through channels with a few localized states, and resonant tunneling at $T < 0.1$ K. Since R_0 increases as the temperature is lowered, this means that the effective sample length L/R_0 is also reduced. The transition between the hopping and tunneling regimes was thus made possible by the variation of temperature. The main and the most striking result of our work, however, is the observation of an almost exponential decrease of the sample resistance with the length in the resonant tunneling regime. Although no satisfactory theoretical explanation has been proposed yet, we believe that this effect is an indication of transport through channels with $N > 1$.

The measurements were carried out on a series of n-channel rectangular metal-oxide-semiconductor field-effect transistors fabricated on the (100) surface of silicon doped to a level of $\approx 3 \times 10^{14}$ acceptors/cm^3. The gate oxide thickness was 500 Å and the amount of oxide charge, estimated from the threshold voltage at 77 K, was $< 10^{10}$ cm^{-2}. All devices had ion-implanted contacts and came from the same wafer. The sample conductances were measured in a dilution refrigerator with a current preamplifier and a lock-in technique. The samples were driven by ~ 10-Hz excitation voltage of 0.2-μV rms amplitude. The results presented here were obtained from eight samples with $1 \leq L \leq 8$ μm and width-to-length ratios $3 \leq W/L \leq 81$ in the temperature range from 0.01 to 1.6 K; nine more devices with $1 \leq L \leq 127$ μm and $3 \leq W/L \leq 32$ were measured at 4.2 and 77 K. All samples exhibit reproducible conductance fluctuations with the chemical potential u for temperatures up to

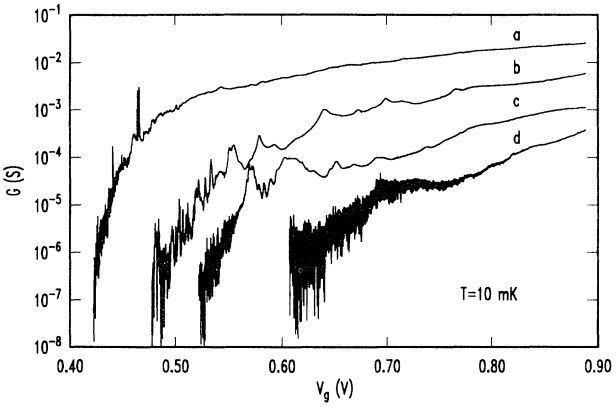

FIG. 1. Conductance G vs gate voltage (or, equivalently, chemical potential μ) for four different samples: Curves a, $L=8$ μm, $W=71$ μm; b, $L=3.5$ μm, $W=11.5$ μm; c, $L=1.5$ μm, $W=11.5$ μm; and d, $L=1.0$ μm, $W=11.5$ μm. All samples exhibit conductance fluctuations for temperatures up to about 1.6 K.

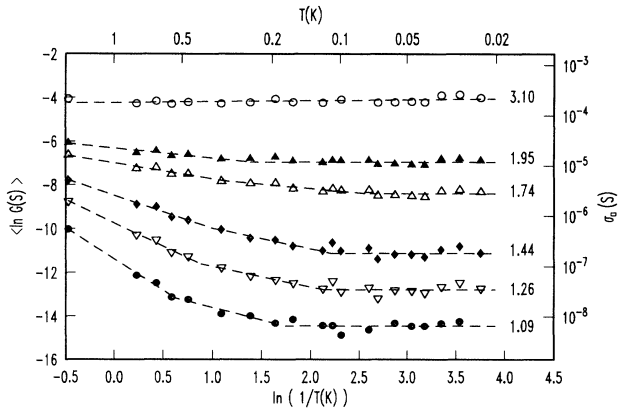

FIG. 2. ln$G$, averaged over the chemical potential, vs ln$(1/T)$ for the sample with $L=2$ μm and $W=162$ μm. The numbers next to the curves show the corresponding carrier densities $N_s(10^{11}$ cm$^{-2})$, ranging from the strongly to weakly localized regime. The dashed lines are guides to the eye. All samples show this kind of power-law dependence of conductance on temperature, in qualitative agreement with the theory of Glazman and Matveev [8].

about 1.6 K. Representative results are shown in Fig. 1, where we plot conductance versus gate voltage V_g (and hence μ and carrier density N_s). A more detailed examination of the fluctuations reveals an incredibly rich structure: in many samples, conductance fluctuates on three scales in μ. This is also reflected in the structure of the autocorrelation function $C(\Delta V_g)=\langle F(V_g+\Delta V_g)F(V_g)\rangle$, where $F(V_g)=\ln G(V_g)-\langle \ln G(V_g)\rangle$ and $\langle\cdots\rangle$ denotes the averaging over V_g (or, equivalently, chemical potential) [13].

In order to study the temperature dependence of the conductance in this regime, one must average ln$G$ over different impurity realizations or gate voltages (ensemble average). The averaging was carried out by fitting ln$G$ by a (typically) third-order polynomial in V_g with the least-squares method. $\langle \ln G\rangle$, found in this way, is displayed as a function of ln$(1/T)$ in Fig. 2, for one of the samples at different carrier densities, ranging from the strongly to weakly localized regime. It is immediately apparent that the temperature dependence is very weak and, in fact, below about 0.1 K, we find that conductance does not depend on temperature at all. This indicates that, in this temperature range, conduction proceeds via resonant tunneling. Above 0.1 K, conductance acquires a weak, power-law dependence on temperature, with the power increasing as temperature goes up, as manifested by the increasing slopes of the linear portions of the curves. This behavior is consistent with theoretical predictions [8] based on a model of channels with a few localized states. For hopping through channels with $N=2$, the slope is expected [8] to be equal to $-\frac{4}{3}$. For the sample used for Fig. 2, we find the slope of -1.1 ± 0.1 for $0.18\ \text{K}\lesssim T\lesssim 0.5$ K and $N_s=1.09\times10^{11}$ cm^{-2}. The agreement between the experimental and theoretical values is even better in other samples (e.g., -1.2 ± 0.1, -1.4 ± 0.1). For hopping through channels with $N=3$ and $N=4$, slopes of -2.5 and -3.6 are expected [8], respectively. We find the slope of -2.9 ± 0.2 for the sample used for Fig. 2, at $T\gtrsim0.5$ K and the same N_s as above. Similar values for slopes (e.g., -2.7, -2.5) are found in other samples in the same temperature range. These results seem to indicate that conduction occurs via VRH through channels with mostly two, and then, most likely, three and four localized states. The agreement with the theory gets worse at higher carrier densities, as one approaches metallic conduction; this is consistent with the assumption of strong localization in the calculations. Further quantitative comparison between our results and the theory is not particularly instructive since, strictly speaking, the latter is not quite adequate for our experimental situation for various reasons. Most importantly, the theory assumes *complete* self-averaging; the temperature dependence is given for conductance obtained by averaging over all possible positions and energies of localized states in channels. On the other hand, we measure *partially* self-averaged conductance and then perform the averaging of ln$G$ over different configurations of localized states. It is not obvious *a priori* that these two procedures are equivalent.

The study of the length dependence of conductance was first carried out for temperatures below about 0.1 K, where conduction occurs via resonant tunneling. In Fig. 3 we show $\langle \ln(G/W)\rangle$ as a function of the sample length L for one carrier density in the strongly localized regime, and two carrier densities in the crossover region between strong and weak localization. Two samples with the same length ($L=2$ μm) and very different widths ($W=11.5$ and 162 μm) have been used. $\langle \ln(G/W)\rangle$ is more or less

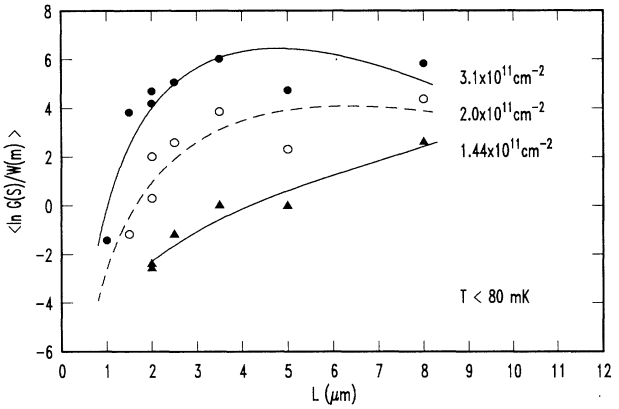

FIG. 3. A considerable *increase* in conductance (normalized to the sample width W), with the increasing length L, for three carrier densities in the resonant tunneling regime. The lines are guides to the eye.

the same for these two samples, i.e., $G \sim W$. The most striking feature of these data, however, is a considerable *increase* in conductance with the sample length, which is not only totally counterintuitive but it has also not been explained by the theory so far. In particular, for resonant tunneling through a single state in each channel, Larkin and Matveev [14] predicted that the average conductance should decrease exponentially with L [$\langle G\rangle \sim \exp(-L/\xi)$, where ξ is the localized length]. In case of resonant tunneling through channels with $N=2$, it was found [14] that $\langle G\rangle \sim L^{\alpha}\exp(-L/\xi)$, where $\alpha=2$ in 3D. The origin of the prefactor lies in averaging over all possible positions and energies of localized states in the channels. It seems plausible that α would be even larger for $N>2$. In that case, the quantity in the exponential would probably also depend on N. It is important to note, however, that this kind of functional dependence has a maximum, i.e., it allows for an increase of conductance with the sample length. This seems to indicate that in our samples, for $T<0.1$ K, the conduction proceeds via resonant tunneling through channels that contain more than one localized state. Although we have no way of determining ξ in these devices, localization length of the order of 500 Å is typically found [15] in silicon inversion layers, which is much smaller than the length of our shortest sample ($L=1$ μm). This also suggests, indirectly, that tunneling through more than one state is required in order to give a significant contribution to the sample conductance. Although a detailed study of the fluctuations has not been made yet, the preliminary results suggest that the shortest scale fluctuations are associated with tunneling through a single state, whereas fluctuations on two larger scales might be related to tunneling through channels with $N=2$ and $N=3$. The comparison between the experiment and the theory [14] is plagued by many problems again. The question of the differences in the averaging is still present. Also, the theory predicts a maximum

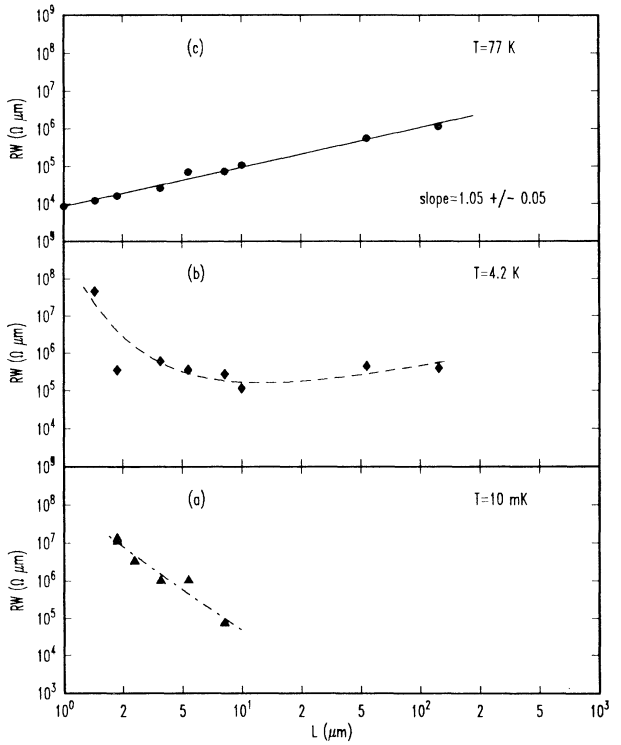

FIG. 4. Resistance as a function of sample length for $N_s=1.44\times10^{11}$ cm^{-2} at (a) $T=10$ mK, (b) $T=4.2$ K, and (c) $T=77$ K. While resistance *decreases* with increasing L in the resonant tunneling regime ($T=10$ mK), the behavior is completely reversed at high enough temperatures (77 K), where the usual $R \sim L/W$ is recovered.

when L is of the order of ξ but, in that range, the theory may not be applicable. Obviously, a much firmer theoretical basis is required in order to interpret the experimental results adequately and to gain full understanding of the conduction processes in these systems.

Figure 4 shows RW as a function of sample length at three different temperatures and the same carrier density in the strongly localized regime. Resistance R is just the inverse of conductance G measured at higher temperatures (4.2 and 77 K), where conductance fluctuations have already disappeared. At lower temperatures, $RW=\exp[-\langle\ln(G/W)\rangle]$. Figure 4(b) shows that the initial decrease of the resistance with the sample length is still present even at 4.2 K but that, in sufficiently long samples, resistance begins to increase with L. At 77 K, the behavior is completely reversed and the resistance is perfectly linear in L [Fig. 4(c)], i.e., we find the usual $R \sim L/W$. The existence of the minimum in resistance at relatively high temperatures (4.2 K) may seem surprising at first. The measurements of the temperature dependence have shown (Fig. 2), however, that the model of hopping through channels with a few localized states is valid at least up to 1.6 K. The average conductance cal-

culated in this model [8] also contains a prefactor proportional to L, which multiplies the exponentially decreasing component. This may allow, in principle, for the existence of the minimum in RW vs L. As the metallic regime is approached, the minimum shifts towards lower values of L. It is interesting that it still exists in the crossover region between strong and weak localization.

In summary, we have measured the conductance of a short and wide two-dimensional electron gas in a strongly localized regime. We have observed the weak, power-law dependence of $\langle \ln G \rangle$ on $\ln(1/T)$, in a qualitative agreement with the model of conduction through channels with a few localized states [8]. Recently we have learned of another experiment [16] in this tunneling regime, where similar, but much smoother temperature dependence was observed. Most importantly, we have found an order-of-magnitude decrease of the sample resistance with increasing length at all temperatures up to 4.2 K, that is, in both the resonant tunneling and hopping regimes. This effect is most likely due to an increased number of possible tunneling paths in longer samples. A similar, but much smaller decrease (about a factor of 3) of the resistance with the increasing barrier thickness was observed in thin a-Si films at room temperature [10], while the low-temperature data showed only an exponential decrease of conductance with the thickness, consistent with the resonant tunneling through a single state. The observed resistance anomaly was thus associated with the inelastic processes. Our results show conclusively that this effect is not an attribute of incoherent processes only, but that it is most likely a statistical effect.

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[a]Present address: Department of Physics and Astronomy, University of North Carolina, Chapel Hill, NC 27599.

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