Disappearance of Flow in Intermediate-Energy Nucleus-Nucleus Collisions

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The disappearance of transverse collective flow for ^{40}Ar+^{27}Al collisions is studied with an improved Boltzmann-Uehling-Uhlenbeck equation. For collisions at impact parameters less than 3 fm, the predicted energy of balance, E_{bal}, is very sensitive to the in-medium nucleon-nucleon cross section, but insensitive to the equation of state. At larger impact-parameter collisions, the sensitivities to both the in-medium nucleon-nucleon cross section and the equation of state at subnuclear density become comparable. Comparisons with experimental data indicate an in-medium nucleon-nucleon cross section in the range of 25–45 mb.

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Flow, or the in-plane transverse momentum distribution of emitted particles, can carry important information about the reaction dynamics of nucleus-nucleus collisions and the nuclear equation of state. At incident energies of a few tens of MeV per nucleon, the interaction between nucleons is dominated by the attractive part of the nuclear mean field and the particles are deflected to negative angles [1]. At energies of a few hundred MeV to 1 GeV/nucleon, the individual nucleon-nucleon scattering and the repulsive part of the nuclear mean field become important and the particles are emitted to positive angles [2–4]. At a certain intermediate incident energy, E_{bal}, referred to as the energy of balance [5–7], the attractive part and the repulsive part of the interactions are expected to balance each other and the flow crosses zero, changing from a negative sign at low energies to a positive sign at high energies [8,9]. Measurements of flow at different energies could, in principle, provide quantitative information concerning the nuclear equation of state at both low (attractive) and high (repulsive) densities. Significant efforts were made recently to study the equations of state at high densities [2–4, 8, 10–17]. Whether there is any noticeable sensitivity of flow to equations of state at low densities remains unclear.

The disappearance of flow and its change to negative angles at a finite incident energy was first predicted by Bonasera and Csernai [8] based on a fluid-dynamical scaling study of LBL Bevalac data, before experimental data were available. This prediction was based on scaling violations due to viscosity [14,18] and due to the phase transition in the nuclear equation of state. The qualitative behavior of the energy dependence of flow was also studied by Bertsch *et al.* within the context of the Boltzmann-Uehling-Uhlenbeck (BUU) equation [9]. Because of the importance of Coulomb interactions and nuclear surface effects, and the significant dependence of E_{bal} upon the masses of projectile and target, it was rather difficult to compare the earlier calculations with available data.

To investigate the sensitivities of E_{bal} to the equation of state and to the in-medium nucleon-nucleon cross section and to determine whether one can separate these dual dependences, we have performed improved BUU calculations for ^{40}Ar+^{27}Al collisions. In our improved calculations [19,20], we have included Coulomb interactions and have used a lattice Hamiltonian method [21] to propagate test particles. This method provides accurate energy conservation [19–21] and a reasonable nuclear surface [22]. The calculations indicate that the energy of balance E_{bal} is insensitive to the equation of state, but very sensitive to the in-medium nucleon-nucleon cross section at small impact parameters, $b \leq 3$ fm. The sensitivities to the equation of state at subnuclear density and to the in-medium nucleon-nucleon cross section become comparable at larger impact parameters. Compared with experimental data, the calculations suggest an in-medium nucleon-nucleon cross section in the range of 25–45 mb.

We solve the Boltzmann-Uehling-Uhlenbeck equation [18]

$$\frac{\partial f_1}{\partial t}+\mathbf{v}\cdot\nabla_r f_1-\nabla_r U\cdot\nabla_p f_1=\frac{4}{(2\pi)^3}\int d^3k_2\,d^3k_3\,d\Omega\,\frac{d\sigma_{NN}}{d\Omega}v_{12}\times[f_3f_4(1-f_1)(1-f_2)-f_1f_2(1-f_3)(1-f_4)]\,\delta^3(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4)\,, \qquad (1)$$

with the lattice Hamiltonian method of Lenk and Pandharipande [21]. In Eq. (1), $d\sigma_{NN}/d\Omega$ and v_{12} are the in-medium cross section and relative velocity for the colliding nucleons, and U is the total mean-field potential consisting of the Coulomb potential and a nuclear potential with isoscalar and symmetry terms. The isoscalar mean-field potential U_0 (in MeV) is approximated by

$$U_0=A\rho/\rho_0+B(\rho/\rho_0)^{\gamma}\,, \qquad (2)$$

where $\rho_0=0.17\ \text{fm}^{-3}$ and $\rho=\rho(\mathbf{r})$ is the local density of nuclear matter. In our calculations, values of $A=-356$,

$B=303$, and $\gamma=\frac{7}{6}$ correspond to a soft nuclear equation of state (EOS) with compressibility coefficient $K=200$ MeV; while values of $A=-124$, $B=70.5$, and $\gamma=2$ correspond to a stiff EOS with $K=375$ MeV. The symmetry potential $U_{\rm sym}$ is represented by

$$U_{\rm sym}=32[(\rho_n-\rho_p)/\rho_0]\tau_z \text{ MeV}, \tag{3}$$

where ρ_n and ρ_p are the neutron and proton densities and τ_z is the isospin operator with eigenvalues $+1$ and -1 for neutrons and protons, respectively. For simplicity, $\sigma_{NN}=\int(d\sigma_{NN}/d\Omega)d\Omega$ is chosen to be isotropic and energy independent [18]. The mean-field and the Pauli-blocking factors in the collision integral are averaged over an ensemble of 80 parallel simulations.

Figure 1 shows the calculated in-plane transverse momentum distributions of free nucleons as a function of rapidity for $^{40}\mathrm{Ar}+{}^{27}\mathrm{Al}$ collisions at an impact parameter $b=1.6$ fm and with a stiff equation of state. Nucleons are considered free when the local densities are less than 7% ρ_0. The flow shown in the figure is evaluated at $t=120$ fm/c. This time is comparable to the freezeout time used in the study of residues [20,21]. At this time, distinguishable projectilelike and targetlike residues were already well separated. Evaluation of flow at later times indicates little change for the transverse momenta in the midrapidity region. To accumulate sufficient statistics for emitted nucleons, about 700 events were collected for each given input parameters. A total of 4000 computer CPU hours on a VAXstation 3100 were used to generate more than 30000 events.

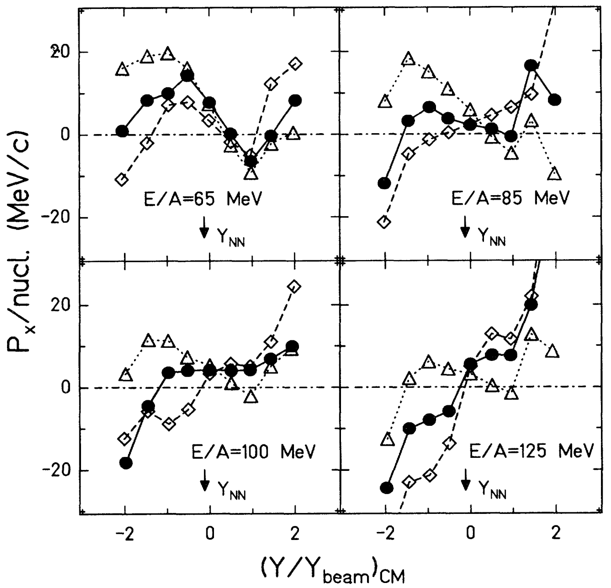

FIG. 1. The transverse momentum distribution as a function of longitudinal rapidity for $^{40}\mathrm{Ar}+{}^{27}\mathrm{Al}$ collisions calculated with a stiff equation of state at impact parameter $b=1.6$ fm and incident energies $E/A=65$ (upper-left panel), 85 (upper right), 100 (lower left), and 125 (lower right) MeV, respectively. The triangles, circles, and diamonds indicate calculations with $\sigma_{NN}=25$, 35, and 45 mb, respectively. The lines are used to guide the eye. The arrows indicate the nucleon-nucleon center-of-mass rapidity Y_{NN}.

The upper-left panel in Fig. 1 shows the in-plane transverse momentum distributions of free nucleons for an incident energy $E/A=65$ MeV with an in-medium nucleon-nucleon cross section of $\sigma_{NN}=25$ mb (triangles), 35 mb (circles), and 45 mb (diamonds), respectively. For all values of σ_{NN}, the momentum distributions are characterized by negative slopes in the midrapidity region, indicating the importance of the attractive mean field at this energy. Calculations with larger σ_{NN} yield less steep slopes since nucleon-nucleon scattering tends to make the emission more isotropic. The evolution of flow with incident energy can be clearly seen in the calculations with $\sigma_{NN}=35$ mb (circles). In the midrapidity region, the slope becomes less negative at $E/A=85$ MeV (upper-right panel), flat at $E/A=100$ MeV (lower-left panel), and changes to positive sign at $E/A=125$ MeV (lower-right panel). This gradual change reflects the increasing importance of the repulsive part of the nuclear equation of state. The higher the incident energy, the larger the nuclear compression, and, therefore, the stronger the repulsive mean field. The role of nucleon-nucleon scattering is clearly evident in all incident energies: More positive emission is observed with a larger value of σ_{NN}. It is interesting to note here that for this asymmetric system the momentum distribution becomes flat at a nonzero value (in the projectile side), in contrast to collisions between symmetric nuclei [5].

To better characterize flow for this asymmetric system and to allow for a comparison with data, we follow Ref. [7] and define the flow parameter as the slope in the midrapidity region multiplied by $(Y_{\rm beam}-Y_{NN})/Y_{\rm beam}$. Here Y_{NN} indicates the nucleon-nucleon center-of-mass rapidity and the slope is extracted by a linear fit of the momentum distribution within $|Y/Y_{\rm beam}|_{\rm c.m.}\leq 1$ (c.m. denotes the nucleus-nucleus center of mass). In Fig. 2, we display the flow parameters as a function of incident energy for $^{40}\mathrm{Ar}+{}^{27}\mathrm{Al}$ collisions at impact parameters $b=1.6$, 3, and 5 fm, respectively. The solid triangles, circles, and diamonds correspond to calculations with $\sigma_{NN}=25$, 35, and 45 mb, respectively, and with a stiff equation of state. The open circles are the calculated results with $\sigma_{NN}=35$ mb and a soft equation of state. Clearly, at all impact parameters, the calculated flow parameter is very sensitive to the in-medium nucleon-nucleon cross section. For the calculations at $b=1.6$ and 3 fm, the flow parameter appears to be insensitive to the equation of state, allowing σ_{NN} to be determined from the data. The sensitivities to the equation of state and to σ_{NN} become comparable at impact parameters $b\gtrsim 5$ fm. This sensitivity to the EOS at large impact parameters could arise from the difference of the EOS at low densities [20]. A stiffer EOS has higher surface tensile strength [23] and the emitted nucleons are therefore deflected to more neg-

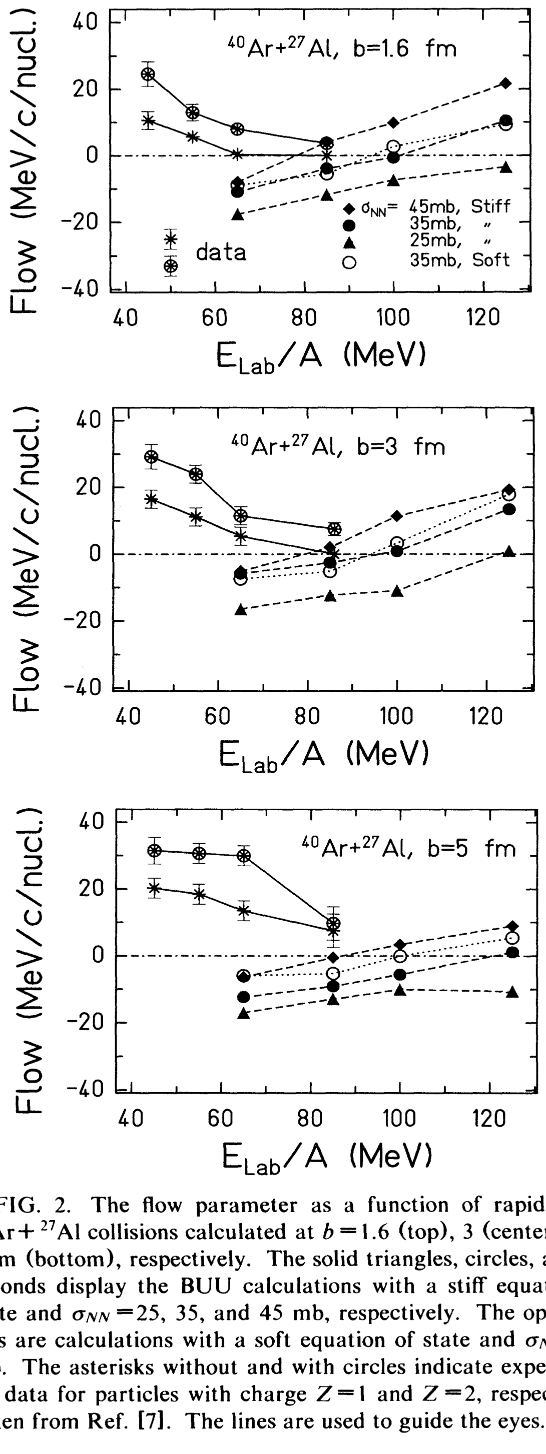

FIG. 2. The flow parameter as a function of rapidity for ^{40}Ar+^{27}Al collisions calculated at $b=1.6$ (top), 3 (center), and 5 fm (bottom), respectively. The solid triangles, circles, and diamonds display the BUU calculations with a stiff equation of state and $\sigma_{NN}=25$, 35, and 45 mb, respectively. The open circles are calculations with a soft equation of state and $\sigma_{NN}=35$ mb. The asterisks without and with circles indicate experimental data for particles with charge $Z=1$ and $Z=2$, respectively, taken from Ref. [7]. The lines are used to guide the eyes.

ative angles (the corresponding $F_{\rm bal}$ is higher). At smaller impact parameters, the sensitivity to the EOS at subnuclear density is washed out because (1) collisions among nucleons are more frequent and (2) surface effects due to the different EOS are reduced (the average density in the participant region is closer to ρ_0, where the gradients of mean field for both EOS are about zero). If σ_{NN} could be determined from the flow in small-impact-parameter collisions, the equation of state could, in turn, be extracted from the flow in large-impact-parameter collisions.

The asterisks without and with circles in Fig. 2 depict the experimental data for particles with charge $Z=1$ and $Z=2$, respectively, taken from Ref. [7]. Because of experimental limitations [2], only the absolute values were extracted. The average flow per nucleon should be intermediate between the flow extracted for $Z=1$ and $Z=2$ since these two charges constitute the dominant part of the observed multiplicities [24,25]. Based on coalescence models [26,27], one also expects that particles with charge $Z=1$ and $Z=2$ have the same value of $E_{\rm bal}$, at which the flow vanishes. The calculations with $\sigma_{NN}=35$–45 mb yield a value of $E_{\rm bal}/A \approx 80$–100 MeV at $b=1.6$ and 3 fm, consistent with the experimental data. The calculations seem to exclude values of σ_{NN} below 25 mb. Clearly, more data at higher energies are needed to determine the upper limit of $E_{\rm bal}$ (lower limit of σ_{NN}). Experimental tests of whether particles with different charges and masses produce different values of $E_{\rm bal}$ would also provide a stringent test for coalescence models.

In Ref. [9], the flow was characterized by the average transverse momentum, $\langle P_x\rangle/A$, in the projectile rapidity region $(Y/Y_{\rm beam})_{\rm c.m.} \gtrsim 0$. We have investigated this quantity for *free* nucleons emitted in ^{40}Ar+^{27}Al collisions. For this asymmetric system, the sensitivity of the calculated $\langle P_x\rangle/A$ of free nucleons to both the equation of state and σ_{NN} is significantly reduced. This is mainly due to the presence of a large projectilelike residue which carries away a major transverse momentum transfer. This residue is deflected close to 0° and is usually not measured. The average transverse momentum of free nucleons in the target rapidity region still shows a significant sensitivity to the in-medium nucleon-nucleon cross section and the nuclear equation of state. For these two quantities, however, final-stage particle evaporation from both the projectilelike and targetlike residues, which is not incorporated in the BUU calculations, could significantly complicate the observed flow, making it difficult to compare with model calculations [6,7]. In contrast, the flow defined by the slope in the midrapidity region has the advantage that it is relatively insensitive to final-stage equilibrium emission. Indeed, we have evaluated the flow at a slightly later time and found that the momentum distribution changes little in the midrapidity region, while in the projectile rapidity region it changes quite noticeably with time, indicating the importance of equilibrium emission. A full exploration of this issue requires the coupling of dynamical models with evaporation or fragmentation models [28].

In summary, we have studied the disappearance of flow for ^{40}Ar+^{27}Al collisions with improved BUU calculations. The influence of the attractive and repulsive parts of the nuclear mean field, and of the in-medium nucleon-nucleon scattering to the flow is investigated. The qualitative behavior of the calculated disappearance of flow is consistent with earlier estimates [8,9]. Within the present parametrization for the EOS, the calculated energy of balance E_{bal} is insensitive to the equation of state but quite sensitive to the in-medium nucleon-nucleon cross section, at impact parameters of less than 3 fm. This lack of sensitivity to the nucleon equation of state could provide a direct observable to extract the in-medium nucleon-nucleon cross section from experimental data. The sensitivity to both the equation of state and σ_{NN} becomes comparable at larger impact parameters. Although the change of flow from a negative sign at low energies to a positive sign at high energies reflects the increasing importance of the repulsive part of the EOS, the sensitivity of E_{bal} to the EOS at large impact parameters turns out to be determined by the EOS at *low* densities. Compared with the available data, the calculations indicate an in-medium nucleon-nucleon cross section in the range of $\sigma_{NN} \approx 25$–45 mb. Clearly, more experimental data in the high-energy region are called for to determine the lower limit of σ_{NN}. On the theoretical side, further calculations are needed to investigate the influence of the detailed algorithm of Pauli blocking and of the surface energy to the predicted flow.

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