Tests of Signal Locality and Einstein-Bell Locality for Multiparticle Systems

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(Received 29 April 1991)

For multiparticle systems we formulate the notions of signal locality (i.e., absence of faster-than-light signaling) and Einstein-Bell locality (or local realism) and obtain inequalities between experimental correlation functions to test them. Quantum theory obeys signal locality but violates Einstein-Bell locality by a factor $2^{(n-1)/2}$ for n-particle systems.

PACS numbers: 03.65.Bz

The Einstein-Podolsky-Rosen (EPR) paradox [1] was based on the following locality postulate [2]: "The real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former." Bell [3] made the discovery (Bell's theorem) that quantum mechanics contradicts this postulate. Experiments [4] apparently violate the Bell-Clauser-Horne-Shimony-Holt (CHSH) inequalities [5] following from Einstein-Bell locality.

On the other hand, in quantum field theory observables at spacelike separation commute. This implies "signal locality," i.e., the absence of faster-than-light signals [6]. Signal locality can be formulated [7-9] independently of quantum theory and also leads to inequalities which can be experimentally tested [9]. Signal locality has previously been called "simple locality" by Ballentine and Jarrett [8] and "parameter independence" by Shimony [8].

EPR correlations for multiparticle systems have recently acquired great interest. Eberhard [10] proposed a model which implies small violations of both Einstein-Bell locality and signal locality for n-particle systems, if $n \geq 3$. We presented at a conference [11] three-particle Bell inequalities and signal locality inequalities. At the same conference Greenberger, Horne, and Zeilinger [12] (GHZ) presented a proof of Bell's theorem without inequalities using a state of four spin-$\frac{1}{2}$ particles. Motivated by the GHZ state, Mermin [13] recently derived an elegant n-particle Bell inequality.

The purpose of this work is to present experimental tests of signal locality as well as Einstein-Bell locality for multiparticle systems. The n-particle Bell inequalities we derive are violated by quantum mechanics by a factor $2^{(n-1)/2}$ for both even and odd n. The improvement with respect to Mermin [13] is by a factor $\sqrt{2}$ for even-n experiments. It results from our inequalities involving a continuum of apparatus parameters. The largest violation is not always in the GHZ state used by Mermin; this fact may also be important for experiments.

Consider a system of particles $1,2,\ldots,n$ flying apart from an apparatus producing them. Let physical quantities $A_1,A_2,\ldots,A_n$ be measured for them by instruments in spacelike separated regions, settings of the instrument (orientations of polarizers, magnets, etc.) measuring the jth particle being collectively denoted by a_j. Suppose that by the very definition of the A_j, $|A_j| \leq 1$ (e.g., $A_j = +1$ for transmission through a polarizer and -1 for nontransmission). In any theory, the configuration of the n particles may be characterized by the apparatus parameters $a_1,a_2,\ldots,a_n$ plus possible additional variables λ, with $\rho(\lambda;a_1,a_2,\ldots,a_n)$ being the normalized, nonnegative probability distribution of the configuration, and $\bar{A}_j(\lambda;a_1,a_2,\ldots,a_n)$ being the expectation value of A_j in this configuration. Hence, the m-particle correlation functions are given by

$$\left\langle \prod_{r=1}^{m} A_{i_r} \right\rangle = \int d\lambda\, \rho(\lambda;a_1,a_2,\ldots,a_n) \times \prod_{r=1}^{m} \bar{A}_{i_r}(\lambda;a_1,a_2,\ldots,a_n)\,, \tag{1}$$

where

$$|\bar{A}_j| \leq 1\,. \tag{2}$$

Signal locality.—We define [7-9] signal locality to mean that the expectation value of a physical quantity measured in one spacetime region cannot depend on apparatus settings in spacelike separated regions. Hence,

$$\left\langle \prod_{r=1}^{m} A_{i_r} \right\rangle = P_{i_1 i_2 \cdots i_m}(a_{i_1},a_{i_2},\ldots,a_{i_m})\,, \tag{3}$$

where the right-hand side is independent of apparatus settings not appearing in the argument of P because they refer to spacelike separated regions. Clearly, by changing apparatus settings in spacelike separated regions, the postulate (3) can be directly tested experimentally.

We show below that the signal locality postulate also implies interesting inequalities between different correlation functions that could be measured in independent experiments. Consider the consequence of signal locality,

$$\int d\lambda\, \rho(\lambda;a_1,a_2,\ldots,a_n) \prod_{j\in J} [1+\eta_j \bar{A}_j(\lambda;a_1,a_2,\ldots,a_n)]$$
$$= 1 + \sum_{j\in J} \eta_j P_j(a_j) + \sum_{j\neq k,\, j,k\in J} \eta_j \eta_k P_{jk}(a_j,a_k) + \cdots + \left(\prod_{j\in J} \eta_j\right) P_{j_1 j_2 \cdots j_m}(a_{j_1},a_{j_2},\ldots,a_{j_m}) \tag{4}$$
$$\equiv I(\eta_{j_1},\eta_{j_2},\ldots,\eta_{j_m};a_{j_1},a_{j_2},\ldots,a_{j_m})\,, \tag{5}$$

where $J=\{j_1,j_2,\ldots,j_m\}$ is a subset of the integers $\{1,2,\ldots,n\}$, and $\eta_j^2=1$ for all j. In Eq. (4) the signal locality postulate (3) has been used to restrict the arguments of the correlation functions P. Using $\rho\geq 0$, and $|\bar{A}_j|\leq 1$, we then have the following 2^m ($\because \eta_j=\pm 1$) experimental tests of the signal locality hypothesis for each choice of J, and of the apparatus parameters a_j (for $j\in J$):

$$I(\eta_{j_1},\eta_{j_2},\ldots,\eta_{j_m};a_{j_1},a_{j_2},\ldots,a_{j_m})\geq 0\,. \tag{6}$$

A violation of the inequalities (6) would imply a violation of signal locality. It would also imply a violation of quantum theory which respects signal locality. The inequalities should be tested by the apparatus used to test Bell's inequalities.

Einstein-Bell locality.— Bell demanded [3,5] that in Eq. (1), the probability distribution ρ for the configuration be independent of the apparatus parameters $a_1,\ldots,a_n$, and the expectation value $\bar{A}_j$ for the jth apparatus be independent of orientations a_k of the other spacelike separated apparatus. Thus,

$$\begin{aligned}&\bar{A}_j(\lambda;a_1,a_2,\ldots,a_n)=\bar{A}_j(\lambda,a_j),\ \forall j;\\ &\rho(\lambda;a_1,a_2,\ldots,a_n)=\rho(\lambda)\,.\end{aligned} \tag{7}$$

These are locality requirements for each value of the variables λ. They imply on integration over λ the weaker requirement of signal locality, Eq. (3), and hence the inequalities (6). In addition, they imply generalized Bell's inequalities. Consider

$$P_{i_1i_2\cdots i_m}(a_{i_1},a_{i_2},\ldots,a_{i_m})=\int d\lambda\,\rho(\lambda)\prod_{r=1}^{m}\bar{A}_{i_r}(\lambda,a_{i_r})\,. \tag{8}$$

Using the elementary inequality

$$|x(y+y')|+|x'(y-y')|\leq 2\max(|x|,|x'|)\max(|y|,|y'|)\,, \tag{9}$$

together with $|\bar{A}_j(\lambda,a_j)|\leq 1$, and the definition

$$\begin{aligned}A_{kl}(\lambda,a_{kl})\equiv{}&\eta_{kl}\bar{A}_k(\lambda,a_k)[\bar{A}_l(\lambda,a_l)+\bar{A}_l(\lambda,a_l')]\\ &+\eta_{kl}'\bar{A}_k(\lambda,a_k')[\bar{A}_l(\lambda,a_l)-\bar{A}_l(\lambda,a_l')]\,,\end{aligned}$$

where $\eta_{kl}^2=\eta_{kl}'^2=1$, and $a_{kl}\equiv\{a_k,a_k',a_l,a_l',\eta_{kl},\eta_{kl}'\}$, we prove that $|A_{kl}(\lambda,a_{kl})|\leq 2$. Multiplying by $\rho(\lambda)$, using $\rho(\lambda)\geq 0$, and integrating over λ, we obtain the Bell-CHSH inequalities. Now, define

$$\begin{aligned}A_{i,kl}(\lambda,a_{i,kl})\equiv{}&\eta_{i,kl}\bar{A}_i(\lambda,a_i)[A_{kl}(\lambda,a_{kl})+A_{kl}(\lambda,a_{kl}')]\\ &+\eta_{i,kl}'\bar{A}_i(\lambda,a_i')[A_{kl}(\lambda,a_{kl})-A_{kl}(\lambda,a_{kl}')]\,,\end{aligned}$$

where $\eta_{i,kl}^2=\eta_{i,kl}'^2=1$, and $a_{i,kl}\equiv\{a_i,a_i',a_{kl},a_{kl}',\eta_{i,kl},\eta_{i,kl}'\}$, where a_{kl}' denotes a different set of values of the parameters a_{kl} defined before. Then, using the inequality (9), $|\bar{A}_i|\leq 1$, and $|A_{kl}|\leq 2$ derived before, we prove the generalized Bell inequalities [11],

$$\int d\lambda\,\rho(\lambda)A_{i,kl}(\lambda,a_{i,kl})\leq 4\,, \tag{10}$$

where the left-hand side is seen to be a combination of sixteen three-particle correlation functions. More generally, let $A_x^{(m)}(\lambda,a_x)$ and $A_y^{(n)}(\lambda,a_y)$ be m- and n-particle functions of the series

$$\bar{A}_i(\lambda,a_i),A_{kl}(\lambda,a_{kl}),A_{i,kl}(\lambda,a_{i,kl}),A_{ij,kl}(\lambda,a_{ij,kl}),A_{i,jkl}(\lambda,a_{i,jkl}),\ldots .$$

Then, another member of the series is

$$A_{x,y}^{(m+n)}(\lambda,a_{x,y})\equiv\eta_{xy}A_x^{(m)}(\lambda,a_x)[A_y^{(n)}(\lambda,a_y)+A_y^{(n)}(\lambda,a_y')]+\eta_{xy}'A_x^{(m)}(\lambda,a_x')[A_y^{(n)}(\lambda,a_y)-A_y^{(n)}(\lambda,a_y')]\,,$$

where $\eta_{xy}^2=\eta_{xy}'^2=1$. The corresponding $(m+n)$-particle generalized Bell inequality we obtain is

$$\int d\lambda\,\rho(\lambda)A_{x,y}^{(m+n)}(\lambda,a_{x,y})\leq 2^{m+n-1}\,, \tag{11}$$

of which the two-, three-, and four-particle Bell inequalities given before are special cases. It is cumbersome but straightforward to see that for the quantum-mechanical states of spin-$\frac{1}{2}$ particles,

$$|12\rangle_{\pm}\equiv(|1\rangle_+|2\rangle_-\pm|1\rangle_-|2\rangle_+)/\sqrt{2}\,,$$

$$|123\rangle_{\pm}\equiv(|12\rangle_+|3\rangle_-\pm|12\rangle_-|3\rangle_+)/\sqrt{2}\,,$$

$$|1234\rangle_{\pm}\equiv(|12\rangle_+|34\rangle_-\pm|12\rangle_-|34\rangle_+)/\sqrt{2}\,,$$

where $|j\rangle_{\pm}$ denote eigenstates of the Pauli matrix $\sigma_z^{(j)}$ with eigenvalue ± 1; the quantum analogs of the left-hand side of (11) can be made equal to $2\sqrt{2}$, 8, and $16\sqrt{2}$, respectively, for $n=2$, 3, and 4 by suitable choices of the apparatus parameters, whereas the right-hand side analogs are 2, 4, and 8, respectively. Thus the n-particle Bell inequalities are violated by quantum mechanics by a factor $2^{(n-1)/2}$ for $n=2$, 3, and 4. The fact that this holds for general n will now be proved using generalizations of the elegant method of Mermin.

Let us define, for $\eta_j^2=1$, and $j\in J=$set of n positive integers specifying the particles involved,

$$A_J^{(n)}(\lambda,a_J)\equiv\mathrm{Im}\prod_{j\in J}[\bar{A}_j(\lambda,a_j)+i\eta_j\bar{A}_j(\lambda,a_j')]\,,$$

$$B_J^{(n)}(\lambda,a_J)\equiv\mathrm{Re}\prod_{j\in J}[\bar{A}_j(\lambda,a_j)+i\eta_j\bar{A}_j(\lambda,a_j')]\,,$$

where $a_J\equiv\{a_j,a_j',\eta_j|j\in J\}$. Both the functions are linear in each of the $2n$ quantities $\bar{A}_j(\lambda,a_j)$, $\bar{A}_j(\lambda,a_j')$, which can vary between -1 and $+1$. Hence,

$$|A_J^{(n)}(\lambda,a_J)|\leq p_n,\quad |B_J^{(n)}(\lambda,a_J)|\leq p_n\,, \tag{12}$$

where $p_n=2^{(n-1)/2}$ for odd n and $p_n=2^{n/2}$ for even n. We immediately obtain generalized Bell inequalities on

n-particle correlation functions,

$$\left|\int d\lambda\,\rho(\lambda)A_J^{(n)}(\lambda,a_J)\right|\le p_n\,,$$
$$\left|\int d\lambda\,\rho(\lambda)B_J^{(n)}(\lambda,a_J)\right|\le p_n\,. \tag{13}$$

Further, we can combine n- and m-particle inequalities of the form (12) using Eq. (9) to obtain $(m+n)$-particle inequalities. Let $x_J^{(n)}(\lambda),x_J'^{(n)}(\lambda)$ be chosen to be any two out of the set

$$\{A_J^{(n)}(\lambda,a_J),A_J^{(n)}(\lambda,a_J'),B_J^{(n)}(\lambda,a_J),B_J^{(n)}(\lambda,a_J')\}\,,$$

and $y_K^{(m)}(\lambda),y_K'^{(m)}(\lambda)$ be any two out of the set

$$\{A_K^{(m)}(\lambda,a_K),A_K^{(m)}(\lambda,a_K'),B_K^{(m)}(\lambda,a_K),B_K^{(m)}(\lambda,a_K')\}\,,$$

where the n-particle set J and the m-particle set K contain no particle in common. Then we derive the following $(4c_2\times 4c_2=36)$ generalized Bell inequalities on $(m+n)$-particle correlation functions for each choice of the particle sets J,K:

$$\left|\int d\lambda\,\rho(\lambda)x_J^{(n)}(\lambda)[y_K^{(m)}(\lambda)+y_K'^{(m)}(\lambda)]\right|+\left|\int d\lambda\,\rho(\lambda)x_J'^{(n)}(\lambda)[y_K^{(m)}(\lambda)-y_K'^{(m)}(\lambda)]\right|\le 2p_np_m\,. \tag{14}$$

A particularly simple special case of this with the choice $J=1,2,\ldots,n-1$, $K=n$, yields the generalized Bell inequality

$$I_n=\left|\int d\lambda\,\rho(\lambda)A_J^{(n-1)}(\lambda,a_J)[\bar{A}_n(\lambda,a_n)+\bar{A}_n(\lambda,a_n')]\right|+\left|\int d\lambda\,\rho(\lambda)B_J^{(n-1)}(\lambda,a_J)[\bar{A}_n(\lambda,a_n)-\bar{A}_n(\lambda,a_n')]\right|\le 2^{n/2}\,, \tag{15}$$

if n is even. In a quantum state $|\psi\rangle$ the correlation functions corresponding to the left-hand sides of (13) are represented by

$$\int d\lambda\,\rho(\lambda)[A_J^{(n)}(\lambda,a_J),B_J^{(n)}(\lambda,a_J)]\to\langle\psi|[A_J^{(n)}(a_J),B_J^{(n)}(a_J)]|\psi\rangle\,,$$

where

$$[iA_J^{(n)}(a_J),B_J^{(n)}(a_J)]=\tfrac{1}{2}\left[\left(\prod_{j\in J}\boldsymbol{\sigma}^{(j)}\cdot(\mathbf{a}_j+i\eta_j\mathbf{a}_j')\right)\mp\text{H.c.}\right],$$

and H.c. denotes a Hermitian conjugate. Similarly, the correlation function corresponding to the left-hand side of (15) is represented quantum mechanically by

$$I_n^{(\psi)}=|\langle\psi|A_J^{(n-1)}(a_J)\boldsymbol{\sigma}^{(n)}\cdot(\mathbf{a}_n+\mathbf{a}_n')|\psi\rangle|$$
$$+|\langle\psi|B_J^{(n-1)}(a_J)\boldsymbol{\sigma}^{(n)}\cdot(\mathbf{a}_n-\mathbf{a}_n')|\psi\rangle|\,. \tag{16}$$

We use the n-particle quantum states,

$$|\phi_\pm^{(n)}\rangle\equiv\frac{1}{\sqrt{2}}(|\uparrow\uparrow\cdots\uparrow\rangle\pm i|\downarrow\downarrow\cdots\downarrow\rangle)\,,$$
$$|\chi_\pm^{(n)}\rangle\equiv\frac{1}{\sqrt{2}}(|\uparrow\uparrow\cdots\uparrow\rangle\pm|\downarrow\downarrow\cdots\downarrow\rangle)\,,$$
$$(|\psi^{(n)}\rangle,|\psi'^{(n)}\rangle)\equiv\frac{1}{\sqrt{2}}[(|\phi_\mp^{(n-1)}\rangle,|\chi_\mp^{(n-1)}\rangle)|\downarrow\rangle$$
$$-(|\phi_-^{(n-1)}\rangle,|\chi_-^{(n-1)}\rangle)|\uparrow\rangle],$$

and choose $\mathbf{a}_j=\hat{\mathbf{x}}$, $\mathbf{a}_j'=\hat{\mathbf{y}}$, $\eta_j=1$ for $j\le n-1$ to obtain

$$[A_J^{(n-1)}(a_J)\mp 2^{n-2}]|\phi_\pm^{(n-1)}\rangle=0\,,$$
$$[B_J^{(n-1)}(a_J)\mp 2^{n-2}]|\chi_\pm^{(n-1)}\rangle=0\,, \tag{17}$$

$$\langle\psi^{(n)}|\{A_J^{(n-1)}(a_J),B_J^{(n-1)}(a_J)\}\boldsymbol{\sigma}^{(n)}\cdot\mathbf{a}_n|\psi^{(n)}\rangle$$
$$=-2^{n-2}\{(a_n)_z,(a_n)_y\}\,. \tag{18}$$

Further, choosing $\mathbf{a}_n$ and $\mathbf{a}_n'$ in the y-z plane with components $\mathbf{a}_n=(0,1/\sqrt{2},1/\sqrt{2})$, $\mathbf{a}_n'=(0,\mp 1/\sqrt{2},\pm 1/\sqrt{2})$ for the states $|\psi^{(n)}\rangle$, $|\psi'^{(n)}\rangle$, we obtain from Eqs. (16) and (18)

$$I_n(\psi^{(n)})=2^{(2n-1)/2}=I_n(\psi'^{(n)})\,, \tag{19}$$

which violate the generalized Bell inequality (15) for even n by a factor $2^{(n-1)/2}$. Letting $n\to n+1$ in Eqs. (17) and comparing with the generalized Bell inequalities (13) we see that they are violated by a factor $2^{(n-1)/2}$ for odd n. [For the first inequality of (13) this is Mermin's result.]

Our inequalities for large n add another dimension to the large body of previous research on quantum mechanics at the macroscopic level. As summarized by Leggett [14], "we are all familiar with the idea that while the microscopic description of the physical world requires quantum mechanics, at the macroscopic level a classical description suffices.... In sum, it is claimed that at the macroscopic level, ... we never have to deal with states in which a macroscopic variable is not even approximately defined, i.e., a quantum superposition of macroscopically different states." Leggett and others [14] have considered tests of this claim in solid-state physics, hoping either to see quantum interference at the macroscopic level or to discover serious evidence against the universality of the quantum description. Ghirardi, Rimini, and Weber [15] have proposed modifications of quantum theory designed to limit superpositions of macroscopically different states to very small time intervals. Our results show that in the absence of such modifications, quantum theory will violate classical mechanics by a factor of $2^{(n-1)/2}$ for macroscopic systems, too.

For small n (=3,4, for example), it seems feasible to experimentally produce photonic analogs of states such as $\phi\pm$, $\chi\pm$, ψ, and ψ' via cascade photon decays of atomic systems. Real experiments for multiparticle systems could then be designed using suggestions of Greenberger *et al.* [16].

We wish to thank A. Aspect, D. M. Greenberger, and A. Shimony for conversations during the 1989 Erice Conference and record our deep gratitude to John S. Bell for his many incisive comments. One of us (S.M.R.) wishes to thank the Theoretical Physics Institute at University of Alberta, particularly A. Z. Capri, for excellent hospitality and acknowledges their financial support during part of this work.

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