Critical Fluctuations in Strongly Type-II Quasi-Two-Dimensional Superconductors

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The critical fluctuations near $H_{c2}(T)$ in the mixed phase of layered type-II superconductors are related to an equivalent system of classical particles with many-body "gauge" interactions. The scale invariance of this dense vortex plasma leads to a universal character of the vortex solid-liquid melting line where the transition to the superconducting state occurs. The relation to experimental situation in high-T_c and other layered superconductors and superconducting thin films is discussed.

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Thermal fluctuations in the mixed phase of high-temperature superconductors (HTS) have been of considerable interest recently [1-5]. HTS are examples of strongly type-II ($\kappa \gg 1$) layered systems, with the interlayer coupling ranging from moderate (in Y-Ba-Cu-O) to very weak (in Bi or Tl compounds) [6]. Because $\kappa \gg 1$, the regime in which the average separation between vortices, r_s, is larger than the penetration depth $\lambda(T)$ is confined to the vicinity of $H_{c1}(T)$ [7] (region of hatching in Fig. 1). The remaining portions of the phase diagram are region A, near $H_{c2}(T)$, where vortices are "dense" with r_s being of the order of the core size, and region B, far from $H_{c2}(T)$ and near $H_{c1}(T)$, where vortices are dilute and r_s is much larger than the core size, with magnetic induction $\mathbf{B}(\mathbf{r})$ being essentially uniform. This division can be made more precise by expanding the order parameter $\Psi(\mathbf{r})$ as $\Psi(\mathbf{r})=(2\pi l^2)^{1/2}\sum_{j,m} b_{jm}\varphi_{jm}(z,z^*)$, where $l=(c/e^*B)^{1/2}$ is the magnetic length, $z=(x+iy)/l$, and $\varphi_{jm}(z,z^*)$ are the Landau eigenfunctions for charge $e^*=2e$ in a suitable gauge. In region A the dominant contribution comes from the lowest Landau level (LLL), $j=0$ [8]. The fluctuations from higher Landau levels $\{b_{j\neq 0,m}\}$ have a gap and are not dynamically important near $H_{c2}(T)$. In region B contributions from all Landau levels are required to maintain the constant amplitude of $\Psi(\mathbf{r})$ over large distances. The crossover from A to B starts at some $H_{\text{cross}}\sim \frac{1}{7}H_{c2}(T)$ [9] (Fig. 1).

We start from the Ginzburg-Landau (GL) partition function [10]:

$$Z=\int \mathcal{D}\Psi_n(\mathbf{r})\mathcal{D}\Psi_n^*(\mathbf{r})\int \mathcal{D}\mathbf{a}(\mathbf{R})\exp(-\{F_s[\Psi_n(\mathbf{r}),\mathbf{a}(\mathbf{R})]+F_m[\mathbf{a}(\mathbf{R})]\}/k_BT)\,, \tag{1}$$

where n is the layer index,

$$F_s=\sum_{n,\sigma}\int d^2r\left[\alpha(T)|\Psi_n|^2+\frac{\beta}{2}|\Psi_n|^4+\frac{n_e}{2m^*}\left|\left(\nabla+\frac{2ei}{c}(\mathbf{A}+\mathbf{a}_n)\right)\Psi_n\right|^2+\frac{\eta}{2}\left|\Psi_{n+\sigma}-\Psi_n\exp\left(-\frac{2ei}{c}da_{\zeta n}\right)\right|^2\right]$$

$\mathbf{r}=(x,y)$, $\mathbf{R}=(\mathbf{r},\zeta)$, n_e is the electron density, d is the interlayer separation, $\sigma=\pm 1$, $\nabla\times\mathbf{A}=\mathbf{H}$, and F_m is the magnetic free energy. Here we only consider the case $\mathbf{H}\parallel\hat{\mathbf{c}}$. We neglect fluctuations in $\mathbf{a}$, which is justified for $\kappa\gg 1$ [11].

In the mean-field treatment of (1) the thermodynamic superconducting transition takes place at $H_{c2}(T)$ below which the lattice of vortices is formed [12,13]. The critical fluctuations near $H_{c2}(T)$ have been studied in the past by replacing $\Psi(\mathbf{r})=(2\pi l^2)^{1/2}\sum_m b_{0m}\varphi_{0m}(z,z^*)$ in (1) and using the perturbation theory in the quartic piece of F_s, representing the interaction among $\{b_{0m}\}$ [8,14,15]. This approach leads to a good description of fluctuations above $H_{c2}(T)$. However, the essential part of the physics is missing. The perturbative expansion does not show any sign of a transition to the Abrikosov lattice, even when carried out to a large order [14]. This is in stark contrast with Abrikosov's mean-field theory and harmonic fluctuations around it [2,3] which suggest that some type of positional ordering exists below $H_{c2}(t)$. On the other hand, the elastic theory, which breaks the symmetry from the outset, cannot itself be used to describe fluctuations close to and above $H_{c2}(T)$. Consequently, a complete description of critical behavior is not available.

In this Letter we devise a theory of fluctuations in the critical region near $H_{c2}(T)$ which overcomes the above difficulties. Our approach provides a unified framework for both vortex "solid" and "liquid" phases within a many-body "algorithm" which contains a detailed description of the critical behavior from the perturbative regime far above $H_{c2}(T)$ to the spontaneous formation of the Abrikosov lattice far below $H_{c2}(T)$. To proceed we note that, for $\mathbf{H}=0$, it is useful to write $\Psi(\mathbf{r})=R(\mathbf{r})\times\exp[i\chi(\mathbf{r})]$ and study fluctuations in amplitude (R) and phase (χ) separately. The physics is dominated by the singular part of χ, corresponding to the motion of vortices and antivortices interacting via familiar pairwise logarithmic interactions: We call this system a dilute vortex plasma (dVP). The dVP picture remains correct in low fields [far from $H_{c2}(T)$] except now only vortices will be present. Such a dVP description of quasi-2D supercon-

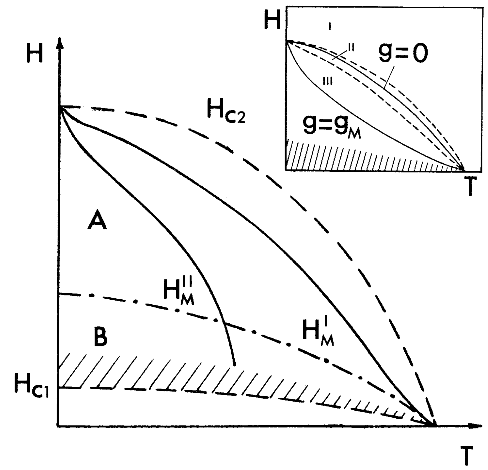

FIG. 1. A schematic H-T phase diagram for layered and thin-film type-II superconductors. The dense vortex regime (A) is separated from the dilute vortex regime (B) by a dash-dotted $H_{\rm cross}$ line. As θ_M increases, $H_M(T)$ evolves from $H_M^I(T)$ to $H_M^{II}(T)$ (see text). In the region of hatching, $r_s > \lambda$ and $H_M(T)$ in layered systems turns toward $H_{c1}(0)$ [1]. Inset: The same phase diagram in the DVP scaling form. Regions I, II, and III are defined in the text.

ductors in magnetic field has been studied in [16] and [17]. In the critical region near $H_{c2}(T)$ there is a *qualitative* change in this picture. When the dominant part of $\Psi(\mathbf{r})$ comes from the LLL the phase fluctuations alone are not sufficient and the amplitude fluctuations become essential. The confinement to the LLL provides a stringent constraint which couples the fluctuations of R and χ. This nonlinear constraint in (1) can be enforced *exactly* if we use the *symmetric* gauge:

$$\Psi(\mathbf{r}) = (2\pi l^2)^{1/2} \sum_{m=0}^{N} b_{0m}\varphi_{0m}(z,z^*)$$
$$= \Phi \prod_{i=1}^{N} (z-z_i)\exp(-|z|^2/4)\,, \qquad (2)$$

where $\varphi_{0m}(z,z^*) = z^m(2\pi l^2 m! 2^m)^{-1/2}\exp(-|z|^2/4)$ and $N = \Omega/2\pi l^2$, Ω being the area of the system. The fluctuations in Φ and $\{z_i\}$, which are in highly nonlinear relation to $\{b_{0m}\}$, are the "natural" way of representing two distinct tendencies in $\Psi(\mathbf{r})$ near $H_{c2}(T)$: The "global" superconducting correlations are embodied in Φ, while the "local" amplitude and phase fluctuations, coupled by the confinement to the LLL, arise from the unrestricted motion of $\{z_i\}$.

We now use these new variables, Φ and $\{z_i\}$, to rewrite the partition function (1). We first consider the $\eta=0$ case and drop the layer index n. This corresponds to very anisotropic systems or thin superconducting films. The transformation in Eq. (2) involves a Jacobian $\propto \prod_{i<j}^{N}|z_i - z_j|^2(\Phi\Phi^*)^N$. After this change of variables we note the following important fact: The integration over Φ in the partition function can be performed *exactly* in the thermodynamic limit. This is a nontrivial point since this integral contains contributions from the nonperturbative sector of (1). This finally leads to

$$Z = \left[\int \prod_{i=1}^{N} \frac{dz_i\,dz_i^*}{2\pi} \langle f^4\rangle^{-(N+1)/2} \prod_{i<j}^{N} |z_i - z_j|^2\right]^{-1} \int \prod_{i=1}^{N} \frac{dz_i\,dz_i^*}{2\pi} \langle f^4\rangle^{-(N+1)/2} \prod_{i<j}^{N} |z_i - z_j|^2$$
$$\times \exp[\tfrac{1}{2}NV^2 - \tfrac{1}{2}NV(V^2+2)^{1/2} - N\sinh^{-1}(V/\sqrt{2})]\,, \qquad (3)$$

where

$$V(\{z_i\}) \equiv \frac{g\langle f^2\rangle}{\langle f^4\rangle^{1/2}},\quad \langle f^p\rangle = \int \frac{dz\,dz^*}{2\pi N}\exp(-p|z|^2/4)\prod_i |z-z_i|^p,\quad g = \tilde{\alpha}\left[\frac{2\pi l^2 d}{2\beta k_B T}\right]^{1/2},\quad \tilde{\alpha} = \alpha\left[1 - \frac{H}{H_{c2}(T)}\right],$$

and d is either the interlayer separation or the thickness of the film. In (3), Z is normalized to its $g=0$ form.

Equation (3) is our main result. It describes a classical incompressible many-body system of particles $\{z_i\}$ (vortices). We call this system a dense vortex plasma (DVP). The basic dynamical interaction among $\{z_i\}$ in DVP, $V(\{z_i\})$, is long ranged and contains multiple-body forces reflecting the dense vortex limit (unlike the dilute limit, the forces cannot be reduced to two-body terms only). We recognize the variance $\langle f^2\rangle/\langle f^4\rangle^{1/2}$ as a square root of an inverse of Abrikosov's β_A except that now the $\{z_i\}$ do not form a lattice as in [12] but are allowed to move all over the x-y plane. $\langle f^2\rangle/\langle f^4\rangle^{1/2}$ varies between 0 and 0.928, for the triangular lattice. DVP provides a paradigm for the study of critical behavior in type-II systems near $H_{c2}(T)$ where the main contribution to (1) comes from many-body effects of the vortex core overlap and should be contrasted with a dVP description which is appropriate for low fields.

Because of the scale invariance of (3), cooperative phenomena in dense vortex systems take place at $g(T,H) = g_X$, where g_X is some number. There are three physical regimes of this DVP illustrated in the inset of Fig. 1. For $g \gg \sqrt{2}$, one is in regime I far above $H_{c2}(T)$, with interactions being very weak. In this regime the perturbation theory of Refs. [8], [14], and [15] should work well. As one approaches $H_{c2}(T)$ ($g=0$) one enters regime II where, for $-\sqrt{2} \ll g \ll \sqrt{2}$, the effective interaction goes as $+\sqrt{2}NV + N\mathcal{O}(g^2)$. Finally, for $g \ll -\sqrt{2}$, the effective interaction is $-NV^2 - N\ln(\sqrt{2}V) + N\mathcal{O}(g^0)$. These nonperturbative correlations are recovered only through the integration over Φ. Note that the leading term in the interaction becomes exactly the Abrikosov mean-field free energy if the $\{z_i\}$ are frozen into a triangular lattice. DVP, therefore, correctly reproduces all the relevant fluctuation regions.

DVP in Eq. (3) should undergo a liquid-solid transition at some finite g. We can write down the equation for the transition line $H_M(T)$:

$$g(T,H_M)=g_M\,, \tag{4}$$

where $g_M<0$ is a pure number. $H_M(T)$ derived from (4) has the qualitative form shown in Fig. 1 as $H_M^l(T)$. This melting transition in DVP corresponds to a true thermodynamic superconducting transition which in the mean field occurs at $H_{c2}(T)$ ($g_M=0$). To determine the value of this "universal" number g_M we have performed a numerical Monte Carlo simulation of DVP in Eq. (3). All computations were done on a Cray Y-MP supercomputer and Stardent Titan-III mini-supercomputer and will be reported in detail elsewhere. We find $g_M^2=63\pm12$. The origin of the uncertainty in g_M^2 is the relatively small system size (we are limited to 224 particles due to multiple-body interactions in DVP) and the softness of the potential V, requiring long equilibration times.

The large size of $|g_M|$ indicates that the transition occurs deep in the nonperturbative region III and is *inherently very far* from the mean-field $H_{c2}(T)$. Such strong nonperturbative character arises from the infinite degeneracy of the LLL [18]. Within our numerics we are unable to tell whether the transition is of the Kosterlitz-Thouless-Berezinskii-Halperin-Nelson-Young (KTBH NY) type or possibly weak first order due to long-range forces. The estimate of $H_M(T)$ based on the KTBHNY theory agrees with our exact numerical determination within factors of order unity [19].

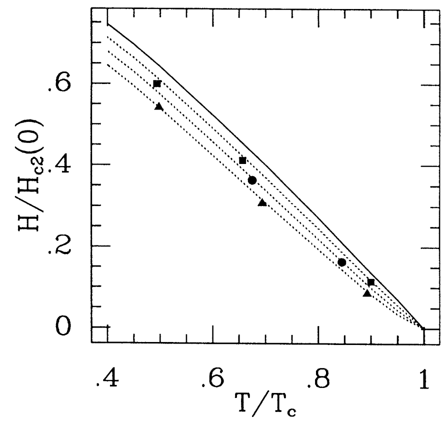

FIG. 2. $H_M(T)$ for three different thicknesses in thin films of Nb_3Ge. The data points are from Ref. [20]. The dashed lines are obtained from (4) with $g_M^2=67$.

The above prediction for the "universality" of $H_M(T)$ can be tested on recent experiments with films of Nb_3Ge [20]. There are no fitting parameters here (the only uncertainty left is in our numerical determination of g_M). The experimental data points in Fig. 2 agree very well with the theoretical result (4), using a single value $g_M^2=67$. Such apparent universality in $H_M(T)$ has been noted by the authors of Ref. [20], who compared their results with those on thin films of other materials. Our theory provides an explanation for this empirical observation.

We now turn to the effect of Josephson coupling $\eta\neq0$. The partition function in Eq. (3) is simply generalized to the case of coupled layers by including the additional Josephson interaction:

$$\sum_n\sum_{i=1}^N\left\{-\eta\frac{2\pi l^2 d|\tilde{a}|}{\beta N}\left[\left(\frac{\langle f_n^2\rangle\langle f_{n+\sigma}^2\rangle}{\langle f_n^4\rangle\langle f_{n+\sigma}^4\rangle}\right)^{1/2}\mathrm{Re}\langle f_n^* f_{n+\sigma}\rangle+(\sigma\to-\sigma)-\frac{2\langle f_n^2\rangle^2}{\langle f_n^4\rangle}\right]\right\}. \tag{5}$$

Finite η will make the positional ordering three dimensional [21]. $H_M(T)$ itself should change in some continuous fashion as η increases and for $\eta\ll1$ should not be very different from a 2D case. In fact, we notice that the interaction in Eq. (5) remains scale invariant. Consequently, $H_M(T)$ for $\eta\neq0$ can be obtained from

$$g(H,T)=g_M\left(\frac{\eta}{\tilde{a}(T,H)}\right), \tag{6}$$

where $g_M(\eta/\tilde{a}(T_M,H_M))$ is now a number different from $g_M(0)$ appearing in Eq. (4) for a pure 2D case. Note that $\eta/\tilde{a}=\xi_z^2/d^2$ and is the natural parameter describing the crossover from quasi-2D to 3D behavior. If we write

$$g_M^2\left(\frac{\eta}{\tilde{a}(T_M,H_M)}\right)=g_M^2(0)-\gamma\left(\frac{\eta}{\tilde{a}(T_M,H_M)}\right),$$

where $\gamma=0$ for $\eta=0$, and assume the KTBHNY-type correlations below g_M, we find $\gamma(\eta/\tilde{a})\approx C_1^2/\ln^2(\tilde{a}/C_2^2\eta)$, where C_1 and C_2 are constants of order unity. Thus, unless $H_M(T)$ is very close to $H_{c2}(T)$, the T and H dependence of $g_M^2(\eta/\tilde{a}(T_M,H_M))$ is weak and $H_M(T)$ is still approximately of the universal form (4) but with g_M^2 shifted to some lower value [$H_M(T)$ moves toward $H_{c2}(T)$]. This is the situation in very anisotropic systems where the identity of a single vortex is not maintained from layer to layer and where the physical picture is that of a set of 2D DVP's moving in a random potential provided by the adjacent layers through Eq. (5) [22]. If $H_M(T)$ gets too close to $H_{c2}(T)$, so that $\eta/\tilde{a}(T_M,H_M)=\xi_z^2/d^2\geq1$, deviations from (4) will become significant and a description in terms of the anisotropic 3D GL theory will be more appropriate, with vortices in DVP becoming continuous lines [23].

In this paper we have developed a description of critical fluctuations near $H_{c2}(T)$. One should keep in mind that this DVP description holds in region A and cannot be extended to low fields [region B, near $H_{c1}(T)$]. This relationship to the low-field dVP behavior can be illustrated on the example of $H_M(T)$. Equation (4) can be rewritten as $f(t,h)=\theta_M th$, where $t\equiv T/T_{c0}$, $h\equiv H/H_{c2}(0)$, $f(t,h)$ is the BCS condensation energy normalized to its value E_c at $T=H=0$, and $\theta_M=g_M^2 T_{c0}/2\pi\xi_0^2 dE_c$. For $\theta_M\ll 1$, $H_M(T)$ is close to $H_{c2}(T)$. Since $\theta_M\sim g_M^2(k_F^2\xi_0 d)^{-1}$, H_M will be near H_{c2} in most superconductors and the DVP description is appropriate. This behavior is denoted by $H_M^I(T)$ in Fig. 1. However, for $1-t\ll\theta_M$, $H_M(T)$ *always* drops below $H_{\rm cross}$ and the low-field dVP has to be used. In dVP the transition line is largely independent of H [17]: $T_M\cong T_{c0}/(1+\theta_M^C)$, $\theta_M^C\cong\theta_M$. Thus, if θ_M is small, the DVP-dVP crossover in $H_M(T)$ will occur at rather low fields, only very close to T_{c0}. As θ_M increases, $H_M(T)$ in Fig. 1 evolves from $H_M^I(T)$ to $H_M^{II}(T)$, where the DVP-dVP crossover occurs at moderate fields. This may take place in strongly fluctuating systems like Bi-Sr-CaCu-O or in very thin films. Finally, disorder will play a role in real superconductors [4] and can be readily incorporated in DVP.

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