# Global Analysis of $\pi N \to \pi\pi N$ Data Near Threshold and Chiral-Symmetry Breaking

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An analysis has been made of all available data near threshold for the reactions $\pi N \to \pi\pi N$ in terms of isospin amplitudes. The experiments for different channels are found to be consistent with each other and with the chiral-symmetry-breaking model of Olsson and Turner. The analysis yields values for the $\pi\pi$ scattering lengths which are in good agreement with a calculation based on the Weinberg $\pi\pi$ interaction corrected for the effects of the $f_0(975)$ scalar meson.

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Chiral symmetry is an important feature of elementary-particle dynamics in the low-energy region. As implemented in the theories of current algebra and the partially conserved axial current, it provides a basis for the calculation of many low-energy properties of strongly interacting systems in the region where QCD becomes nonperturbative, and has shown impressive agreement with experiment [1-7]. In addition, chiral symmetry is of interest in its own right as a rather good fundamental symmetry, relevant to both weak- and strong-interaction physics and relating phenomena in these areas. The topic is described in detail in several review articles [5-7].

The nature of chiral-symmetry breaking can be studied by measurements of the $\pi\pi$ scattering amplitudes at zero relative momentum, which vanish in the chiral limit. Experimentally, $\pi\pi$ scattering can be probed through the pion production reactions $\pi N \to \pi\pi N$; the $\pi\pi$ interaction at zero relative momentum is related to the amplitude for $\pi N \to \pi\pi N$ at threshold. An important development in the understanding of $\pi N \to \pi\pi N$ reactions was made by Olsson and Turner [3,4], who calculated the leading contributions near threshold, using an almost model-independent Lagrangian. They showed, on rather general assumptions, that for $\pi N \to \pi\pi N$ reactions, the nature of the chiral-symmetry breaking can be characterized by a single parameter, $\xi$.

Many experiments on $\pi N \to \pi\pi N$ reactions at low momenta have been reported, which were directed towards a study of chiral-symmetry breaking in $\pi\pi$ scattering. Several of these were analyzed in the framework of Olsson and Turner's model; threshold amplitudes were deduced by extrapolation from the measured cross sections. The values extracted for $\xi$ from these analyses are not particularly consistent, and it is not clear whether the inconsistencies are the result of problems in the data, of difficulties in extraction of $\xi$ from the data (for example, the problem of extrapolating the amplitude of threshold), or of basic limitations in the theoretical model.

A summary of the data available in 1984 is given by Manley [8], who published a careful analysis of all data available at that time. Since Manley's paper appeared, several other measurements have been published or are in progress, on $\pi^- p \to \pi^+\pi^- n$ (Refs. [9-11]), $\pi^- p \to \pi^-\pi^0 p$ (Ref. [12]), $\pi^+ p \to \pi^+\pi^0 p$ (Ref. [13]), $\pi^+ p \to \pi^+\pi^+ n$ (Refs. [14,15]), and $\pi^- p \to \pi^0\pi^0 n$ (Ref. [16]).

There are five experimentally accessible charge states for $\pi N \to \pi\pi N$ reactions and only four independent isospin amplitudes. Further, two of these amplitudes are constrained to zero at threshold where the final-state pions must be in an $s$ state and therefore have even isospin. With the new experiments, for the first time a complete set of data close to threshold is available. Thus the problem is now overdetermined and an analysis of the combined data can give better values for the threshold amplitude and at the same time provide a consistency check on the data and the model. This Letter reports such an analysis.

The four isospin channels are $(I,I_{\pi\pi})=(\frac{3}{2},2)$, $(\frac{3}{2},1)$, $(\frac{1}{2},1)$, and $(\frac{1}{2},0)$. The two amplitudes with $I_{\pi\pi}=1$ vanish at threshold. The total amplitude for $\pi N \to \pi\pi N$ can be written for each channel in the form [17]

$$\mathcal{M}=\bar{u}\{\mathcal{A}\gamma^5+\mathcal{B}\gamma^5 \not{p}+\mathcal{C}\gamma^5 \not{q}\}u\ ,$$

where $p$ and $q$ are, respectively, the four-momenta of the final-state nucleon in the overall center-of-mass frame and of the $\pi\pi$ relative motion in the $\pi\pi$ rest frame. The initial and final nucleon spinors are $u$ and $\bar{u}$. The coefficients $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ can be functions of any Lorentz-invariant variable with suitable analytic properties. This form of the amplitude results in terms with spin dependences $\chi_f^\dagger(\boldsymbol{\sigma}\cdot\mathbf{Q})\chi_i$, $\chi_f^\dagger(\boldsymbol{\sigma}\cdot\mathbf{q})\chi_i$, $\chi_f^\dagger(\boldsymbol{\sigma}\cdot\mathbf{p})\chi_i$, and $\chi_f^\dagger(\boldsymbol{\sigma}\cdot\mathbf{p})(\boldsymbol{\sigma}\cdot\mathbf{q})(\boldsymbol{\sigma}\cdot\mathbf{Q})\chi_i$, where $\mathbf{Q}$ is the center-of-mass momentum of the initial pion. The term in $\boldsymbol{\sigma}\cdot\mathbf{q}$ corresponds to $l=1$ for the relative motion of the final-state pions and hence to the amplitudes with $I_{\pi\pi}=1$. In the fits described below, terms in $(\boldsymbol{\sigma}\cdot\mathbf{p})$ were not included. The amplitude then takes the form

$$\mathcal{M}=(A_{32}+A_{10})\chi_f^\dagger(\boldsymbol{\sigma}\cdot\mathbf{Q})\chi_i+(A_{31}+A_{11})\chi_f^\dagger(\boldsymbol{\sigma}\cdot\mathbf{q})\chi_i\ .$$

The four coefficients $A_{2I,I_{\pi\pi}}$ were taken to be linear in the total center-of-mass energy $W$. Unitarity constrains [18] the phases of the pion production amplitudes to be equal to those for elastic scattering for the corresponding state. The relevant phases were taken from the Particle Data Group compilation [19].

The data sample taken initially for the fits consisted of all the data in Refs. [9,12–16,20–24]. We have carried out two analyses of this data set.

(A) The first of these is a model-independent fit, relying only on charge independence to relate the cross sections for the five charge states to the isospin amplitudes. Each of the four isospin amplitudes $A_{2I,I_{\pi\pi}}$ was characterized by two parameters (a threshold amplitude and a slope) so that eight parameters were varied in the fit.

The first fits showed that much of the contribution to $\chi^2$ came from some of the highest-momentum points. This presumably reflects an inadequacy in our assumption of a simple linear dependence in $W$ for the amplitudes. A more complicated energy dependence was used by Manley [8] in that he included explicitly the effects of the Roper resonance. In our analysis, the main object was to study the amplitudes close to threshold rather than to investigate their momentum dependence. Therefore, instead of elaborating the fitting function to include higher-momentum points, we simply imposed a cutoff momentum and excluded all points above this momentum from the fit. The parameters of the fit were essentially unchanged provided this cutoff momentum was below 425 MeV/$c$. In the fits below, a value of 400 MeV/$c$ was used.

A further difficulty was evident for some points for the $\pi^+p \rightarrow \pi^+\pi^+n$ channel in that there are discrepancies between some of the experimental data in the region $p_\pi \sim 310$–$370$ MeV/$c$. Rather than excluding data from particular experiments, to avoid any possible bias, all data points for this channel in this momentum region were dropped from the data set.

With the remaining data points, therefore, we believe we have a data set which contains no obviously problematic points and which covers a momentum range for which our simple parametrization of the amplitudes is adequate.

The fit is shown by the solid line in Fig. 1. Three points follow from the fit:

(i) The fit has a $\chi^2$ of 24 for 33 degrees of freedom so the data set is adequately consistent internally. Discrepancies observed previously in the values of $\xi$ extracted from fits to individual channels cannot be ascribed to problems with the data, at any rate for those data points still retained in the data set.

(ii) The $I_{\pi\pi}=2$ term is substantially smaller than that for $I_{\pi\pi}=0$.

(iii) The $I_{\pi\pi}=1$ contribution, which implies $p$ waves in the final $\pi\pi$ system, is surprisingly important, even relatively close to threshold. This is illustrated by the dashed line in Fig. 1, which shows the fit when the $I_{\pi\pi}=1$ terms are dropped.

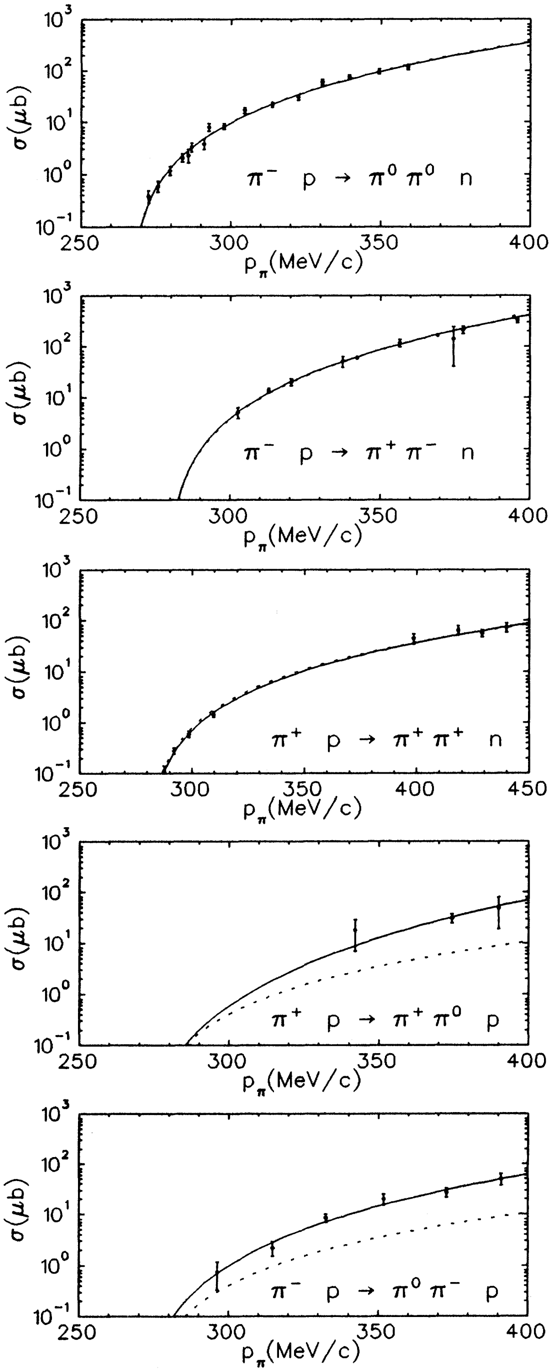


FIG. 1. Fits to data for $\pi N \rightarrow \pi\pi N$ reactions. The solid line shows the model-independent fit using four isospin amplitudes. The dashed line shows the effect of dropping the two amplitudes with $I_{\pi\pi}=1$.

Conclusions (ii) and (iii) are in agreement with Manley's analysis [8]. In the momentum region excluded from the $\pi^+p \to \pi^+\pi^+n$ channel, the Omicron data [14] lie significantly above the fitted curve, though the higher-momentum points from this experiment are fitted reasonably well.

(B) A second fit to the same data set was carried out in which the chiral-symmetry-breaking model of Olsson and Turner [3,4] was incorporated in the analysis. In the context of the present analysis, this model fixes the relation between the $I_{\pi\pi}=0$ and 2 amplitudes at threshold, and each becomes a function of the chiral-symmetry-breaking parameter $\xi$. The invariant amplitude is given by

$$\mathcal{M}=2\sqrt{2}i\left(\frac{G_{NN\pi}}{2M}\right)\left(\frac{1}{f_\pi}\right)^2 a_I(\pi\pi N)\chi_f^\dagger(\sigma\cdot\mathbf{Q})\chi_i\,,$$

where $f_\pi$ is the pion decay constant and $M$ is the nucleon mass. The dimensionless amplitudes $a_I(\pi\pi N)$ are, for total $I=\frac{1}{2}$ and $\frac{3}{2}$,

$$a_{1/2}(\pi\pi N)=3.399-1.058\xi\,,$$

$$a_{3/2}(\pi\pi N)=1.683+0.669\xi\,,$$

using $M=938.9$ MeV and $m_\pi=137.5$ MeV.

Application of Olsson and Turner's model requires values for the coupling constant $G_{NN\pi}$ and for the combination $(M/G_{NN\pi})(g_A/g_V)$ which is related by the Goldberger-Treiman relation to the pion decay constant $f_\pi$. Recent values for the parameters involved have been discussed by Coon and Scadron [25]. They point out that if values for $G_{NN\pi}$, $g_A/g_V$, and $M$ are evaluated at the chiral limit, the Goldberger-Treiman relation gives $f_\pi=90.1$ MeV, which agrees with the experimental value of $f_\pi$ extrapolated to the chiral limit. By comparison, with standard (on-shell) parameters, the Goldberger-Treiman relation gives $f_\pi=88.4$ MeV and the experimental value is 92.6 MeV. There is no obviously correct value to use for $f_\pi$ in this context, but we take the view that using the value $f_\pi=90.1$ MeV, evaluated at the chiral limit, is appropriate for the present analysis. Chiral-symmetry breaking is then included explicitly using the model of Olsson and Turner.

Fitting the data with this constraint gives curves which are close to the solid lines of Fig. 1. The fit has a $\chi^2$ of 33 for 34 degrees of freedom. The value of $\xi$ extracted from the fit is $\xi=-0.60\pm0.10$ and the amplitudes $A_{2I,I_{\pi\pi}}$ are listed in Table I. The error in $\xi$ includes the fitting errors and the effect of errors in the input parameters. It does not include uncertainties from the approach to the choice of parameters discussed in the previous paragraph. It seems that the available data are adequately consistent with Olsson and Turner's model, and discrepancies in the values of $\xi$ between published analyses of individual channels are not due to inadequacies of this model.

TABLE I. The amplitudes $A_{2I,I_{\pi\pi}}$ from fit (B).

| Amplitude | $A_{2I,I_{\pi\pi}}$ at threshold $(m_\pi^{-3})$ | $dA_{2I,I_{\pi\pi}}/dW$ $(m_\pi^{-4})$ |
|---|---|---|
| $A_{32}$ | $2.07\pm0.10$ | $1.98\pm0.33$ |
| $A_{10}$ | $6.55\pm0.16$ | $10.4\pm0.8$ |
| $A_{31}$ | $-5.0\pm2.2$ | $15.0\pm4.2$ |
| $A_{11}$ | $3.3\pm0.8$ | $0.9\pm2.0$ |

TABLE II. $s$-wave $\pi\pi$ scattering lengths.

| Source | $a_0$ $(m_\pi^{-1})$ | $a_2$ $(m_\pi^{-1})$ |
|---|---|---|
| Present results | $0.197\pm0.010$ | $-0.032\pm0.004$ |
| Experimental, $K_{e4}$ decays [26,27] | $0.26\pm0.05$ | ... |
| Chiral perturbation theory [7] | 0.20 | −0.042 |
| Weinberg [1] | 0.156 | −0.045 |
| Jacob and Scadron [28] | 0.201 | −0.028 |

From Olsson and Turner's work [3,4], the value of $\xi$ is related to the $s$-wave $\pi\pi$ scattering lengths $a_I$, for $I=0$ and 2, by

$$a_0=\frac{14-5\xi}{48}\frac{3m_\pi}{4\pi f_\pi^2},\quad a_2=-\frac{\xi+2}{24}\frac{3m_\pi}{4\pi f_\pi^2}\,.$$

These relations, together with our value for $\xi=-0.60\pm0.10$, give values for the $\pi\pi$ scattering lengths of $a_0=(0.197\pm0.010)m_\pi^{-1}$ and $a_2=(-0.032\pm0.004)\times m_\pi^{-1}$.

In Table II, the $s$-wave scattering lengths found here are compared with an experimental determination [26,27] from $K_{e4}$ decays and with three theoretical predictions. The first two of these are the chiral perturbation theory result from Ref. [7] and the current-algebra result of Weinberg [1]. Jacob and Scadron [28] pointed out that there is an important correction to Weinberg's calculation from the contribution of the $f_0(975)$ scalar meson which, they argue, is the most significant contributor. Their prediction, shown in Table II, is in good agreement with the present experimental result.

In summary, (a) apart from a small momentum region in the $\pi^+p\to\pi^+\pi^+n$ channel, all available data on $\pi N\to\pi\pi N$ reactions below 400 MeV/$c$ are internally consistent, (b) Olsson and Turner's model fits the data well, and (c) we extract $s$-wave $\pi\pi$ scattering lengths which agree well with the prediction of Jacob and Scadron (see Table II).

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