Multiple-Scattering Suppression of the Bremsstrahlung Emission of Neutrinos and Axions in Supernovae

Georg Raffelt
Max-Planck-Institut für Physik, Postfach 401212, W-8000 München 40, Germany

David Seckel
Bartol Research Institute, University of Delaware, Newark, Delaware 19716
(Received 14 June 1991; revised manuscript received 27 August 1991)

In a supernova core the nucleon collision rate $\Gamma_{\rm coll}$ far exceeds the oscillation frequency of neutrino or axion radiation originating from nucleon bremsstrahlung processes of the type $NN \to NN + \nu\bar{\nu}$ or a. Therefore, including the imaginary part of the self-energy in the effective nucleon propagator, the rate for such processes is suppressed by a factor $[1+(\Gamma_{\rm coll}/\xi T)^2]^{-1}$, where ξ is a numerical factor and T is the temperature. Previous SN 1987A bounds on axions, right-handed neutrinos, and other particles must be relaxed accordingly.

PACS numbers: 97.60.Bw, 14.60.Gh, 14.80.Gt, 14.80.Ly

A theoretical understanding of stellar evolution requires detailed knowledge of the energy-loss rates by neutrino emission. Conversely, avoiding excessive emission of hypothetical particles such as axions leads to important constraints on their properties [1]. While any practical calculation of standard or exotic energy-loss rates is based on perturbation theory, collective effects of the hot and dense medium must be taken into account to obtain meaningful results. We presently wish to point out that a collective effect which has not previously been discussed in this context will lead to a significant suppression of bremsstrahlung processes in a nuclear medium. Specifically, this includes axion emission, $NN \to NN + a$, which is the basis for the supernova (SN) 1987A constraints on the axion coupling and mass [2,3]. It also includes the neutrino processes, $NN \to NN + \nu\bar{\nu}$ or $nn \to np + e\bar{\nu}_e$, which are important for the late-time cooling of neutron stars [4,5], and possibly for the emission of right-handed neutrinos from supernovae [2,6].

The main idea behind our "new" effect was first stated by Landau and Pomeranchuk [7], who observed that the production of bremsstrahlung radiation requires a nonvanishing "formation time." Therefore, if the density of targets is so large that the emitter suffers additional encounters during this period, these collisions will interfere with the radiation process and eventually suppress it. While Landau and Pomeranchuk discussed their effect in the context of electromagnetic bremsstrahlung by relativistic electrons passing through a medium, we are interested in the simpler case of *nonrelativistic* nucleons emitting axions or neutrino pairs of frequency ω. In this context we may loosely say that the encounter with a target kicks the emitter into an off-shell state which violates energy conservation by an amount ω. The formation time is understood by Heisenberg's uncertainty relation: The emitter will typically reside in this virtual state for a duration $\sim 1/\omega$ before it returns to its mass shell. This suggests that the collision rate of the emitter, $\Gamma_{\rm coll}$, must be less than the frequency of the radiation to be emitted if this process is supposed to occur effectively as in single-target scattering. Because the frequency of the bremsstrahlung radiation of light particles from a thermal medium is on the order of the temperature, $\omega \sim T$, we can ignore multiple scattering only if the condition

$$\Gamma_{\rm coll} \ll T \tag{1}$$

is satisfied.

Our main observation is that condition (1) is not fulfilled in the interior of a SN core for nuclear bremsstrahlung processes. To demonstrate this we consider a newly born neutron star immediately after a SN collapse, with interior temperatures of 20–60 MeV and Fermi momenta of the nucleons of 300–400 MeV. (Throughout, we use natural units $\hbar = c = k_B = 1$.) In order to estimate the nucleon collision rate we use the measured total np scattering cross section [8] which we find can be approximated by $\sigma_{np} \sim (200\ \text{mb})[(300\ \text{MeV})/p]^2$, where p is the relative momentum, applicable for p in the range of 100–500 MeV. (In natural units $\sigma_{np} \sim 50/p^2$.) Because the nucleons are on the verge of degeneracy, typical momenta will be on the order of the Fermi momentum p_F, while the nucleon density is $p_F^3/3\pi^2$. Typical velocities are $v_N \sim p_F/m_N$ so that we estimate $\Gamma_{\rm coll} = \sigma_{np} n_N v_N \sim (50/3\pi^2) p_F^2/m_N$. At nuclear densities we have $p_F \sim 300$ MeV so that $\Gamma_{\rm coll} \sim 200$ MeV. Hence the collision rate exceeds the temperature by a substantial factor. This estimate is very crude because we have neglected all effects of the medium on the collision rate such as nucleon degeneracy and especially the change of the nucleon dispersion relation and of the nucleon interaction potential. These effects could either increase or decrease $\Gamma_{\rm coll}$, but they would also affect the "naive" bremsstrahlung rates.

In order to estimate the impact of collisional damping on nuclear bremsstrahlung processes, we begin with the emission of neutrino pairs, $NN \to NN + \nu\bar{\nu}$, in the limit where $\Gamma_{\rm coll}$ may be neglected. Following Friman and Maxwell [4] we assume that the nucleon interaction is well described by a one-pion exchange potential. If the nucleons are treated nonrelativistically, the spin-summed, squared matrix element for np bremsstrahlung is [4]

$$\sum_{\text{spins}} |\mathcal{M}_{np}|^2 = 64 G_F^2 c_A^2 \left(\frac{f}{m_\pi}\right)^4 \left[\frac{|\mathbf{q}|^4}{(|\mathbf{q}|^2+m_\pi^2)^2} + (\text{similar terms})\right] \frac{\omega_1\omega_2 - (\hat{\mathbf{q}}\cdot\mathbf{k}_1)(\hat{\mathbf{q}}\cdot\mathbf{k}_2)}{\omega^2}, \tag{2}$$

where c_A is the axial weak nucleon charge ($c_A^2 \sim 1$), f/m_π with $f \sim 1$ is the usual parametrization of the pion-nucleon coupling, $\mathbf{q}$ is the momentum transfer between the nucleons, and $k_{1,2} = (\omega_{1,2}, \mathbf{k}_{1,2})$ are the four-momenta of the neutrino and antineutrino. For nn and pp bremsstrahlung a similar expression pertains. An important feature of Eq. (2) is the factor ω^{-2}, where ω is the energy transfer to the radiation ($\omega = \omega_1 + \omega_2$ for $\nu\bar{\nu}$ and $\omega = \omega_a$ for a). The ω^{-1} divergence of the matrix element at soft energy transfers formally arises because the lowest-order nonrelativistic nucleon propagator has the form [4] $iG_N(p) = \pm i\omega^{-1}$, where the sign depends on the time order of the collision and radiation vertices in the amplitude. Thus, the divergence arises because the intermediate state of the emitter is near its mass shell for the emission of soft radiation so that the ω^{-2} behavior of Eq. (2) is generic to this type of process: It does not depend on the specific choice for the nucleon interaction potential.

The energy-loss rate by $\nu\bar{\nu}$ emission can now be calculated by multiplying Eq. (2) with a factor $\omega = \omega_1 + \omega_2$, and integrating over the phase space of the participating particles. It can be expressed as

$$Q_{\nu\bar{\nu}} = A_{\nu\bar{\nu}} \langle \omega^4 \rangle, \tag{3}$$

where we have used the notation

$$\langle \omega^n \rangle \equiv \int_0^\infty d\omega\, F(\omega)\, \omega^n. \tag{4}$$

The function $F(\omega)$ arises after one has performed the integrations over the nucleon phase space; it contains all of the spectral information about the emitted radiation. The expression $A_{\nu\bar{\nu}}$ is a function of the coupling constants, temperature, density, and chemical composition of the medium. The emission rate is well behaved because the ω^{-2} divergence of Eq. (2) is moderated by other factors of ω to yield the ω^4 term in Eq. (3): ω^2 from the weak matrix element, ω because we are calculating an energy-loss rate, ω^5 from the neutrino phase space after integrating over one of the neutrino energies, and a factor of ω^{-2} from the neutrino wave-function normalization, giving a total of ω^6.

We have assumed, of course, that the final-state neutrinos can freely escape. While this assumption applies to cold neutron stars, left-handed (LH) neutrinos are actually trapped in relatively hot SN cores. However, we may consider the production of right-handed (RH) neutrinos by the same processes if a small component of RH weak currents exists. Moreover, if neutrinos have a small Dirac mass, either of the neutrino final states will be RH in a fraction $(m_\nu/2\omega_j)^2$ of all cases ($j=1,2$). Including this factor and assuming that the LH part of the $\nu_L\bar{\nu}_R$ or $\nu_R\bar{\nu}_L$ final state is trapped, the energy-loss rate in RH neutrinos is $Q_{\text{Dirac}} = A_{\text{Dirac}} m_\nu^2 \langle\omega^2\rangle$. This expression has the same form as that for axions, $Q_a = A_a\langle\omega^2\rangle$, as stressed by Turner [6]. Therefore we are led to consider two distinct cases where the "naive" emission rates are of the form $\langle\omega^n\rangle$, namely, $n=2$, representing the emission of axions and RH Dirac neutrinos, and $n=4$, representing the standard emission of $\nu\bar{\nu}$ (see Table I).

For all of the processes under consideration, which involve two-nucleon scattering and an axial-vector coupling of the radiation to nucleons, the $\langle\omega^n\rangle$'s in Eq. (4) are expressed in terms of a common function F. [In the nonrelativistic limit only the axial weak current contributes to neutrino pair emission [4], while the structure of the axion coupling is $(1/2f_a)\bar{N}\gamma_\mu\gamma_5 N\,\partial^\mu a$, where a is the axion field and f_a the axion decay constant. Therefore, all of our examples actually do have the same nuclear part of the matrix element.] Neglecting m_π relative to $|\mathbf{q}|$ and neglecting the momentum transfer to the radiation, while including the energy transfer ω, one finds explicitly for nondegenerate (ND) and degenerate [4] (D) nucleons

$$F_{\text{ND}}(\omega) = e^{-y}\int_0^\infty dx\, e^{-x}(x^2+yx)^{1/2},$$
$$F_{\text{D}}(\omega) = \frac{y + y^3/4\pi^2}{e^y - 1}, \tag{5}$$

where $y \equiv \omega/T$. Both functions are normalized such that $F(0)=1$. Of course, the expressions $A_{\nu\bar{\nu}}$, etc., also take on different forms between the D and ND cases. The average energy transfer to the radiation is $\bar{\omega} = \langle\omega^n\rangle/\langle\omega^{n-1}\rangle$; we give numerical values for our two cases $n=2$ and 4 in Table I.

To account quantitatively for the effect of collisions we proceed by including the imaginary or damping part of the nucleon propagator in the expression for the matrix element. If the intensity of a propagating nucleon wave in the medium decreases as $\exp(-\Gamma t)$, its amplitude decreases as $\exp(-\Gamma t/2)$, leading us to substitute for the nonrelativistic nucleon propagator $iG_N = \pm i/\omega \rightarrow i/(\pm\omega + i\Gamma/2)$. This means that in the squared matrix element

TABLE I. Characteristics of the distributions of radiation energies ω in nucleon bremsstrahlung processes. For a vanishing damping parameter of the nucleon propagator, $\Gamma = 0$, the energy-loss rate is proportional to $\langle\omega^n\rangle$, leading to the indicated average values for the energy transfer $\bar{\omega}$.

Emission	n	Degeneracy	$\bar{\omega}/T$	ξ_∞
a or $\nu_L\bar{\nu}_R$ ($\nu_R\bar{\nu}_L$)	2	ND	2.286	7.63
		D	3.159	10.22
$\nu_L\bar{\nu}_L$	4	ND	4.364	11.74
		D	5.783	15.10

we should substitute

$$\frac{1}{\omega^2} \to \frac{1}{\omega^2+\Gamma^2/4}\,. \tag{6}$$

The damping parameter Γ is a function of the energy E and momentum $\mathbf{p}$ of the virtual nucleon which are related by $(|\mathbf{p}|^2+m_N^2)^{1/2}=E\pm\omega$.

In a more rigorous approach using field theory at finite temperature and density (FTD) our prescription amounts to using a resummed, effective propagator for a "soft" particle line in a thermal Feynman graph [9]. Put another way, on certain internal lines of a (thermal) Feyman graph one must use the full self-energy in the denominator of the particle propagator, including the imaginary part which can be computed by using the cutting rules at FTD [10]. Suppressing the spin structure of the self-energy for our nonrelativistic nucleons, we are thus led to associate Γ in Eq. (6) with the nucleon collision rate $\Gamma_{\rm coll}$. We stress that there is no conceptual problem in defining a collision rate for a *virtual* nucleon because the imaginary part of the self-energy is a well-defined quantity both on and off shell. For a virtual nucleon of energy E and momentum $\mathbf{p}$ one would compute $\Gamma_{\rm coll}$ as for a real nucleon with mass $(E^2-|\mathbf{p}|^2)^{1/2}$ [11].

The interpretation of Γ must be modified when the nucleons are degenerate because back reactions are then important. According to Weldon [11] this leads to $\Gamma=\Gamma_{\rm coll}+\Gamma_{\rm inv}$, where $\Gamma_{\rm inv}$ is the inverse collision rate, i.e., the rate by which the medium scatters nucleons *into* the state $(E,\mathbf{p})$. Following Weldon's general arguments we conclude that these rates are universally related by $\Gamma_{\rm coll}/\Gamma_{\rm inv}=e^{(E-\mu)/T}$, where μ is the nucleon chemical potential. In a nondegenerate medium, $e^{(E-\mu)/T}\gg 1$ so that $\Gamma_{\rm coll}\gg\Gamma_{\rm inv}$ and thus $\Gamma\sim\Gamma_{\rm coll}$, in accordance with intuition because $\Gamma_{\rm inv}$ requires two particles of the dilute medium, while $\Gamma_{\rm coll}$ requires only one. In a degenerate system the collision rate for states far below the Fermi surface is greatly suppressed, but "holes" are easily filled, i.e., $\Gamma_{\rm coll}\ll\Gamma_{\rm inv}$ so that $\Gamma\sim\Gamma_{\rm inv}$. However, in our bremsstrahlung process all initial- and final-state nucleons are near their Fermi surfaces so that the same applies to the intermediate state. Hence $\Gamma_{\rm inv}$ will always be of the same order as $\Gamma_{\rm coll}$ so that as a first approximation we may use $\Gamma\sim 2\Gamma_{\rm coll}$ for the degenerate case.

However, the collision rate will be suppressed if the nucleons are very degenerate. For a nucleon near the Fermi surface, the collision rate is suppressed by a factor proportional to $(T/\hat\mu)^2$, where $\hat\mu=\mu-m_N$. One power of $T/\hat\mu$ comes from requiring the target nucleon to also be near the Fermi surface and the other comes from a restriction on the available scattering angles that both particles end up near the Fermi surface. With this suppression, our figure of merit, Γ/T, decreases proportionally to T in the extreme degenerate limit. Thus, collisional damping should not have an important effect on the cooling of cold neutron stars.

In general, Γ is a function of E and ω, but we do not expect strong variations because the dependence of the NN scattering cross section on the relative momentum is largely washed out by thermal averaging over the target nucleons. As a first approximation we therefore take the collision rate as a constant, allowing us to factor out expression (6) from the nuclear phase-space integration. The modified emission rates can then be put in the simple form

$$Q\propto\left\langle\frac{\omega^{n+2}}{\omega^2+\bar\Gamma^2/4}\right\rangle, \tag{7}$$

where $\bar\Gamma$ should be thought of as an average damping rate depending on the local density, temperature, and chemical composition.

Even without attempting to determine the precise value for $\bar\Gamma$ we can estimate a suppression factor $f_{\rm supp}$ of the bremsstrahlung rates as a function of $\bar\Gamma$. It is defined as the emission rate at a given value of $\bar\Gamma$, divided by the rate at $\bar\Gamma=0$, i.e.,

$$f_{\rm supp}=\left\langle\frac{\omega^{n+2}}{\omega^2+\bar\Gamma^2/4}\right\rangle\frac{1}{\langle\omega^n\rangle}\,. \tag{8}$$

Evaluating Eq. (8) by means of the explicit functions (5) we show in Fig. 1 $f_{\rm supp}(\bar\Gamma)$ for $n=2$ (axions) and $n=4$ (neutrino pairs), treating D and ND nucleons separately. In a SN core the nucleons are partially degenerate so that the relevant suppression factors are expected to lie in the crosshatched bands between the D and ND curves. Axion emission from ND nucleons is the most extreme example: The suppression factor could be as low as $\frac{1}{10}$ for plausible values of $\bar\Gamma$.

The form of Eq. (6) suggests that one write the suppression factor as $f_{\rm supp}\equiv[1+(\bar\Gamma/\xi T)^2]^{-1}$, where ξ is another function of $\bar\Gamma$. From Eq. (8) we find that its limiting value for $\bar\Gamma\to\infty$ is $\xi_\infty=(2/T)[\langle\omega^{n+2}\rangle/\langle\omega^n\rangle]^{1/2}$. We give numerical results for ξ_∞ in Table I. Taking $[1+(\bar\Gamma/\xi_\infty T)^2]^{-1}$ for $f_{\rm supp}$ gives us the correct limiting results for $\bar\Gamma\to 0$ and $\bar\Gamma\to\infty$.

We stress that including the damping part of propaga-

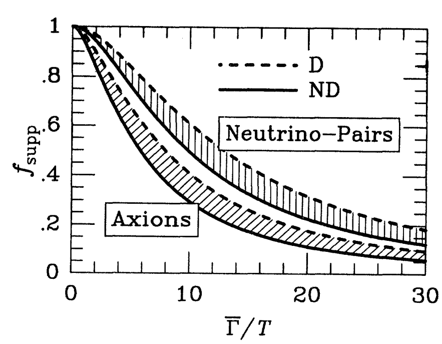

FIG. 1. Suppression factor by collisional damping, $f_{\rm supp}$, defined in Eq. (8), for the emission of axions or right-handed neutrinos ($n=2$) or neutrino pairs ($n=4$). In each case the solid line refers to nondegenerate (ND) and the dashed line to degenerate (D) conditions.

tors will also modify other processes that have been discussed in the literature. One example is closely related to the bremsstrahlung processes, namely, the modified URCA process, $e+np \rightarrow nn+\nu$. Interestingly, it has recently been remarked [12] that this process will be suppressed by collisional damping. Another example is the emission of RH neutrinos from the process $\gamma\gamma \rightarrow \pi^0 \rightarrow \nu_R \bar{\nu}_L$ or $\bar{\nu}_R \nu_L$ which is allowed and proportional to m_ν^2 if neutrinos have a small Dirac mass. Natale [13] argued that because of the high temperatures available in a SN core this reaction can be on resonance and will occur at a large rate. However, the resonant rate involves the total pion width Γ_π, for which Natale used the vacuum value. In a SN core, pions will be damped mostly by nucleon absorption rather than by free decay so that Γ_π in the medium is much larger than in vacuum and Natale's rate will be strongly suppressed. Therefore, we believe that the two-photon production of RH neutrinos is negligible relative to other processes such as spin-flip scattering, $\nu_L + N \rightarrow N + \nu_R$.

In summary, we have pointed out that the frequency of nucleon collisions in the interior of a SN core typically exceeds the oscillation frequencies of thermal neutrinos, photons, or axions, i.e., the collision rate exceeds the temperature by a factor of a few. Therefore bremsstrahlung processes must be computed using effective nucleon propagators which include the imaginary part of the self-energy (essentially the collision rate) in the denominator. This leads to a suppression of the naive rates which may be as large as a factor of a few for the case of axion emission from a SN core. Therefore, the SN 1987A bounds on the axion mass derived by us [2] and by several other groups [3] as well as the bounds on other exotic particles [1] must be relaxed accordingly.

We thank L. Stodolsky for illuminating discussions of the Landau-Pomeranchuk effect and for comments on the manuscript. Discussions with T. Altherr helped to deepen our understanding of the FTD approach to this problem. Also, discussions with G. Steigman and V. Zakharov helped to improve the manuscript. D.S. is partly supported by the DOE at Bartol, and he thanks the Aspen Center for Physics, the NAS, and the AS-U.S.S.R. for hospitality during preparation of the manuscript. G.R. thanks the University Paris VI/VIII for hospitality and financial support while part of this research was conducted.

[1] For a review, see G. Raffelt, Phys. Rep. **198**, 1 (1990).

[2] G. Raffelt and D. Seckel, Phys. Rev. Lett. **60**, 1793 (1988).

[3] M. S. Turner, Phys. Rev. Lett. **60**, 1797 (1988); R. Mayle *et al.*, Phys. Lett. B **203**, 188 (1988); **219**, 515 (1989); R. P. Brinkmann and M. S. Turner, Phys. Rev. D **38**, 2338 (1988); A. Burrows, M. S. Turner, and R. P. Brinkmann, Phys. Rev. D **39**, 1020 (1989).

[4] B. L. Friman and O. V. Maxwell, Astrophys. J. **232**, 541 (1979).

[5] H.-Y. Chiu and E. E. Salpeter, Phys. Rev. Lett. **12**, 413 (1964); J. N. Bahcall and R. A. Wolf, Phys. Rev. Lett. **14**, 343 (1965); Phys. Rev. **140**, 1452B (1965); R. F. Sawyer and A. Soni, Astrophys. J. **230**, 859 (1979).

[6] M. S. Turner, Fermilab Report No. Fermilab-Pub-91/36-A, 1991 (unpublished).

[7] L. D. Landau and I. Ja. Pomeranchuk, Dokl. Akad. Nauk SSSR **92**, 535 (1953); **92**, 735 (1953). For an early review in English, see E. L. Feinberg and I. Pomeranchuk, Nuovo Cimento Suppl. **3**, 652 (1956). For a pedagogical discussion, see L. Stodolsky, in *Proceedings of the Sixth International Colloquium on Multiparticle Reactions* (Rutherford Laboratory, Oxford, 1975).

[8] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).

[9] E. Braaten and R. D. Pisarski, Nucl. Phys. **B337**, 569 (1990). For some relatively simple applications of these ideas, see T. Altherr, Ann. Phys. (N.Y.) **207**, 374 (1991); E. Braaten and T. C. Yuan, Phys. Rev. Lett. **66**, 2183 (1991).

[10] See, for example, R. L. Kobes and G. W. Semenoff, Nucl. Phys. **B260**, 714 (1985).

[11] A. Weldon, Phys. Rev. D **28**, 2007 (1983).

[12] J. M. Lattimer, C. J. Pethick, M. Prakash, and P. Haensel, Phys. Rev. Lett. **66**, 2701 (1991).

[13] A. A. Natale, Phys. Lett. B **258**, 227 (1991).