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Quantum Irreversibility and Chaos

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We study the Hamiltonian of a two-level system interacting with a one-mode radiation field by means of the Wigner method and without using the rotating-wave approximation. We show that a phenomenon of collapses and revival, reminiscent of that exhibited by the Jaynes-Cummings model, takes place in the high-coupling limit. This process appears as irreversible or virtually reversible, according to whether the semiclassical regime is chaotic or not. Thus we find a new mechanism for dissipation in the quantum domain.

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In the past decade there has been a steadily increasing interest in the phenomenon of "collapse and revival" of quantum state vectors [1-5]. Cummings [6] noted in his investigation of a two-level system coupled to an electromagnetic field [7] that over a relatively short time interval the initial wave function collapses as an exponential in t^2, i.e., a Gaussian decay. A transparent discussion of the physical origin of both collapses and revivals is given by Phoenix and Knight [8], who used the concept of entropy to describe the appearance of disorder resulting from the interaction between the two-level system and the electromagnetic field in a coherent state; i.e., the field can be represented as the coherent superposition of an infinite number of discrete photon states.

All these investigations have been carried out by applying the rotating-wave approximation (RWA) to the spin-boson Hamiltonian:

$$\mathcal{H} = -\tfrac{1}{2}\omega_0\hat{\sigma}_z + (g/\sqrt{2\Omega})\hat{\sigma}_x(\hat{b}+\hat{b}^\dagger) + \Omega\hat{b}^\dagger\hat{b}\,, \qquad (1)$$

where b ($b^\dagger$) is the annihilation (creation) operator with the commutation relation $[b,b^\dagger]=1$, and $(\hat{b}+\hat{b}^\dagger)/\sqrt{2\Omega}=\hat{q}$ and $i\sqrt{\Omega/2}(\hat{b}^\dagger-\hat{b})=\hat{p}$ are the coordinate and momentum operators of our oscillator (coherent field) with frequency Ω. The system of equations generated by (1) supplemented by the RWA is usually referred to as the Jaynes-Cummings model (JCM) [7].

It has been pointed out that the suppression of the RWA has dramatic consequences on the semiclassical behavior of the above system and that the inclusion of the counterrotating terms neglected by the RWA back into the equations of motion can produce chaos [9,10]. The purpose of the present paper is to show that this semiclassical chaos is a new source of quantum irreversibility, with properties very distinct from those of the JCM [8].

To establish this crucial property, we adopt a suitable generalization of the Wigner method [11] and write the equation of evolution for the Wigner quasiprobability:

$$\frac{\partial}{\partial t}\rho_W(x_1,x_2,x_3,q,p;t) = (\mathcal{L}_{\text{class}}+\mathcal{L}_{\text{QGD}})\times\rho_W(x_1,x_2,x_3,q,p;t)\,, \qquad (2)$$

where

$$\mathcal{L}_{\text{class}} \equiv \omega_0\left(x_1\frac{\partial}{\partial x_2}-x_2\frac{\partial}{\partial x_1}\right)+2gq\left(x_3\frac{\partial}{\partial x_2}-x_2\frac{\partial}{\partial x_1}\right) + \Omega^2 q\frac{\partial}{\partial p}-p\frac{\partial}{\partial q}+gx_3\frac{\partial}{\partial p}\,, \qquad (3)$$

$$\mathcal{L}_{\text{QGD}} \equiv g\frac{\partial}{\partial p}\left(\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_1}x_1^2-\frac{\partial}{\partial x_2}x_1x_2-\frac{\partial}{\partial x_3}x_1x_3\right). \qquad (4)$$

Like the density matrix, ρ_W describes an ensemble of systems and (2) describes the evolution of that ensemble in phase space. Note that the Wigner formalism was developed precisely to allow the introduction of a phase space in quantum mechanics. The initial state of the ensemble is described by the distribution

$$\rho_W(\mathbf{x},q,p;0)=\delta(\mathbf{x}-x_3)\rho_0(q,p)\,, \tag{5}$$

namely, the product of a dipole distribution (polarization along the z axis) and the initial distribution ρ_0 for the oscillator. The quantum coherent state of the radiation field is simulated by ρ_0 being a Gaussian distribution. However, even in the special case where at $t=0$ the field is found in a single quantum state $|n\rangle$, ρ_0 is not a δ-function distribution. This is in accordance with the uncertainty principle of quantum mechanics. In the special case of semiclassical chaos the sensitive dependence of chaotic orbits on initial conditions will make the average trajectory

$$\langle\hat{\sigma}_i(t)\rangle=\mathrm{Tr}\hat{\sigma}_i\hat{\rho}(t)=\int d\mathbf{x}\,dq\,dp\,x_i\rho_W(\mathbf{x},q,p;t)\,, \tag{6}$$

which is in general qualitatively and quantitatively different from the single-trajectories result, exhibit the character of irreversibility. The connection between the x_i's and the spin operators is given by (6) and similar relations connect the coordinates q and momentum p to the corresponding quantum operators $\hat{q}$ and $\hat{p}$ (see Ref. [12]).

The operator $\mathcal{L}_{\mathrm{class}}$ is identical to the Liouvillian of a classical dipole interacting with a classical oscillator; i.e., this term alone corresponds to the semiclassical set of equations discussed by various authors [9,10]. We refer to the calculations based on the study of the single-trajectory solutions of the nonlinear dynamic equations as the semiclassical predictions. In this regard we point out a significant feature of the analysis that was apparently overlooked in previous investigations having to do with chaotic trajectories. These earlier studies focused on individual trajectories and gave them physical meaning, whereas we see from (6) that even when $\mathcal{L}_{\mathrm{QGD}}$ is neglected, it is only the ensemble that has physical significance not the individual trajectories.

The term $\mathcal{L}_{\mathrm{QGD}}$ in (2) is quite significant. It has a diffusionlike structure, but the state dependence of the diffusion coefficient results in its not being positive definite. It has recently been shown by Roncaglia *et al.* [11] that if the oscillator is coupled to a heat bath so as to transmit to the spin-$\frac{1}{2}$ dipole standard thermal fluctuations, then this term results in the mean value of the z component of the dipole changing from a Langevin (classical) function to the hyperbolic tangent (quantum). In other words, this term, coined the quantization generating diffusion (QGD) by Roncaglia *et al.* [11], insures that the dipole retains its quantum nature. The operator $\mathcal{L}_{\mathrm{QGD}}$ acts as an antidiffusional mechanism, it competes against thermal fluctuations and constrains the dipole to vacillate between two possible orientations, which otherwise would freely diffuse over all possible orientations.

We carry out two different calculations. The first consists of integrating the semiclassical trajectories and of averaging them over the initial condition of (5). This is equivalent to solving (2) with the approximation of neglecting the QGD. The second consists of the "exact" numerical solution of the standard quantum-mechanical Liouville-Von Neumann equation, and it should be equivalent to the exact solution of (2). To describe the disorder of the system we also evaluate the entropy for the spin-$\frac{1}{2}$ dipole

$$S=-\mathrm{Tr}_{\mathrm{spin}}\rho_S\ln\rho_S\,, \tag{7}$$

where ρ_S is the spin distribution obtained by taking the trace of the density matrix over the boson field. All the calculations illustrated in Figs. 1 and 2, with the exception of the dot-dashed curve in Fig. 2(b), refer to the radiation field in a coherent state. The dot-dashed curve in Fig. 2(b) refers to an initial condition with only one photon in a single quantum state.

In Fig. 1, referring to the condition $\omega_0\ll\Omega=2\pi$, we compare the average z component of the spin operator

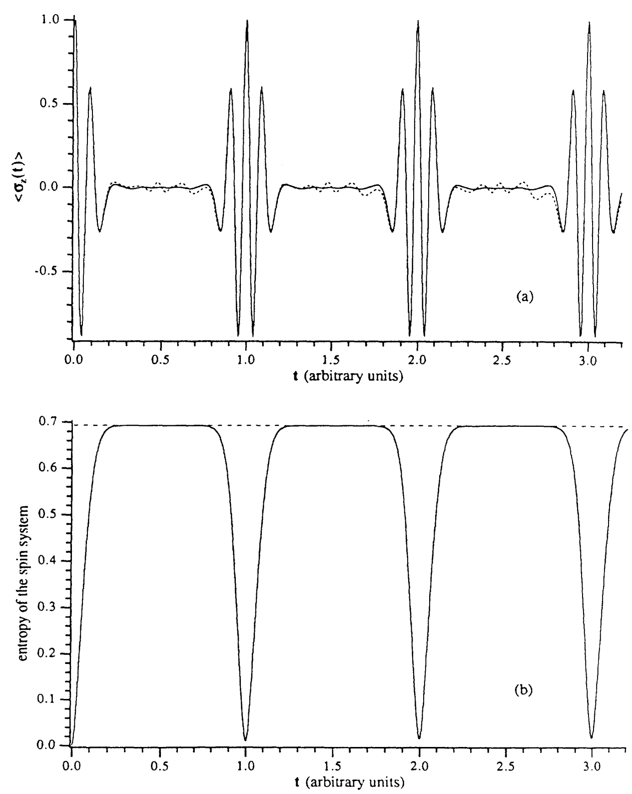

FIG. 1. In all curves $g=20$, $\omega_0=10^{-2}$, $\Omega=2\pi$, and the initial condition of the oscillator is a coherent state with the mean photon number $\langle n\rangle=10$. (a) The solid curve is the analytic prediction of the BGV theory which coincides with the numerically integrated results. The dashed curve is the average z component of the spin obtained by neglecting QGD. (b) The solid curve is the entropy of the spin system numerically calculated. The dashed horizontal line is the theoretical maximum value of the entropy.

calculated using (7) with the theoretical prediction of Bonci, Grigolini, and Vitali [4], which we refer to as BGV. The BGV prediction is closely related to the well-known treatment of quantum dissipation of Leggett *et al.* [13]. According to the detailed analysis of Vitali and Grigolini [14], these linear theories miss the reaction field and the nonlinearity associated with it, thereby suppressing semiclassical chaos [9,10]. Figure 1 refers to a frequency ω_0 so weak as to make the reaction field negligible. This explains why the BGV prediction coincides with the exact, i.e., quantum-mechanical, result of the numerical calculation. It is also remarkable that a satisfactory agreement with these two calculations is obtained by averaging over the semiclassical trajectories using (6). This is so because the QGD, neglected by the dashed curve in Fig. 1, is suppressed by the linear nature of the condition examined here.

According to the BGV interpretation, the collapse observed in Fig. 1 depends on a "multiplicative stochastic" process formally equivalent to Kubo's stochastic oscillator [15]. Semiclassical trajectories with different initial conditions are characterized by slightly different "oscillation frequencies" in the nonchaotic case and the interference between these different members of the ensemble produces a Gaussian-like decay of the average spin-$\frac{1}{2}$ operators [15]. This functional form of the collapse was also obtained by Kubo for his stochastic oscillator, and, as we mentioned earlier, by Cummings for the JCM. Unlike the Kubo oscillator, where the stochastic coefficient (frequency) is based on random fluctuations external to the system, in the present model the multiplicative fluctuations are generated by the dynamics of the single quantum oscillator. This is the reason why the process is not genuinely irreversible and collapses are followed by revivals.

Figure 1(b) shows that the entropy S of (7) monotonically increases until it reaches its maximum value at the end of the collapse process. This means that disorder increases during the collapse process. This is reminiscent of the entropy increase during the JCM collapse [8]. However, since in this high-coupling condition the revivals are much more regular than in the JCM case, we have the correspondingly reversible behavior of the entropy S at each unit of time as shown in Fig. 1(b).

Let us now set $\omega_0 = \Omega = 2\pi$, while leaving unchanged all the other parameters, and first turn our attention to the semiclassical trajectories generated by the dynamic equations corresponding to $\mathcal{L}_{\text{class}}$. In accordance with the results of Refs. [9,10], we find that the individual semiclassical trajectories can exhibit chaos for specific values of the system parameters. Using the method of Benettin, Galgani, and Strelcyn [16], we evaluate the Liapunov coefficient λ, which in the case of Fig. 2 turns out to be of order unity. On the other hand, we have seen that the generalization of the Wigner theory leads us to conclude that single trajectories do not have a direct physical interpretation; only quantities averaged over the initial conditions (5) are physically significant. By definition, chaotic trajectories that are initially close together diverge exponentially in time. Thus the decorrelation mechanism is given by the internal dynamics of the system rather than being external as in the stochastic fluctuations of Kubo.

We see in Fig. 2(a) that at the end of the standard relaxation process the mean value $\langle \sigma_z(t) \rangle$, evaluated with the QGD neglected, reaches a sort of thermodynamic equilibrium and that all revivals after the first are suppressed (dashed curve). It is clear that the collapse is now irreversible due to the subsequent incoherence of the chaotic trajectories. Signs of the original revivals are still visible in the full curve corresponding to the exact calculation, i.e., including also the action of the QGD (solid curve). We note that, correspondingly, the solid line of Fig. 2(b), denoting the entropy S, monotonically approaches a plateau as it should for an irreversible process. However, in the vicinity of the first revival, a ghost is observed in the form of a sudden decrease in the entropy. This dip indicates again a competition between the irreversible effects of chaos and the reversible effects of

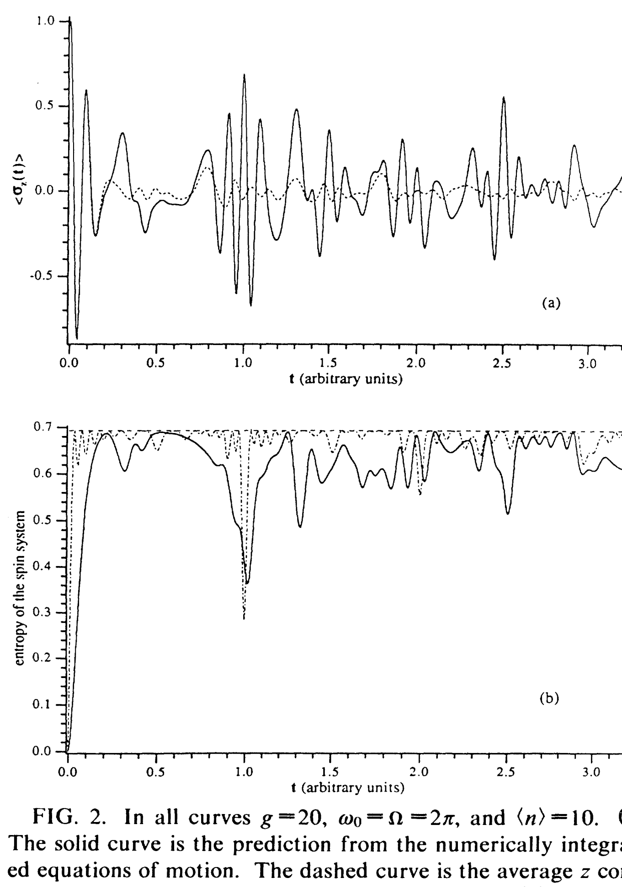

FIG. 2. In all curves $g = 20$, $\omega_0 = \Omega = 2\pi$, and $\langle n \rangle = 10$. (a) The solid curve is the prediction from the numerically integrated equations of motion. The dashed curve is the average z component of the spin calculated by neglecting QGD. (b) The solid and dot-dashed curves denote the entropy of the spin calculated numerically, with an initial condition in a coherent state and in the eigenstate $|n\rangle$, with $n = 10$, respectively. The dashed horizontal line is the theoretical maximum value of the entropy.

QGD, both of which are in the exact calculation. We are led to conclude that the QGD mechanism reacts against the spreading of chaotic semiclassical trajectories and tends to recover, at least in part, the original correlations just as it did for thermal fluctuations [11].

We note that the behavior of the entropy in Fig. 2 is reminiscent of that in Fig. 10 of Ref. [8]. In both cases the entropy increases initially, to reach its maximum possible value around the collapse time, then it exhibits some oscillations and dips a few times, but on the whole there seems to be a trend for the entropy to increase. This apparent similarity (actually the sequel of JCM collapses and revivals take place with a frequency proportional to the interaction strength, whereas the leading frequency of the processes here under study is the field frequency Ω) might imply that irreversibility has the same root in both cases, which would be the wrong conclusion. It is true that the adoption of a coherent state makes the process of collapse depend on the conventional source of irreversibility, that being the infinite number of photon states in the "heat bath." However, this contribution to irreversibility can be totally quenched by adopting an initial condition with a single photon in a single state [dot-dashed curve in Fig. 2(b)]. Within the RWA this initial condition would result in the reversible Rabi oscillation behavior. In contrast, we see here from the dot-dashed curve in Fig. 2(b) that the entropy exhibits a distinctly irreversible behavior, with ghosts of decreasing intensity at $t=2\pi n/\Omega$, with $n=1,2,\ldots$. These ghosts are totally inhibited if the QGD is neglected, thereby confirming again that the QGD competes with the irreversibility generated by semiclassical chaos.

We conclude by contrasting the above results with the traditional picture of a quantum system coupled to the heat bath. In a sense the JCM dissipation [8], which implies the interaction between the two-level atom and a set of an infinite number of quantum states (necessary to reproduce the oscillator in the coherent state), belongs to this traditional picture. Quantum dissipation in these earlier models arises from the infinite numbers of degrees of freedom in the bath, which for technical reasons is generally treated as being linear [17]. In this paper, however, dissipation is found to be a quantum manifestation of the chaos in the semiclassical trajectories. It has nothing to do with a heat bath and is a consequence of the nonintegrability of the spin-boson Hamiltonian. Thus, one kind of quantum irreversibility is herein not a many-body or a many-state effect, but rather an effect of chaos, a mechanism heretofore not identified.

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