Dynamical Localization in Josephson Junctions

R. Graham, M. Schlautmann, and D. L. Shepelyansky[(a)]
Fachbereich Physik, Gesamthochschule Universität Essen, D 4300 Essen 1, Germany
(Received 2 November 1990)

A novel quantum effect, dynamical localization, is predicted for periodically current-driven Josephson junctions: the quantum-mechanical decrease, as opposed to the classical increase, of the intensity of voltage fluctuations with increasing driving amplitude.

PACS numbers: 72.15.Rn, 74.40.+k, 74.50.+r

In the last decade it has become possible to fabricate Josephson junctions sufficiently small that quantum effects due to the quantization of the charge of Cooper pairs and the canonically conjugate variable, the phase difference φ across the junction, become, in principle, observable. The quantization of charge in units of e has already been observed in normal junctions via the Coulomb blockade [1]. The quantum nature of the phase has been observed via the quantized energy levels in Josephson's $\cos\varphi$ potential [2], and, more indirectly, via quantum tunneling in the biased $\cos\varphi$ potential at very low temperatures [3]. Some quantum effects in classically chaotic Josephson junctions have been analyzed theoretically in Ref. [4].

In the present Letter we wish to discuss a new and different effect based on the quantization of φ, namely, dynamical localization. This effect has been discussed in model systems in "quantum chaos" like the kicked rotator [5] and atomic [6] and molecular [7] models. Furthermore, there is experimental evidence [8] that it occurs in hydrogen atoms in Rydberg states driven by a strong microwave field. According to theory it should appear, under appropriate conditions, in periodically driven quantum systems which are chaotic in their classical limit. Here, we will specify the appropriate conditions for current-driven Josephson junctions and show theoretically and by numerical simulation that the effect leaves an unambiguous footprint in the dependence of the intensity of the voltage fluctuations on the amplitude of the driving current: Classically, this intensity is predicted to rise linearly with the current amplitude. Quantum mechanically, we predict the intensity of voltage fluctuations to decrease in inverse proportionality to the current amplitude, above a certain current threshold. The observation of this effect in an array of junctions seems possible. If so, it would not only give a further demonstration of the quantum nature of φ, but would also be a much-sought-for experimental test of some basic ideas of the field of quantum chaos.

We will consider a small (capacitance $C \simeq 10^{-13}$ F) dissipationless Josephson junction driven by an ideal external periodic current source $I = I_0 \sin\Omega t$. The leads from the source are regarded as an internal part of the junction. To avoid dissipation the temperature must be very small compared to the gap, the impedance of the leads has to be kept sufficiently small, and the output power must also be kept low. The latter requirement will be specified more concretely at the end, and is probably met best in an experiment using a large array of independent junctions in series. The Hamiltonian is that of a periodically driven pendulum [9] and can be written in the form

$$H = p^2/2 - k\cos(\phi - \lambda\sin t)\,, \tag{1}$$

where time is rescaled to make $\Omega = 1$, and ϕ is related to φ via $\phi(t) = \varphi(t) + (2eI_0/\hbar C\Omega^2)\sin\Omega t$. The momentum $p = \dot\phi = 2eV/\hbar\Omega$ is connected with the total voltage drop V_t across the junction by $V_t = V - (I_0/C\Omega)\cos\Omega t$. The two parameters $k = 2eI_J/\hbar C\Omega^2$ and $\lambda = 2eI_0/\hbar C\Omega^2$ describe the classical system. I_J is the amplitude of the Josephson supercurrent $I_S = I_J \sin\varphi$. The quantized system contains a third dimensionless constant $\not{k} = (2e)^2/\hbar C\Omega$ via the commutator $[p,\phi] = -i\not{k}$. In our units $\not{k}$ plays the role of Planck's constant and corresponds to one quantum of voltage $2e/C$.

The classical Hamiltonian (1) was analyzed in Ref. [10]. The basic phenomenon described by the Hamiltonian is the crossing, twice during each period $T = 2\pi/\Omega$, of the fundamental resonance at which $p = \dot\phi = \lambda\cos t$. The cases of slow ($\lambda/k < 1$) and fast crossing ($\lambda/k \gg 1$, i.e., $I_0 \gg I_J$) may be distinguished [10,11]. We will consider here only the case of fast crossing, where it is possible to neglect the rate of change ($\simeq k$) of the pendulum frequency ($\simeq\sqrt{k}$) compared to the rate of displacement ($\simeq\lambda$) of the fundamental resonance $\dot\phi = \lambda\cos t$ [11]. Outside each crossing the junction does not interact effectively, but each crossing acts like a sudden kick which randomizes φ (mod2π) and changes p by $\Delta p \simeq -\sqrt{2\pi} \times (k/\sqrt{\lambda})\sin(\varphi \pm \pi/4)$, where the sign depends on the direction of the passing of the resonance. The system can therefore be described, in reasonable approximation, by the standard map [10] of the form $\bar p = p - 2\sqrt{\pi}(k/\sqrt{\lambda})\sin\varphi$, $\bar\varphi = \varphi + 2\pi\bar p$ per period $T = 2\pi$, and from this description the classical-chaos border is derived as $k > \sqrt{\pi\lambda}/40$. As a crossing of the resonance occurs only for $|p| < \lambda$, chaos is essentially restricted to this domain. We shall require a large chaotic domain $\lambda \gg 1$, i.e., $I_0 \gg \hbar C\Omega^2/2e$. In the chaotic domain p diffuses with the diffusion constant $D = 2\langle\Delta p^2\rangle/(2\pi/\Omega)$, where $\langle\Delta p^2\rangle$ is the mean square of the change of p per half period. The above rough estimates yield $D \simeq k^2/\lambda$, but a more quantitative estimate $D=(k^2/\lambda)F(4\pi^2 k/\sqrt{\pi\lambda})$ with known function F of order 1 is available [12] if necessary. Thus, classically the normalized voltage fluctuations p spread diffusively over the entire chaotic domain $|p| \leq \lambda$. Hence voltage fluctuations with root mean square $\langle V^2\rangle^{1/2} \simeq I_0/\sqrt{3}C\Omega$ are classically predicted.

Upon quantization, the classical chaotic diffusion, for the standard map, is replaced by dynamical localization via a quantum-mechanical destructive interference of the amplitudes for transitions with large changes of the quantum number of p [13]. This means that the Floquet states ψ_ν in the $|n\rangle$ representation, with $p|n\rangle = \not{k} n|n\rangle$, fall off exponentially like $|\psi_\nu| \sim \exp(-|n-n_\nu|/l)$, where l is the wave-function localization length. It is given in terms of the diffusion constant D_p of p over one period, $D_p = 2\pi D$, by $l = D_p/2\not{k}^2$. An initial state, like the ground state of the undriven Josephson junction, which is localized near $n=0$ and given by a linear superposition of about l Floquet states, first spreads by classical diffusion and then develops into an exponentially localized distribution $|\psi| \sim \exp(-|n|/l_D)$ with a localization length $l_D \simeq 2l$ [14]. Thus, fluctuations of the excess voltage are quantum mechanically reduced to $\langle p^2\rangle^{1/2} \simeq \not{k} l_D/\sqrt{2}$ or $\langle V^2\rangle^{1/2} \simeq [\sqrt{2}\pi\hbar/(2e)^2] I_J^2/I_0$. For fixed external frequency, they decrease inversely proportionally to I_0, contrary to the classical case. This finding provides us with a clear signature of the effect which should be observable if the classical restriction of the fluctuations by the width of the chaotic domain $(\simeq \lambda/\sqrt{3})$ is larger than the quantum restriction due to dynamical localization $(\simeq \not{k} l_D/\sqrt{2})$, i.e., $I_0 > I_J(\sqrt{6}\pi\hbar C\Omega)^{1/2}/2e$.

In order to demonstrate the effect we have performed some numerical simulations. For simplicity we discretized in time and replaced the Hamiltonian equations following from Eq. (1) by a discrete, periodically time-dependent, two-dimensional map, 300 iterations of which correspond to a single period $2\pi/\Omega$, and we used the quantum version of this map. As a consequence of this discretization the spectrum of resonances of the continuous system is repeated on the frequency axis with a period 300Ω. As we restricted ourselves to values $\lambda \leq 130$ the chaotic domain $|p| < \lambda$ does not overlap with its repeated copies and the discretization therefore does not significantly affect our results. The saving of computer time, on the other hand, is enormous. In Fig. 1 we present an example of $\langle \Delta n^2\rangle$ versus time as measured in periods of the external current for $\lambda = 85.0$, $k=15.0$, and $\not{k} = 1.58$. The initial sharp rise of $\langle \Delta n^2\rangle$ from the initial state at $n=0$ by classical diffusion is followed by a localized regime where $\langle \Delta n^2\rangle$ changes due to random beatings of a finite number of Floquet states. We have found cases where these beatings can contain very long periods which may be caused by interferences between nearly degenerate pairs of doubly humped states, e.g., states localized symmetrically around positive and negative values of

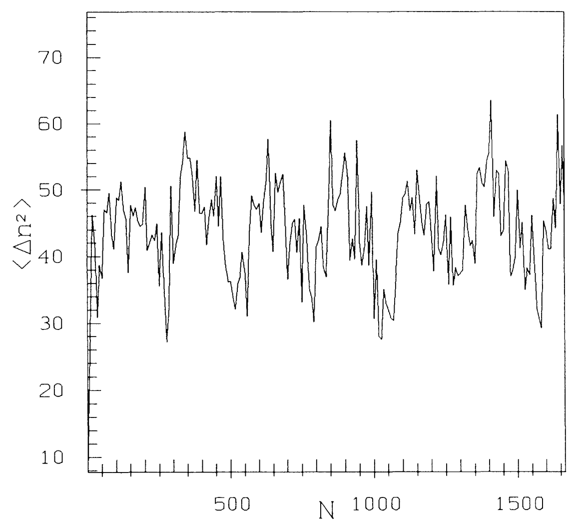

FIG. 1. Mean square of the number of occupied levels of the cosine potential vs the number N of cycles of the external current for $\lambda = 85.0$, $k=15.0$, and $\not{k} = 1.58$.

n, which occur due to the $p \to -p$, $\phi \to -\phi$, $t \to t+\pi/\Omega$ symmetry of the system. In Fig. 2 the time-averaged localized probability distribution over the eigenstates of p which has established itself towards the end of Fig. 1 is shown in a semilogarithmic plot. The dashed lines give the classical border $|n| = \lambda/\not{k}$ and the exponential falloff with the theoretically estimated localization length l_D in good agreement with the numerical result.

In Fig. 3 a numerical example of our experimentally

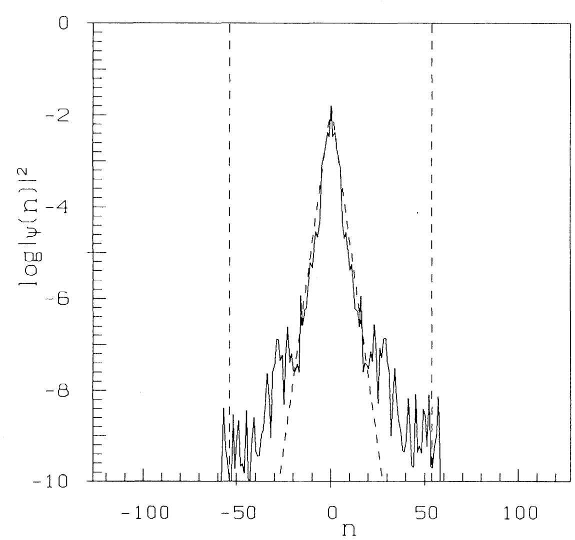

FIG. 2. Logarithm of the time-averaged occupation probability corresponding to Fig. 1. Dashed lines give the border $|n| = \lambda/\not{k}$ of the classical chaotic domain and the exponential falloff with the localization length l_D.

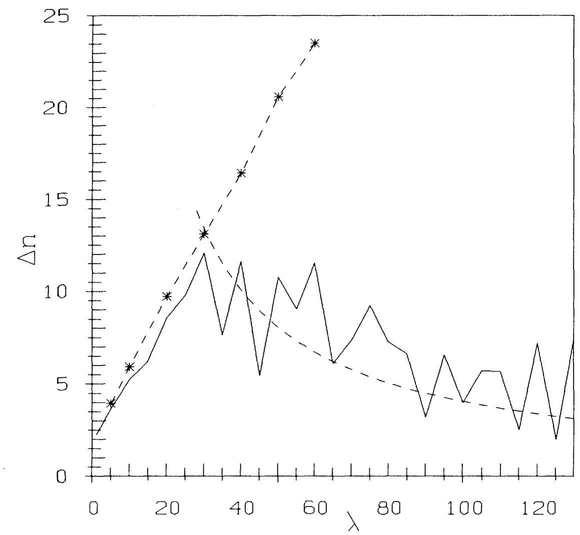

FIG. 3. Root mean square of the number of occupied levels versus the normalized amplitude λ of the driving current for the same values of the parameters $k, \not{k}$ as in Fig. 1. Classical results, indicated by $*$'s, are joined by a dashed line. Another dashed line gives the analytical result for the quantum regime.

accessible prediction is presented—the root-mean-square excess voltage fluctuations, expressed in fluctuations of the quantum number n, versus the amplitude of the external current, expressed in scaled form by λ. The values for k and $\not{k}$ are the same as in Figs. 1 and 2. We also present points of a classical calculation for the same parameter values, joined by a dashed line for convenience, and a dashed curve giving the estimate $\Delta n = l_D/\sqrt{2} = \sqrt{2}\pi k^2/\lambda \not{k}^2$ provided by localization theory. In agreement with the theoretical estimate the transition from classical to quantum behavior takes place at $\lambda_q = (\sqrt{6}\pi k)^{1/2}/\not{k}^{1/2} \simeq 33$. We also tested the theoretically predicted scaling of the localization length with $\not{k}$, according to which $\Delta n \not{k}^2 = \text{const}$, by further numerical calculations for fixed $\lambda = 85.0$, $k = 15.0$, and obtained $10^3 \Delta n \not{k}^2 = 7.3$, 7.0, and 7.6 for $\not{k} = 1.58$, 1.18, and 0.988, respectively, in reasonable agreement.

Let us now summarize the experimental conditions under which the effect shown in Fig. 3 should be observable. If we wish to observe the classical to quantum crossover, we must satisfy the condition of fast crossing of the resonance $I_0 \gg I_J$ already in the classical domain $\lambda/\sqrt{3} < \not{k} l_D/\sqrt{2}$, i.e., for $I_0 < I_J (\sqrt{6}\pi\hbar\Omega C)^{1/2}/2e$. These conditions are compatible only for $\not{k}/\sqrt{6}\pi \ll 1$. For example, for a junction with $C \simeq 10^{-13}$ F, $I_J \simeq 10^{-7}$ A, driven with a frequency $\Omega/2\pi \simeq 10^{10}$ Hz, we have $k \simeq 0.8$, $\not{k} \simeq 0.16$, and $\lambda \simeq 0.8 I_0/I_J$. The amplitude of the driving current should then be varied in the range 0.1–10 μA, the crossover between the classical and quantum domain occurring at about 0.7 μA.

The predicted effect rests entirely on coherence, and therefore dissipation has to be kept sufficiently low. As the localization needs about $l_D/2 = \pi\hbar C I_J^2/I_0(2e)^3$ cycles of the external field to establish itself (cf. Fig. 1) the quality factor of the driven junction (including the leads from the current source and the energy loss due to the measurement performed) must be large compared to $l_D/2$ [15]. For the example given above $l_D \simeq 28$ at the classical quantum crossover and $l_D < 28$ in the quantum regime.

The influence of the impedance due to the leads from the current source and the voltage measurement can be significantly reduced by using an array of N_0 Josephson junctions in series. For example, if P_c is the minimal detectable power, the rate constant R at which energy must be extracted is limited from below by $R > P_c/(N_0 C\langle V^2\rangle/2)$. Estimating $\langle V^2\rangle$ by the localization length and requiring the quality factor $Q = \Omega/2\pi R$ to be larger than $l_D/2$, we obtain the condition $N_0 \gg P_c 4eI_0/\hbar\Omega I_J^2$. For example, in the example given before, for $P_c \simeq 10^{-3}$ erg/sec and $I_0/I_J \simeq 10$, we find $N_0 \gg 10^3$. This value could be reduced by choosing a junction with larger I_J. However, then the localization length, and hence the required quality of the oscillator, increases quadratically with I_J and restrictions on the losses not caused by the measurement, e.g., the losses caused by the leads from the current source, become more severe.

Furthermore, the temperature T has to be sufficiently low such that quasiparticle tunneling is negligible, and such that thermal excitations of p states can be neglected, i.e., $k_B T < C(\hbar\Omega/2e)^2 \not{k}^2/2$, which is the energy spacing of the lowest lying p states. In the example given before this amounts to $T < 0.04$ K.

In summary, we have shown by a simple theory based on the standard map and direct numerical simulation of the Schrödinger equation that dynamical localization appears as a quantum effect in ideal chaotic current-driven Josephson junctions. We have also examined conditions under which this effect might be observable experimentally in an array of real junctions.

The authors would like to thank J. Keymer for a useful discussion. One of us (R.G.) is grateful to M. Devoret for pointing out the importance of the impedance of the leads and another one (D.S.) to S. Fishman and D. Iliescu for stimulating discussions. This work was carried out during the workshop "Quantum Aspects of Nonlinear Systems," Universität Gesamthochschule Essen, funded by the European Science Foundation. Further support of this work came from the Deutsche Forschungsgemeinschaft through its Sonderforschungsbereich No. 237.

[a]Permanent address: Institute of Nuclear Physics, 630090 Novosibirsk, U.S.S.R.

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