Three-Wave Parametric Amplification in Time-Dependent Media, with Application to Stimulated Brillouin Scattering

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A method is presented for evaluating the amplification of a three-wave parametric instability in a medium having a slow, locally linear variation in both time and space, and is applied to the specific case of stimulated Brillouin scattering (SBS) in a plasma undergoing an isothermal rarefaction. Time dependence can substantially alter the gain of such a parametric amplifier. For SBS in a subsonic rarefaction, inclusion of time dependence eliminates the absolute instability.

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Because optical media can support so many different waves, any intense (pump) wave in a plasma or other optical medium can generally decay into other waves. In three-wave parametric instabilities, the pump resonantly decays into two daughter waves, with the three waves having frequencies and wave vectors $\omega_0, \omega_1, \omega_2$ and $\mathbf{k}_0, \mathbf{k}_1, \mathbf{k}_2$, respectively. If the medium is inhomogeneous in space and time, then the decay is only resonant at the spatiotemporal matching surface where $\omega_0 = \omega_1 + \omega_2$ and $\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$. Nonetheless, amplification can occur as the waves transit the matching surface and absolute instability is possible under some conditions [1]. There are several important examples of such instabilities, notably including stimulated Brillouin scattering (SBS) [2]. In SBS in a plasma an electromagnetic (em) pump wave decays into a scattered em wave and an ion-acoustic wave. This instability is of particular practical importance for laser fusion [3], as it acts to reduce the efficiency of the fusion implosion. Other similar instabilities are important to laser-plasma interactions [3], magnetic-fusion plasmas [4,5], and other plasma systems [6].

Most linear analysis of unstable, three-wave decay has followed the formal approach developed by Rosenbluth [1] for time-independent media. Such work has shown that absolute instability is *only* possible when the decay waves have opposed group velocities. However, even then the instability is *nevertheless* convective in media having a uniform pump and a precisely linear spatial gradient of the mismatch of the wave numbers of the three waves. Many specific cases have since been worked out [7], but always on the assumption that the profiles of the plasma parameters do not vary in time. Nonetheless, experiments often produce time-dependent media—plasmas expand, filaments form, and ions ionize further.

The appropriate abstract generalization of previous results to (slowly) time-dependent systems with linear gradients in space and time was recently given in a Comment by Williams [8]. The present work is the first discussion and application of this theory. A method suitable for the evaluation of the gain of any three-wave process is presented. This method is applied to SBS backscatter in an isothermal rarefaction, including the case of a subsonic rarefaction that applies to current laser-plasma experiments using visible and UV lasers. As we shall see, time dependence can alter the gain and the spectrum of three-wave instabilities. For clarity in the following, we discuss only plasmas with one-dimensional spatial variations.

Rosenbluth found [1] that three-wave instabilities are inherently convective in media having a linear spatial gradient of the wave-number mismatch, $\kappa = k_1 + k_2 - k_0 = \kappa' x$. The amplification is e^G, where

$$G = 2\pi \gamma_0^2 / |\kappa' v_1 v_2| . \quad (1)$$

Here the growth rate of the instability in a homogeneous medium is γ_0, and the group velocities of the two decay waves are v_1 and v_2. (For clarity, we restrict the present discussion to systems in which a normally incident pump irradiates a one-dimensional medium.) Under some circumstances, such as that of a parabolic spatial profile of κ, κ' vanishes locally, and the instability becomes absolute. In subsequent work [7,9], it has been seen that almost any deviation of the spatial properties of the plasma system from the conditions assumed to derive Eq. (1) can restore absolute instability to backscattering when the amplification exceeds some threshold.

Meanwhile, the problem of mode conversion in (slowly) time- and spatially varying systems was considered by Friedland, Goldner, and Kaufman [10]. They obtained an equation, similar to Eq. (1), for the transmission of an initial mode through a mode-conversion layer in such a system. Williams [8] connected these two results to show that, for parametric amplification in a time- and spatially varying medium, Eq. (1) must be replaced by

$$G = 2\pi \gamma_0^2 / |B| . \quad (2)$$

The denominator B can be expressed in a form that is easily connected with the denominator in Eq. (1), as follows:

$$B = [\partial_t^2 + (v_1 + v_2)\partial_t \partial_x + (v_1 v_2)\partial_x^2]\phi . \quad (3)$$

Here the group velocities of the waves are denoted by v_i, and ϕ is the mismatch phase, expressed as the difference between the phases of the pump and the two decay waves (Ψ_0,Ψ_1,Ψ_2): $\phi=\Psi_1+\Psi_2-\Psi_0$. As usual, for each of the waves the local wave number k_i is $\partial_x\Psi_i$ and the local frequency ω_i is $-\partial_t\Psi_i$. Thus, the wave-number mismatch is given by $\kappa=\partial_x\phi$, and B becomes $\kappa' v_1 v_2$ for time-independent media.

However, it is clear from the original formulation [8] that it is not necessary to completely evaluate the phase of each wave in order to evaluate B. We can identify the convective derivative of the frequency or wave number of a wave packet of wave i as $\dot\omega_i=(\partial_t+v_i\partial_x)\omega_i$ and $\dot k_i=(\partial_t+v_i\partial_x)k_i$, respectively. These derivatives are readily obtained from the dispersion functions [11]. Specifically, $\dot\omega_i=-\partial_t D_i/\partial_\omega D_i$ and $\dot k_i=\partial_x D_i/\partial_\omega D_i$, in which D_i is the dispersion function of wave i [10,11]. Some algebra then shows that (3) can be written as

$$B=B_1+B_2-B_0\,, \tag{4}$$

in which $B_1=-\dot\omega_1+v_2\dot k_1$, $B_2=-\dot\omega_2+v_1\dot k_2$, where remarkably the contribution due to wave 1 (B_1) depends on the group velocity of wave 2 and vice versa. In addition, one can write B_0 as

$$B_0=\frac{v_1}{v_0}\left[-\dot\omega_0\left(1+\frac{v_2}{v_1}+\frac{v_2}{v_0}\right)+v_2\dot k_0\right] - \partial_t\omega_0\left(1-\frac{v_1}{v_0}\right)\left(1-\frac{v_2}{v_0}\right) \tag{5}$$

to allow simplification in the common case where $v_2 \ll v_1 \approx v_0$. Given a model of the time and spatial dependence of the plasma parameters, it is straightforward to evaluate the convective derivatives here. However, unlike the mode-conversion problem there remains a term proportional to $\partial_t\omega_0$ which is determined by the global variation of the medium and the external pump source. This arises because, in the parametric approximation, the pump field is externally imposed and is effectively a property of the medium. Variation of the pump frequency at a fixed position in the plasma can arise from an external chirp in the vacuum pump frequency and/or from a propagational Doppler shift deriving from a time-varying optical path [12].

A study of Eqs. (4) and (5) suggests that effects of time dependence may be significant if the group velocity of either daughter wave is not large compared to the characteristic velocity of the plasma defined as the ratio of its characteristic length and time scales. This is the case for SBS backscatter from an expanding, laser-produced plasma, which we now consider, where the characteristic velocity is of order of the ion sound speed c_s. SBS has been discussed for various circumstances, including that of flowing, supersonic, inhomogeneous but time-independent expansions [13–15]. For the sake of definiteness, we will treat a plasma undergoing a planar, isothermal, self-similar rarefaction [16] This model is reasonable for experiments in which the laser spot is larger than the product of c_s and laser-pulse duration. Let the electron density be given by $n=n_s e^{\xi}$ and the flow velocity by $u=c_s(\xi-1)$, with n_s being the density at the sonic point and with the similarity variable relating space (x) and time (t) by $\xi=x/c_s t$. The expanding plasma extends from $\xi=1$ to $-\infty$ and attaches to the undisturbed plasma at $\xi=1$. We take waves 1 and 2 to be the electromagnetic, scattered-light wave and the ion-acoustic wave, and designate them as em and ia, respectively. Table I shows the values of $\dot\omega_{\rm em}$, $\dot\omega_{\rm ia}$, $\dot k_{\rm em}$, and $\dot k_{\rm ia}$ implied by this plasma model, and substituting 0 for em in the table gives $\dot\omega_0$ and $\dot k_0$. The electron Debye length is λ_D. Note the relativistic effect that the dispersion function of the electromagnetic waves properly *does not* include a Doppler shift.

We compute the terms B_0, $B_{\rm em}$, and $B_{\rm ia}$ to lowest order in c_s/c, keeping $k_{\rm ia}\lambda_D$ finite, with results also shown in Table I. The contribution from $\partial_t\omega_0$, which as discussed above includes the effect of the global variation of the medium on the pump frequency, is first order in c_s/c and

TABLE I. Ray parameters for SBS in an isothermal rarefaction.

Scattered-light wave

$$D_{\rm em}(\omega_{\rm em},k_{\rm em},x,t)=\omega_{\rm em}^2-\omega_{pe}^2-c^2k_{\rm em}^2$$

$$v_{\rm em}=-\partial_k D_{\rm em}/\partial_\omega D_{\rm em}=c^2k_{\rm em}/\omega_{\rm em}$$

$$\dot\omega_{\rm em}=-\partial_t D_{\rm em}/\partial_\omega D_{\rm em}=\frac{-\omega_{pe}^2\xi}{2\omega_{\rm em}t}$$

$$\dot k_{\rm em}=\partial_x D_{\rm em}/\partial_\omega D_{\rm em}=\frac{-\omega_{pe}^2\xi}{2\omega_{\rm em}x}$$

$$B_{\rm em}=-B_o=\frac{\omega_o}{t}\frac{1}{2}\frac{n}{n_c}\left\{1-\left(1+k_{ia}^2\lambda_D^2\right)^{-3/2}\right\}$$

Ion-acoustic wave

$$D_{ia}(\omega_{ia},k_{ia},x,t)=\omega_{ia}-k_{ia}u-\left\{k_{ia}c_s\left(1+k_{ia}^2\lambda_D^2\right)^{-1/2}\right\}$$

$$v_{ia}=-\partial_k D_{ia}/\partial_\omega D_{ia}=u+c_s\left(1+k_{ia}^2\lambda_D^2\right)^{-3/2}$$

$$\dot\omega_{ia}=\frac{-\partial_t D_{ia}}{\partial_\omega D_{ia}}=\frac{-k_{ia}c_s\xi}{t}\left\{1+\tfrac{1}{2}k_{ia}^2\lambda_D^2\left(1+k_{ia}^2\lambda_D^2\right)^{-3/2}\right\}$$

$$\dot k_{ia}=\frac{\partial_x D_{ia}}{\partial_\omega D_{ia}}=\frac{-k_{ia}}{t}\left\{1+\tfrac{1}{2}k_{ia}^2\lambda_D^2\left(1+k_{ia}^2\lambda_D^2\right)^{-3/2}\right\}$$

$$B_{ia}=\frac{2\omega_o}{t}\left(1-\frac{n}{n_c}\right)\left\{1+\tfrac{1}{2}k_{ia}^2\lambda_D^2\left(1+k_{ia}^2\lambda_D^2\right)^{-3/2}\right\}$$

is unimportant to lowest order. B is then given by

$$B=\frac{\omega_0}{t}\left[1+\left(1-\frac{n}{n_c}\right)-\frac{n}{n_c}[(1+k_{\rm ia}^2\lambda_D^2)^{-1/2}]+\frac{k_{\rm ia}^2\lambda_D^2}{(1+k_{\rm ia}^2\lambda_D^2)^{3/2}}\right], \tag{6}$$

where n_c is the critical density of the pump wave. For comparison, if we had neglected the contribution of the time derivatives to B, keeping only the usual $\kappa' v_{\rm ia} v_{\rm em}$ we would have obtained

$$\kappa' v_{\rm ia} v_{\rm em}=\frac{\omega_0}{t}\left\{1+\left(1-\frac{n}{n_c}\right)-\frac{n}{n_c}\left[\ln\left(\frac{n}{n_s}\right)+(1+k_{\rm ia}^2\lambda_D^2)^{-1/2}\right]+\frac{k_{\rm ia}^2\lambda_D^2}{(1+k_{ia}^2\lambda_D^2)^{3/2}}\right\}. \tag{7}$$

The use of Eq. (7) might be a reasonable approximation for plasmas produced by small laser spots of diameter D that develop a spherically diverging, stationary flow during the laser pulse, if one substituted $t \sim D/c_s$. However, this would only be an approximation as the model used here does not accurately describe the profiles for stationary spherical flow. Note also that the WKB approximation breaks down as t approaches zero, but that the tendency for B to increase and for the gain to decrease is physically correct.

Figure 1 shows the gain factor G evaluated from Eqs. (6) and (7) for two cases of interest, with γ_0 from Kruer [17]. Previous theory of SBS has considered the case of supersonic flow in an inhomogeneous plasma with $n_s = n_c$ [13]. Such a flow profile may plausibly be present in plasmas produced by infrared lasers, and Fig. 1(a) shows results for this case. The impact of time dependence is to increase G by a factor of up to $\frac{3}{2}$ near the critical surface. In contrast, plasmas produced by moderate-intensity, visible or UV light have $n_s < n_c$. Figure 1(b) shows results for this case. For steady plasmas, SBS becomes absolutely unstable in an inhomogeneous plasma with inhomogeneous, subsonic flow. The effect of time dependence is to eliminate the absolute instability, through the dephasing of the light waves in time.

One can apply the method just demonstrated to evaluate the gain for any other three-wave parametric instability in any time-dependent medium. For example, similar results to those obtained for SBS are obtained for the ion-acoustic decay instability [18]. In addition, time-dependent effects can in principle cause B to vanish when it otherwise would not. This requires more rapid time variation, as may be produced for example by the fast ionization of an inhomogeneous plasma by a short laser pulse [19]. One should note that the significance of divergence of the gain G, arising from a zero of B, is less clear cut in the time-dependent case than otherwise. It is then necessary to keep higher-order terms in the Taylor expansion of the phase mismatch. One anticipates that the convective gain will turn out to be finite, but there is the additional possibility of an instability of an absolute nature, that is, of the existence of a temporally growing mode [20]. However, the growth is not time asymptotic as in the stationary case because in general the Taylor expansion in time will cease to be valid.

In summary, any time dependence of the medium should be considered when evaluating the amplification of three-wave parametric instabilities. A method for doing so has been presented here, and the specific case of SBS backscatter in an isothermal rarefaction has been worked out as an example. Further studies of instabilities in

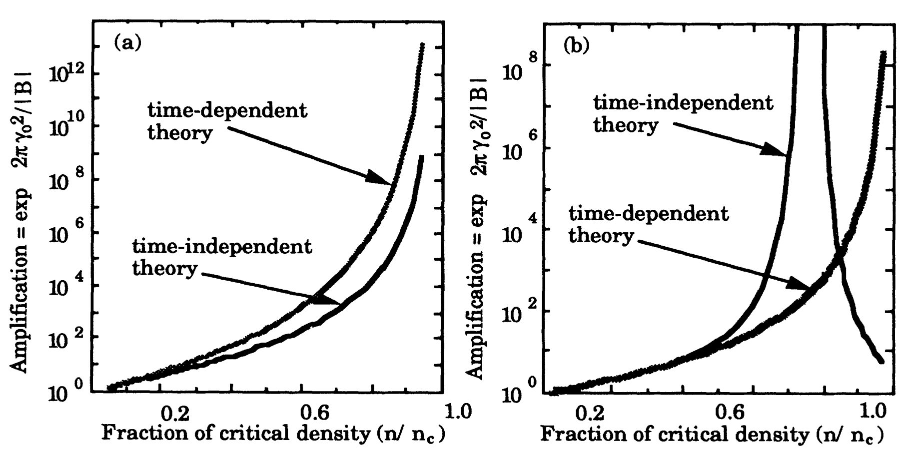

FIG. 1. Amplification of SBS backscatter vs density for an isothermal rarefaction from a CH target after 0.5 ns. (a) Supersonic case ($n_s = n_c$), with electron temperature $T_e = 2$ keV, pump wavelength $\lambda_0 = 1.05$ μm, and pump intensity $I_L = 10^{14}$ W/cm^2. (b) Subsonic case ($n_s = 0.4 n_c$), with $T_e = 1$ keV, $\lambda_0 = 0.351$ μm, and $I_L = 10^{14}$ W/cm^2.

time-dependent media are clearly warranted. Such work should include analysis of the growth rate of any absolute instabilities, determination of the angular distribution of the scattering, evaluation of instability behavior in systems with nonlinear profiles and in three dimensions, consideration of the effects of pump bandwidth and damping, and scaling experiments to isolate these effects.

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[16] Kruer (Ref. [3]), p. 116.

[17] Kruer (Ref. [3]), p. 90. In our notation the formula is $\gamma_0 = (k_{ia} v_0/4)\{\omega_{pi}/(\omega_{em} k_{ia} c_s)^{1/2}\}$.

[18] Kruer (Ref. [3]), Chap. 6.

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[20] The present treatment based on Eq. (2) ignores the case in which some source of external feedback drives an amplifier absolutely unstable. In such cases the effect of time dependence on the amplifier and on the feedback must be considered.