Squeezing and Anomalous Moments in Resonance Fluorescence

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A scheme for measuring squeezing in resonance fluorescence from a single trapped and cooled ion is proposed, which is based on the observation of photon pair correlations after beating the fluorescence with a local oscillator. In addition to squeezing, anomalous moments and sub-Poissonian photon statistics of the fluorescence contribute to the nonclassical behavior of the light in homodyne detection.

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Resonance fluorescence experiments turned out to play a vital role in testing fundamental predictions of modern quantum physics. Typical examples are the observations of photon antibunching [1,2], sub-Poissonian statistics [3], and quantum jumps [4]. Moreover, according to theory squeezing should appear in the fluorescence of a single atom [5] or a regularly arranged system of atoms [6]. While measurements of antibunching and sub-Poissonian statistics confirmed the theoretical predictions of these nonclassical effects of the fluorescence light, the observation of phase-sensitive squeezing is the most fundamental open problem with respect to the quantum properties of the resonance fluorescence radiation.

Two facts are expected to complicate an observation of squeezing in resonance fluorescence. First, the atomic motion produces phase shifts which destroy the squeezing effect. In order to overcome this problem the atom must be localized within a region small compared with the optical wavelength of the light. Experiments with a single trapped and cooled ion are a possible way to realize such a localization. Second, one has to find an observation scheme which yields a significant effect due to squeezing of the fluorescence. Based on a quantitative analysis of the sub-Poissonian nature of the light in a homodyne measurement, Mandel has shown that squeezing gives rise to effects which are even smaller than the small sub-Poissonian noise reduction directly observed in the fluorescence light [7]. Such a scheme, in which the detection efficiency (including the collection of the fluorescence) limits the observable effect, seems to be incapable of detecting squeezing.

In the present Letter we propose the detection of squeezing of the resonance fluorescence from a single trapped and cooled ion based on the observation of the photon pair correlations (at equal times) in a homodyne measurement. The detection efficiency does not limit the observable squeezing in such a scheme. Moreover, anomalous moments [8], which are phase sensitive and cannot be measured directly in photodetection of the fluorescence light, may be observed. Squeezing, anomalous moments, and sub-Poissonian statistics of the fluorescence light may contribute to the nonclassical photon pair correlations of the superposition light after beating the fluorescence with the local oscillator. A separation of all these effects is possible in view of their different dependences on the local oscillator phase. We will consider the situation for the fluorescence of a coherently driven two-level atom undergoing radiative damping. In the case of multilevel systems, smaller squeezing effects are expected to occur. Moreover, we deal with the stationary fluorescence; a nonstationary experiment seems to be more complicated.

In the case of a short-time measurement, a given light field yields a sub-Poissonian counting statistics when the joint probability for the simultaneous detection of two photons is less than the product of the independent counting events. Based on Glauber's detection theory [9] this condition can be formulated as follows:

$$\Gamma^{22} < 0\,, \tag{1}$$

where

$$\Gamma^{22} = \mu^2\{\langle [E^{(-)}(\mathbf{r},t)]^2 [E^{(+)}(\mathbf{r},t)]^2\rangle - \langle E^{(-)}(\mathbf{r},t)E^{(+)}(\mathbf{r},t)\rangle^2\}\,, \tag{2}$$

$E^{(-)}$ ($E^{(+)}$) is the negative (positive) frequency part of the operator of the electric-field strength, and μ is the product of the detection efficiency and the measurement time. Beating the fluorescence field E_{fl} by means of a beam splitter with the local oscillator field E_{lo} yields

$$E^{(-)}(\mathbf{r},t) = (E_{\text{fl}}^{(-)} + E_{\text{lo}}^{(-)})\exp\{i(\omega_1 t - \mathbf{k}\cdot\mathbf{r})\}/\sqrt{2}\,, \tag{3}$$

with ω_1 being the frequency of the pump laser (from which the local oscillator is derived) [10]. Inserting Eq. (3) into Eq. (2) leads to

$$\Gamma^{22} = \Gamma_0^{22} + \Gamma_1^{22} + \Gamma_2^{22}\,, \tag{4}$$

where the index i ($i = 0,1,2$) in Eq. (4) denotes the ith-order contribution with respect to the local oscillator field. Introducing the amplitude $\hat{E}_{\rm lo}$ and the phase $\varphi_{\rm lo}$ of the local oscillator, which is assumed to be in a coherent state, we derive

$$\Gamma_2^{22}=(\mu^2/4)\hat{E}_{\rm lo}^2\langle:[\Delta E_{\rm fl}(\varphi_{\rm lo})]^2:\rangle\,, \tag{5}$$

$$\Gamma_1^{22}=(\mu^2/2)\hat{E}_{\rm lo}\{\exp(-i\varphi_{\rm lo})[\langle E_{\rm fl}^{(-)}E_{\rm fl}^{(+)}E_{\rm fl}^{(+)}\rangle-\langle E_{\rm fl}^{(-)}E_{\rm fl}^{(+)}\rangle\langle E_{\rm fl}^{(+)}\rangle]+\text{c.c.}\}\,, \tag{6}$$

and Γ_0^{22} is obtained from Eq. (2) by substituting $E_{\rm fl}/\sqrt{2}$ for E. Any of these terms may contribute to negative values according to Eq. (1), that is to a sub-Poissonian statistics of the superposition light. The corresponding contribution of Γ_2^{22} is related to squeezing of the fluorescence, viz.,

$$\langle:[\Delta E_{\rm fl}(\varphi_{\rm lo})]^2:\rangle<0\,, \tag{7}$$

provided that the phase of the local oscillator is appropriately chosen. In the normally ordered variance of the fluorescence field [Eqs. (5) and (7)], the local oscillator phase appears instead of the phase $\omega_l t-\mathbf{k}\cdot\mathbf{r}$. Sub-Poissonian statistics of the fluorescence with negative values of Γ_0^{22} is a well-known phenomenon measured some years ago [3].

So far our results are closely related to those of Mandel [7]. In the following, however, our study differs from that of Ref. [7] in two important points: (i) Instead of dealing with a quantitative measure for the sub-Poissonian statistics (Mandel's Q parameter), the relative photon pair correlations are analyzed. In this manner we overcome the obstacle that the observable squeezing is limited by both the detection and the collection efficiencies. (ii) We study the situation in which the intensities of the local oscillator and the fluorescence may be of the same order of magnitude. It turns out that in this case the maximum effect due to squeezing of the fluorescence is observed. Additionally, anomalous moments of the fluorescence appear under such conditions. To our knowledge nonclassical contributions arising from anomalous moments of the fluorescence light have not been studied yet.

Usually in squeezing measurements the local oscillator is strong compared with the squeezed light under study [11]. In such a case Γ_2^{22} is the leading contribution to Γ^{22}, cf. Eqs. (4)-(6). Let us first deal with this situation. In order to disregard the contributions of zeroth and first order in the local oscillator field the ratio of the local oscillator intensity to the fluorescence intensity $I_{\rm lo}/I_{\rm fl}$ should be at least 10^2. In this case we find approximately

$$\Gamma^{22}\approx(\mu^2/4)I_{\rm lo}\langle:[\Delta E_{\rm fl}(\varphi_{\rm lo})]^2:\rangle$$
$$=(\mu^2/2)I_{\rm lo}[I_{\rm fl,inc}-I_{\rm fl,coh}\cos(2\varphi)]\,, \tag{8}$$

where $I_{\rm fl}=\langle E_{\rm fl}^{(-)}E_{\rm fl}^{(+)}\rangle$ and $I_{\rm fl,coh}=|\langle E^{(-)}\rangle|^2$, respectively, are the fluorescence intensity and its coherent part; the incoherent part is given by $I_{\rm fl,inc}=I_{\rm fl}-I_{\rm fl,coh}$. The phase φ in Eq. (8) differs from the local oscillator phase by the fact that it includes the phase shifts produced by the atomic polarization. To get some insight into the magnitude of the measurable squeezing we have to compare the value of Γ^{22} according to Eq. (8) with the signal level from which this effect must be derived. A suitable measure for this level is the uncorrelated (product) counting rate $R_{\rm unc}$ of the two detectors, which is approximately given by $\mu^2I_{\rm lo}^2/4$. Thus the following observable effect occurs:

$$\Gamma^{22}/R_{\rm unc}=2[I_{\rm fl,inc}-I_{\rm fl,coh}\cos(2\varphi)]/I_{\rm lo}\,. \tag{9}$$

Note that the quantity $\Gamma^{22}/R_{\rm unc}$ may be regarded as the relative photon pair correlation; negative values indicate nonclassical properties of the light under study. The maximum effect is obtained in the limit of weak pump field (in this case the incoherent part of the fluorescence is negligibly small). Adopting $I_{\rm lo}/I_{\rm fl}=100$ we arrive at

$$\Gamma^{22}/R_{\rm unc}=-0.02\cos(2\varphi)\,, \tag{10}$$

that is, the squeezing of the fluorescence leads to a relative effect of 2% in the homodyne measurement. Although this effect is rather small it is insensitive to the collection efficiency of the fluorescence light. It should be noted that the condition for maximum observable squeezing in the proposed observational scheme does not agree with the maximum negative value of the normally ordered field variance of the fluorescence light which appears for somewhat stronger pump fields [5,6].

To get larger effects due to squeezing of the fluorescence we now study the more general case with the local oscillator intensity being of the same order of magnitude as the fluorescence intensity. Under such conditions all the terms in Eq. (4) must be taken into account. Let us consider the weak-field limit in which the largest effects are expected to occur. We readily derive

$$\Gamma_2^{22}=-(\mu^2/2)I_{\rm lo}I_{\rm fl}\cos(2\varphi)\,, \tag{11}$$

$$\Gamma_1^{22}=-\mu^2(I_{\rm lo}I_{\rm fl})^{1/2}I_{\rm fl}\cos(\varphi)\,, \tag{12}$$

$$\Gamma_0^{22}=-(\mu^2/4)I_{\rm fl}^2\,. \tag{13}$$

The uncorrelated counting rate is now given by

$$R_{\rm unc}=(\mu^2/4)[I_{\rm fl}+I_{\rm lo}+2(I_{\rm lo}I_{\rm fl})^{1/2}\cos(\varphi)]^2\,. \tag{14}$$

Choosing $\varphi=2\pi n$ and $I_{\rm lo}=I_{\rm fl}=I$ we obtain the following contributions to nonclassical (negative) photon pair correlations of the superimposed light:

$$\Gamma_2^{22}/R_{\rm unc}=-0.125\,, \tag{15}$$

$$\Gamma_1^{22}/R_{\rm unc}=-0.25\,, \tag{16}$$

$$\Gamma_0^{22}/R_{\rm unc}=-0.0625\,. \tag{17}$$

We find that squeezing of the fluorescence leads to an effect of 12.5%, which is encouraging for performing an experiment. Moreover, the anomalous moments contribute an effect of 25%. Provided that we are mainly interested in this phenomenon, an effect of about 42% may be caused by the anomalous moments when the ratio of the fluorescence intensity to the local oscillator intensity is increased. The three contributions to Γ^{22} are easily separated from each other because of the different dependences on the local oscillator phase; see Eqs. (11)–(13). Thus one may record Γ^{22} as a function of the phase and analyze the result with respect to the various phase-sensitive contributions.

Summarizing the results, we have proposed a scheme which allows the measurement of both squeezing and anomalous moments in resonance fluorescence. The method is not limited by the detection and collection efficiencies. Squeezing of the fluorescence may lead to a relative effect of 12.5% in the homodyne experiment under study. Moreover, under such conditions anomalous moments of the fluorescence contribute 25% to the nonclassical photon pair correlations of the superimposed light. The separation of these effects can be based on the phase sensitivity of the measured signal.

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[8] Usually, in photodetection normally ordered moments with the same number of annihilation and creation operators (or positive and negative frequency parts of field operators) are observed, cf. Ref. [9]. In our context anomalous moments are expectation values containing different numbers of both kinds of operators.

[9] R. J. Glauber, *Quantum Optics and Electronics* (Gordon and Breach, New York, 1965); P. L. Kelley and W. H. Kleiner, Phys. Rev. **136**, A316 (1964).

[10] In our observational scheme the local oscillator is derived from the laser beam pumping the atom. After introducing a variable phase shift the local oscillator is combined by means of a beam splitter with the fluorescence light. The superposition light in one output port of this beam splitter is analyzed in a photon pair correlation measurement. In this manner the statistical properties of the light in the two channels of the correlation device are identical.

[11] A collection of papers on squeezing can be found in the October issue of J. Opt. Soc. Am. B **4** (1987).