Reconciliation of Neutron-Star Masses and Binding of the Λ in Hypernuclei

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By relating scalar and vector couplings of the hyperons to the inferred empirical binding of the Λ hyperon in saturated nuclear matter, we obtain compatibility of this binding energy with neutron-star masses. Using the observational constraint on the lower bound on the maximum neutron-star mass and the upper bound of the couplings that are compatible with hypernuclear levels, we place bounds on the reduction in neutron-star mass that hyperons produce. For the best current estimate of nuclear-matter properties, the reduction in mass due to conversion of nucleons to hyperons is $(0.71\pm0.15)M_\odot$. Neutrons comprise a slight majority population in neutron stars with mass $\sim1.5M_\odot$.

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It has long been recognized that hyperons will be present in the dense cores of neutron stars, unless preempted by the conversion to quark matter of that density domain which they would otherwise populate [1–5]. The conversion of some nucleons to hyperons will occur through the weak interaction, and is energetically favorable in dense matter because the Fermi energy of the highest-lying nucleons will otherwise exceed the mass of hyperons, plus any associated interaction energy and the mass of leptons required by charge conservation. Since the conversion relieves the Fermi pressure exerted by the baryons, the equation of state is softened and the maximum mass of the neutron-star sequence associated with the equation of state is lowered. The neglect of hyperons therefore always leads to an overestimate of the maximum possible mass. However, the magnitude of the reduction has not hitherto been reconciled with any empirical observation, leading to great uncertainty in the theoretical maximum mass [6,7]. This uncertainty was the particular focus of a recent Letter [7]. It is the purpose of this Letter to show how the binding of the Λ hyperon in nuclear matter and hypernuclear levels can be used to resolve the uncertainty. For later reference we note that the most accurately measured mass (but not necessarily the maximum possible mass) is that of PSR 1913+16 with $M/M_\odot=1.442\pm0.003$ [8]. There is another relevant measurement, that of 4U0900-40 with $M/M_\odot=1.85\pm0.3$ [9]. So it appears that the lower bound on the maximum neutron-star mass is $\sim1.5M_\odot$.

Recent work on the role of hyperons in neutron stars has been done in the framework of relativistic nuclear field theory [4–7,10], the same framework in which analysis of hypernuclear levels had been performed [11–14]. The Lagrangian of the theory is

$$\mathcal{L}=\sum_B\bar\psi_B(i\gamma_\mu\partial^\mu-m_B+g_{\sigma B}\sigma-g_{\omega B}\gamma_\mu\omega^\mu-\tfrac12 g_{\rho B}\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu)\psi_B+\tfrac12(\partial_\mu\sigma\partial^\mu\sigma-m_\sigma^2\sigma^2)-\tfrac14\omega_{\mu\nu}\omega^{\mu\nu}+\tfrac12 m_\omega^2\omega_\mu\omega^\mu$$
$$-\tfrac14\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu}+\tfrac12 m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu-\tfrac13 bm_n(g_\sigma\sigma)^3-\tfrac14 c(g_\sigma\sigma)^4+\sum_{e^-,\mu^-}\bar\psi_\lambda(i\gamma_\mu\partial^\mu-m_\lambda)\psi_\lambda\,. \qquad (1)$$

We regard it as an effective theory to be solved at the mean-field level, and with coupling constants adjusted, as described below, to nuclear-matter properties. The baryons B are coupled to the σ,ω,ρ mesons. The sum on B is over all the charge states of the lowest baryon octet $(p,n,\Lambda,\Sigma^+,\Sigma^-,\Sigma^0,\Xi^-,\Xi^0)$ as well as the Δ quartet. However, the latter are not populated up to the highest density in neutron stars, nor are any other baryon states, save those of the lowest octet for reasons given elsewhere [5]. The last term represents the free-lepton Lagrangians. How the theory can be solved in the mean-field approximation for the ground state of charge-neutral matter in general beta equilibrium (neutron-star matter) is described fully in Ref. [5]. There are five constants here that are determined by the properties of nuclear matter, three that determine the nucleon couplings to the scalar, vector, and vector, isovector mesons, $g_\sigma/m_\sigma, g_\omega/m_\omega, g_\rho/m_\rho$, and two that determine the scalar self-interactions, b,c. The nuclear properties that define their values are the saturation values of the binding, baryon density, symmetry energy coefficient, compression modulus, and nucleon effective mass. Exactly the same choices for their values were made in Refs. [6,10]. In addition, in this paper we examine the effects of exploring their likely range. The hyperon couplings are not relevant to the ground-state properties of nuclear matter, but information about them can be gathered from levels in hypernuclei. We shall assume that all hyperons in the octet have the same coupling as the Λ. They are expressed as a ratio to the above-mentioned nucleon couplings,

$$x_\sigma = g_{H\sigma}/g_\sigma, \quad x_\omega = g_{H\omega}/g_\omega, \quad x_\rho = g_{H\rho}/g_\rho . \tag{2}$$

In most analyses of hypernuclei, the first two ratios are taken to be equal, and the third is undetermined by Λ hypernuclei, since the Λ has zero isospin. With the constraint $x_\sigma = x_\omega = x$, it is found that x *must* be small, in the range 0.21–0.4 by various authors [11–14]. These values lead to maximum neutron-star masses that are too small [6,7]. For $x = 0.21$ the maximum-mass star is $1.2M_\odot$ and for $x = 0.4$ it is $1.35M_\odot$, if the nuclear matter value of the compression modulus is taken to be $K = 300$ MeV. The masses are smaller if K is taken smaller. When the hypernuclear levels are analyzed without the constraint of $x_\sigma = x_\omega$ there are large correlation errors and their determination is highly uncertain, ±100% [13] and ±55% [14].

What we do to resolve the discrepancy between data on hypernuclei and neutron-star masses in this work is to employ the accurately extrapolated value of the Λ hyperon binding in saturated nuclear matter [15]. This procedure avoids the idiosyncrasies of individual nuclear levels in favor of bulk nuclear matter, which is, after all, the material of which neutron stars are made (in its charge-neutron form, of course). We derive now an expression for this binding in our model. From the Weisskopf [16] relation at saturation between the Fermi energy and the energy per nucleon of a self-bound system, $e_F = (\epsilon/\rho)_0$, which is a special case of the Hugenholtz–Van Hove theorem [17], we obtain for the binding energy of the lowest Λ level in nuclear matter,

$$(B/A)_\Lambda = x_\omega V + m^*_\Lambda - m_\Lambda = x_\omega V - x_\sigma S , \tag{3}$$

where $S = g_\sigma \sigma$ and $V = g_\omega \omega_0$ are the values of the scalar and vector field strengths at saturation. The Fermi energy of the lowest Λ level is the $k = 0$ value of the Dirac eigenvalue of the theory, $e_\Lambda(k) = g_{\omega\Lambda}\omega_0 + (k^2 + m_\Lambda^{*2})^{1/2}$. Equation (3) yields a continuous ambiguity in the *pair* of values x_σ, x_ω, each pair of which yields the same Λ binding of −28 MeV. Combined with neutron-star masses, the ambiguity is bounded from below by $M_{\max} \sim 1.5M_\odot$. Combined with the reasonable assumption that the hyperon coupling constants are less than those of the nucleon, based both on the observation that the lowest s-state nucleon is bound by approximately twice as much as the Λ hyperon, and also on the basis of quark counting [18], the ambiguity is bounded from above to be less than $x_\sigma < 0.9$ (see Table I). It is even more stringently bounded from above from the fit to hypernuclear levels [14]. In this case, it was found that $x_\sigma = 0.46 \pm 0.26$, $x_\omega = 0.48 \pm 0.32$, so that we may take $x_\sigma < 0.72$.

The results of the analysis are shown in Fig. 1. We show several curves of maximum neutron-star mass as a function of the ratio of the hyperon-to-nucleon coupling to the scalar meson, corresponding to different values of K and m^*. This is because of some ambiguity in the empirical values of K, which is taken to lie in the range 240–300 MeV [19–22], and the effective (Dirac) nucleon mass at saturation density, $m^*_{\rm sat}/m$, which is taken to be in the range 0.7–0.78, corresponding to the empirical nonrelativistic effective mass in the range 0.74–0.83, which to a good approximation [23] has been identified as the Landau effective mass [24]. The related coupling constants are listed in Table II. (For the cases where c is small and negative, ϵ/ρ is well behaved and linear in ρ at $100\rho_0$, far beyond where we use the theory.) The three curves span the range of uncertainty in these parameters. For each value of x_σ, the value of x_ω is chosen in each case to yield the Λ hyperon binding in saturated nuclear

TABLE I. Values of the hyperon-to-nucleon scalar and vector coupling that are compatible with the binding of −28 MeV for Λ hyperons in nuclear matter for two values of the nucleon (Dirac) effective mass at saturation density.

	x_ω	
x_σ	$m^*/m = 0.7$	$m^*/m = 0.78$
0.2	0.131	0.091
0.3	0.261	0.233
0.4	0.392	0.375
0.5	0.522	0.517
0.6	0.653	0.568
0.7	0.783	0.800
0.8	0.913	0.942
0.9	1.04	1.08
1	1.17	1.23

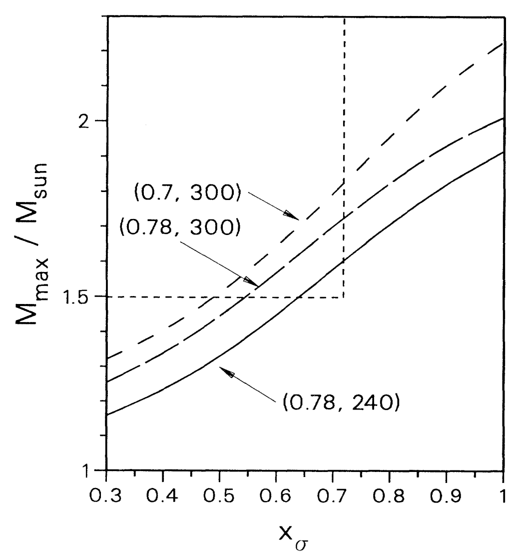

FIG. 1. Maximum neutron-star mass as a function of hyperon scalar coupling, with vector coupling chosen so as to yield correct Λ binding in nuclear matter. Numbers in parentheses are m^*/m and K in MeV. The maximum mass in the absence of hyperons would be $2.35M_\odot$, $2.08M_\odot$, and $2.02M_\odot$, respectively.

TABLE II. Coupling constants that yield binding $B/A = -16.3$ MeV, density $\rho = 0.153$ fm^{-3}, and symmetry energy coefficient, $a_{sym} = 32.5$ MeV, for saturated nuclear matter with the compression K and effective mass m^*.

K (MeV)	m^*/m	$(g_\sigma/m_\sigma)^2$ (fm^2)	$(g_\omega/m_\omega)^2$ (fm^2)	$(g_\rho/m_\rho)^2$ (fm^2)	b	c
300	0.7	11.79	7.149	4.411	0.002947	−0.001070
300	0.78	9.148	4.820	4.791	0.003478	0.01328
240	0.78	9.927	4.820	4.791	0.008659	−0.002421

matter of -28 MeV. The correspondence is given in Table I. Arbitrarily, we set $x_\rho = x_\sigma$. This choice is not a sensitive one, since an alternative choice $x_\rho = x_\omega$ yields essentially the same results. The acceptable ranges of M and x_σ, as discussed above, lie in the boxed area in the upper left of the figure. For the hyperon-to-nucleon scalar coupling x_σ we find a minimum allowable value of ~ 0.5 from the lower bound on the maximum neutron-star mass that is *also* consistent with the Λ binding in nuclear matter, while hypernuclear levels yield a somewhat uncertain upper bound of ~ 0.72. The corresponding values of x_ω can be found in Table I.

It will be noticed that the weaker the hyperon coupling the lower the maximum-mass star. Since the shift in baryon populations from nucleons to hyperons occurs only when it softens the equation of state, the lower mass implies that the hyperons actually participate to a higher degree the weaker their coupling. This seemingly paradoxical behavior is easy to understand and has been explained in Refs. [6,10].

We can gauge the contribution of the hyperons to the determination of the star mass by noting that in their absence the maximum mass would be $2.36 M_\odot$ for $K = 300$ MeV and $m_n^*/m_n = 0.7$. Within the uncertainty of their coupling as limited by the above considerations (boxed region) the hyperons reduce this to the range $M/M_\odot \sim 1.5$–1.8. This is an even greater reduction than it at first seems, because the least value of maximum mass that theory provides is that of noninteracting neutrons, i.e., a star supported solely by the Fermi pressure. This value is $0.75 M_\odot$ [25]. So the actual relevant scale within which the hyperons act is $(0.75$–$2.36) M_\odot$.

Choosing $K = 300$ MeV, $m^*/m = 0.7$, which we consider to be the best empirical values [19–23], we show in Fig. 2 the sequence of stars obtained under three different circumstances: (1) hyperons are neglected; (2) they are taken into account with the coupling $x_\sigma = 0.6$, which falls in the middle of the range discussed above and all particle species are in equilibrium; (3) hyperons are introduced as free baryons, interacting only through the weak interaction so that the system is in equilibrium. For case (2) we show the populations in the maximum-mass star in Fig. 3. Integrated over the star the baryon population is 59% neutrons, 17% protons, and 24% hyperons.

In future work on hypernuclear levels, it is clear from our discussion that the large correlation error found in the determination of x_σ, x_ω when they were treated independently can be circumvented by requiring that they

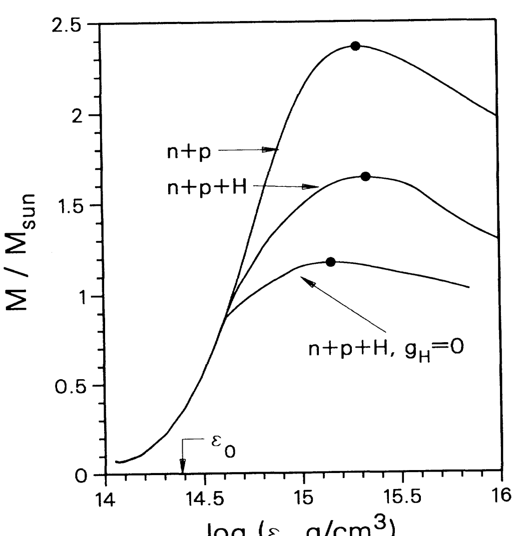

FIG. 2. Sequences of neutron stars for the three cases discussed in the text.

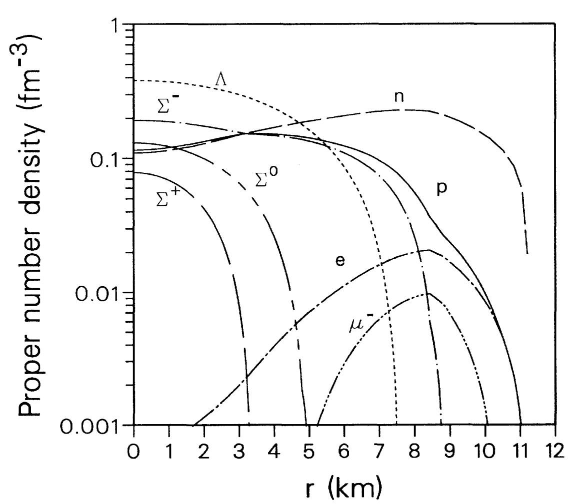

FIG. 3. Composition of the maximum-mass star for the case discussed in the text.

obey Eq. (3) so as to assure that the binding of the Λ in nuclear matter also be correctly given. This may produce a unique solution rather than the poorly determined one presently available and quoted above.

In conclusion, we find that (1) neutron-star masses, (2) the Λ binding in nuclear matter, and (3) hypernuclear levels are mutually compatible and rather narrowly constrain the hyperon couplings and their populations in neutron stars. A key element in resolving the problem was the release of the scalar and vector hyperon-to-nucleon couplings from the constraint of equality, and, instead, choosing their relationship as defined by Eq. (3) to be in accord with the very-well-inferred value of the Λ hyperon binding in saturated nuclear matter. With the hyperon couplings determined as above, albeit within a range of values, we find that the hyperons in neutron stars reduce the maximum mass that they could otherwise have very considerably, by $(0.63 \pm 0.23)M_{\odot}$. This folds in also uncertainties in the compression and effective nucleon mass at saturation. For the preferred values of these parameters, the reduction is $(0.71 \pm 0.15)M_{\odot}$, the uncertainty now residing solely in the hyperon couplings. It should be possible in the future to determine these couplings much more precisely if the relation given above that connects them to the Λ binding in nuclear matter is employed in the analysis of hypernuclear levels. This should permit an even more precise determination of the hyperon role in neutron stars. Even so, the role of hyperons is fairly narrowly delimited, and comprises a large correction to theories of neutron-star structure that neglect them.

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