Vortex-Lattice States at Strong Magnetic Fields

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At strong magnetic fields, Landau quantization invalidates the semiclassical approximations which underly the Ginzburg-Landau (GL) theory of the mixed states of type-II superconductors. We have solved the *microscopic* mean-field equations for the case of a two-dimensional electron system in the strong magnetic-field limit. For delta-function attractive interactions there exist $n+1$ pairing channels in the nth Landau level. For $n>0$, two channels share the maximum T_c and the order parameter differs markedly from expectations based on GL theory.

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The microscopic justification [1] of the Ginzburg-Landau (GL) description of the mixed state of type-II superconductors [2] requires a semiclassical approximation for the magnetic field which completely neglects Landau quantization of the kinetic energy. The possible importance of Landau quantization was recognized some time ago [3] when it was realized that de Haas–van Alphen-like oscillations could occur in the superconducting transition temperature $T_{c2}(H)$ in an external magnetic field. Experimental evidence for the importance of Landau quantization was later found by Graebner and Robbins [4], who observed de Haas–van Alphen oscillations in the mixed state of the layered dichalcogenide 2H-$NbSe_2$. Recently, Rasolt and co-workers [5,6] have brought attention to novel aspects of superconductivity in the limit where few Landau levels are occupied and T_{c2} can be comparable [7] to the zero-field T_c. The emphasis of previous theoretical work in this regime has been on studying the influence of Landau quantization on [6,8] the superconducting transition temperature [9] or on the inclusion of Landau quantization in the estimation [6,8] of the two-particle and four-particle kernels of the Gor'kov equations. In this Letter we present the results of the first self-consistent solution of the microscopic mean-field equations for the superconducting state in the strong-field regime. We obtain results for the spatial dependence of the order parameter and for the quasiparticle band structure of the vortex-lattice state which is of interest in connection with recent experimental [10] and theoretical [11] studies of the local density of states in a vortex core [12].

We consider a two-dimensional [13] electron system in a uniform magnetic field [14] with a delta-function attractive interaction of strength V which acts only between states whose bare energies ϵ satisfy $|\epsilon-\mu|<\hbar\Omega_B$, where μ is the chemical potential and Ω_B is the cutoff frequency for the attractive interaction. The strong-field limit [15] is reached when $\omega_c \equiv eB/m^*c$ exceeds $\Omega_B/2$ so that pairing can occur only within a single Landau level. To describe the vortex-lattice state it is convenient to choose a Landau gauge $[\mathbf{A}=(0,Bx,0)]$ and expand the quasiparticle amplitudes in the form $u(\mathbf{r})=\sum_X u_X\phi_X(\mathbf{r})$, $v(\mathbf{r})=\sum_X v_X\phi_X^*(\mathbf{r})$, where $\phi_X(\mathbf{r})=\exp(ik_y y)\psi_n(x-X)/(L_y)^{1/2}$ denotes the single-particle eigenstate in the nth Landau level with guiding center X. (Here ψ_n is a harmonic-oscillator eigenstate at frequency ω_c and $l^2=\hbar c/eB$ and $X=-k_y l^2$.) In this representation the Bogoliubov–de Gennes (BdG) equations [16] in the strong-field limit take the form [17]

$$\sum_X \{\delta_{X',X}[\hbar\omega_c(n+\tfrac{1}{2})-\mu-E]\}u_X+F_{X',X}v_X=0\,, \tag{1}$$

$$\sum_X \{-\delta_{X',X}[\hbar\omega_c(n+\tfrac{1}{2})-\mu+E]\}v_X+F^*_{X',X}u_X=0\,, \tag{2}$$

where

$$F_{X',X}=V\int d\mathbf{r}\,\phi^*_{X'}(\mathbf{r})\phi^*_X(\mathbf{r})\Delta(\mathbf{r})\,. \tag{3}$$

In Eq. (3)

$$\Delta(\mathbf{r})=\sum_{X',X}\Delta_{X',X}\phi_{X'}(\mathbf{r})\phi_X(\mathbf{r})\,, \tag{4}$$

where

$$\Delta_{X',X}=\sum_m \tanh(\beta E_m/2)v^*_{X',m}u_{X,m}\,, \tag{5}$$

and the sum over m is over positive-energy solutions of the BdG equations. (We discuss only the case of the zero g factor here. Similar results can be obtained whenever the spin splitting and the Landau-level splitting are commensurate. Strong-field solutions can also be found, for parallel-spin pairing within a spin-polarized Landau level [18], which do not require a special value for the g factor.)

Many qualitative features of our results follow from the following identity [18]:

$$\int d\mathbf{r}\,\phi^*_{X+Y/2}(\mathbf{r})\phi^*_{X-Y/2}(\mathbf{r})\phi_{X'+Y'/2}(\mathbf{r})\phi_{X'-Y'/2}(\mathbf{r}) =\frac{\delta_{X,X'}4\pi l}{L_y}\sum_{j=0}^{n}(-V^H_{2j}/V)\chi_j(Y)\chi_j(Y')\,, \tag{6}$$

where $-V^H_{2j}/V$ is the Haldane pseudopotential [19] for angular momentum $2j$ in the nth Landau level for an attractive delta-function potential of unit strength. Equation (6) can be derived by making a transformation to center-of-mass and relative coordinates for a pair of particles in a strong magnetic field [18]. In Eq. (6),

$$\chi_j(Y) = (-1)^j (l/\sqrt{2})^{1/2} \psi_{2j}(Y/\sqrt{2}) , \qquad (7)$$

which are orthonormal and complete for even functions, and $V^H_{2j} = -VN_jN_{n-j}/4\pi l^2 = -\lambda N_j N_{n-j} \hbar\omega_c/2$, where $N_j = (2j-1)!!/2^j j!$ and $\lambda = V/\hbar\omega_c 2\pi l^2$. Expanding $\Delta_{X+Y/2,X-Y/2} = \sum_{j=0}^{\infty} \Delta_j \chi_j(Y)$, $F_{X+Y/2,X-Y/2} = \sum_{j=0}^{\infty} F_j \times \chi_j(Y)$, it follows from Eq. (6) that $F_j = -V^H_{2j}\Delta_j$, and that $F_j = 0$ for $j > n$. Pairing occurs in even relative angular momenta from 0 to $2n$. Although we have derived the relationship between F_j and Δ_j in terms of Haldane pseudopotentials here for the special case of delta-function attractive interactions, it is valid [18] for interactions of any range.

Solutions of the BdG equations may be labeled by total momentum along the y direction. Solutions corresponding to different total momenta are degenerate. *Pairing in total momentum zero is not preferred.* The condensation energy per electron corresponding to a solution at fixed total momentum vanishes in the thermodynamic limit. In order to get a solution with an extensive condensation energy in mean-field theory, it is necessary to break translational symmetry and have pairing at a discrete set of total momenta; the state with this property is the vortex-lattice state, which we discuss below. We first consider the solution at some particular total momentum, which we label as $2X$. Then in Eqs. (1) and (2) $u_{X-Y/2}$ is coupled only to $v_{X+Y/2}$ and the BdG equations reduce to the familiar 2×2 form of BCS theory. This leads to

$$\Delta_j = \sum_{j'=0}^{n} K_{j,j'} \Delta_{j'} , \qquad (8)$$

where

$$K_{j,j'} = -\tfrac{1}{2} V^H_{2j'} \int dY \chi_j(Y) \{\tanh[\tfrac{1}{2}\beta E(Y)]/E(Y)\} \chi_j{}'(Y) , \qquad (9)$$

$E(Y) = (|F_{X-Y/2,X+Y/2}|^2 + \xi^2)^{1/2}$, and $\xi = \hbar\omega_c(n+\tfrac{1}{2}) - \mu$. The solution of the BdG equations is completely specified by the $n+1$ Δ_j parameters which are easily determined by solving Eqs. (8) and (9) iteratively. The order parameter for this solution is given by [18] $\Delta(\mathbf{r}) = \sum_j \Delta_j \Delta_X^{(j)}(\mathbf{r})$, where

$$\Delta_X^{(j)}(\mathbf{r}) = (1/4\pi l^2)(\sqrt{2} N_{n-j} N_j l)^{1/2} \exp(-i2Xy/l^2) \times \psi_{2(n-j)}(\sqrt{2}(x-X)) . \qquad (10)$$

For this solution the order parameter is nonzero only near the line $x = X$.

To solve for T_c we linearize Eqs. (8) and (9). In this limit $K_{j,j'}$ is independent of the Δ_j and diagonal, so that in channel j, T_c satisfies

$$[\tanh(\xi/2k_BT_c)/\xi]^{-1} = \tfrac{1}{4}\hbar\omega_c \lambda N_{n-j} N_j . \qquad (11)$$

T_c is largest in the $j=0$ and $j=n$ channels which have the same T_c. In Fig. 1(a) we plot Δ_j vs ξ at $T=0$, and in Fig. 1(b) we plot Δ_j vs T at $\xi=0$ for the $n=0$ and $n=1$ Landau levels. Solutions occur at sufficiently low T as long as $|\xi| < \lambda\hbar\omega_c N_n/4$. For $n=1$ there are two solutions of the gap equations corresponding to the two different channels within which pairing occurs. Since both channels have the same T_c, they remain strongly mixed even as T approaches T_c. One solution is dominantly d wave ($j=1$), but the lowest-energy solution has its largest amplitude in the s-wave channel whose order parameter has the more complicated spatial dependence.

We now turn to the vortex-lattice case. Following GL theory we expect to find solutions of the BdG equations in which the superconducting order parameter can be written as the sum of solutions at different guiding centers separated by lattice constant a_x. We write

$$\Delta(\mathbf{r}) = \sum_{t,j} \Delta_j \exp(i\phi t^2) \Delta^{(j)}_{X=ta_x}(\mathbf{r}) , \qquad (12)$$

where the Δ_j need not be exactly the same as those obtained from the single-guiding-center calculation because of interference between solutions at different total momentum. ($\phi=0$ provides the most convenient description of the square vortex lattice and $\phi=\pi/2$ describes a class of lattices including the triangular vortex lattice.) For $\phi=0$ the BdG equations can be reduced to 2×2 form by making a unitary transformation from the guiding center representation to a representation of magnetic

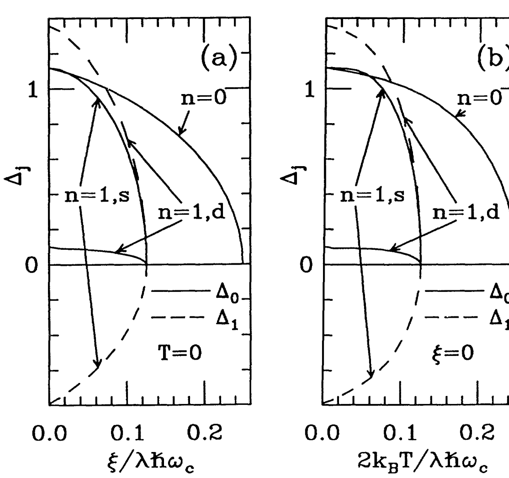

FIG. 1. Solution of the BdG equations for a fixed total momentum in the $n=0$ and $n=1$ Landau levels: (a) order parameters in s and d channels vs ξ at $T=0$; (b) order parameters in s and d channels vs T at $\xi=0$. The quasiparticle energies and $\Delta(\mathbf{r})$ can be expressed in terms of these parameters (see text).

Bloch states:

$$\phi_{k_x,k_y}(\mathbf{r})=(1/\sqrt{S})\sum_s \exp(ik_x s a_x)\phi_{-l^2k_y+sa_x}(\mathbf{r}). \qquad (13)$$

In Eq. (13), $|k_x|<\pi/a_x$, $|k_y|<a_x/2l^2$. $S=L_x/a_x$. It follows from summing solutions over the magnetic Brillouin zone that the assumed form for $\Delta(\mathbf{r})$ is self-consistent [18] and that the gap equation is of the same form as in Eq. (8) with $K_{j,j'}$ replaced by

$$K^L_{j,j'}=\frac{-V^H_{2j'}2\pi l a_x}{L_x L_y}\sum_{\mathbf{k}}\frac{\tanh[\beta E(\mathbf{k})/2]}{E(\mathbf{k})}\chi_j^*(\mathbf{k})\chi_{j'}(\mathbf{k}), \quad (14)$$

where $E(\mathbf{k})=(\xi^2+|F_\mathbf{k}|^2)^{1/2}$ defines the quasiparticle band structure. Here $F_\mathbf{k}=\sum_j F_j\chi_j(\mathbf{k})$ and

$$\chi_j(\mathbf{k})=\sum_t \chi_j(2k_y l^2+2ta_x)\exp(i2tk_x a_x).$$

(A similar expression can be derived [18] for $\phi=\pi/2$.) Again, the full solution, including the quasiparticle band structure, can be expressed in terms of a small number of parameters whose values are easily determined. In the linearized regime, Eq. (14) reduces to Eq. (9), so that the transition to the vortex-lattice state is second order as expected.

In Fig. 2 we present contour maps of the order parameter $\Delta(\mathbf{r})$ for the square vortex-lattice solution of the BdG equations at $T=0$ and $\xi=0$ in the $n=1$ Landau level. (The $n=0$ order parameter has the form assumed in GL theory.) The lowest-energy solutions for both square and triangular lattices are purely s wave. Whenever a substantial s-wave component is present $\Delta(\mathbf{r})$ will differ qualitatively from expectations based on GL theory. For the square lattice each unit cell contains two vortices, one with vorticity 2 and one with vorticity -1. The solution in the triangular lattice case [18] has three vortices per unit cell, two with vorticity 1 and one with vorticity -1. For $n=1$ the triangular lattice has a larger condensation energy than the square lattice. The lowest-energy solution for $n=1$ occurs for a rectangular cell with an aspect ratio $(2a_x/a_y)$ of ~ 5.4, compared to ~ 1.73 for the triangular lattice, and has mixed s-wave and d-wave character [18].

The surprising existence of superconductivity at extremely strong magnetic fields is due to the increase in the density of states at the Fermi level which tends to destabilize the normal state of electrons. The tendency toward destabilization is especially severe in layered structures where the density of states diverges as the interlayer hopping weakens. In the two-dimensional limit treated here, the BCS model reduces to one in which we need to treat attractive interactions among electrons which share the same kinetic energy. Except for the sign of the interaction, the problem is exactly that encountered in the fractional quantum Hall effect. In the repulsive interaction case, the mean-field state [20] is a charge-density wave which breaks translational invariance. Except at small filling factors, however, the true

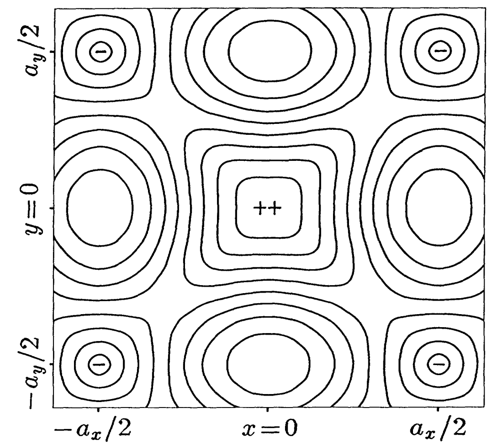

FIG. 2. Contour maps of the absolute value of $\Delta(\mathbf{r})$ obtained by solving the BdG equations in the strong-field limit at $\xi=T=0$ within the $n=1$ Landau level for square vortex-lattice states. The + and − signs indicate the vorticity at zeros of the order parameter. $a_x a_y=\pi l^2$.

ground state is a liquid in which the probability of finding electrons in states of low relative angular momentum is minimized. It is likely that the mean-field states found here are sometimes preempted by liquid states, possibly like the hollow core model state found by Haldane and Rezayi [21], in which the probability of finding electrons in states of low relative angular momentum is maximized. If so, this strong-field regime may provide a concrete example of quantum melting of the vortex lattice. It may be that, as in the repulsive interaction case, the physics is controlled by the commensuration between particle density and vortex density which can lead to peculiar types of off-diagonal long-range order [22].

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[1] L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **36**, 735 (1959) [Sov. Phys. JETP **9**, 1364 (1959)].
[2] See, for example, *Superconductivity*, edited by R. D. Parks (Dekker, New York, 1969).
[3] A. K. Rajagopal and R. Vasudevan, Phys. Lett. **23**, 539 (1966); L. W. Gruenberg and L. Gunther, Phys. Rev. **176**, 606 (1968).
[4] J. E. Graebner and M. Robbins, Phys. Rev. Lett. **36**, 422 (1976).
[5] M. Rasolt, Phys. Rev. Lett. **58**, 1482 (1987).
[6] Zlatko Tešanović and M. Rasolt, Phys. Rev. B **39**, 2718 (1989); Zlatko Tešanović, M. Rasolt, and L. Xing, Phys. Rev. Lett. **63**, 2425 (1989); Phys. Rev. B **43**, 288 (1991).

[7] For more recent work, see M. R. Norman, Phys. Rev. B **42**, 6762 (1990); C. T. Rieck, K. Scharnberg, and R. A. Klemm, Physica (Amsterdam) **170C**, 195 (1990).

[8] T. Maniv, R. S. Markiewicz, I. D. Vagner, and P. Wyder, Physica (Amsterdam) **165 & 166B**, 361 (1990); (unpublished).

[9] For a more detailed discussion of the conditions necessary for Landau quantization to be important in layered systems and of the consequences of a finite spin splitting, see M. R. Norman, H. Akera, and A. H. MacDonald, *Physical Phenomena at High Magnetic Fields* (Addison-Wesley, Reading, MA, 1991).

[10] H. F. Hess, R. B. Robinson, R. C. Dynes, J. M. Valles, and J. V. Waszczak, Phys. Rev. Lett. **62**, 214 (1989); H. F. Hess, R. B. Robinson, and J. V. Waszczak, Phys. Rev. Lett. **64**, 2711 (1990).

[11] Joel D. Shore, M. Huang, Alan T. Dorsey, and James P. Sethna, Phys. Rev. Lett. **62**, 3089 (1989); F. Gygi and M. Schlüter, Phys. Rev. B **41**, 822 (1990).

[12] Quasiparticle states in a vortex lattice have recently been discussed by François Gygi and Michael Schlüter, Phys. Rev. Lett. **65**, 1820 (1990), who treat neighboring vortex cores as weak perturbations; and also by Michael J. Stephen, Phys. Rev. B **43**, 1212 (1991), who assumes the Abrikosov form for the order parameter of the lattice states and examines the quasiparticle states that result.

[13] The generalization to the three-dimensional case requires wave-vector labels to be introduced for motion along the field but causes no essential complication. The effects of Landau quantization are greater in two-dimensional or in layered systems. Some interlayer coupling will be required to suppress fluctuations if the mean-field order is to survive.

[14] In the strong-field limit the penetration depth is much longer than the vortex-lattice period so that the flux density remains nearly constant when the screening currents are included.

[15] The approach we use may be generalized to allow microscopic calculations to be performed at the weaker magnetic fields where GL theory is valid. Landau quantization effects become important at much weaker fields than those required for the strong-field limit considered here to apply.

[16] J. R. Schrieffer, *Superconductivity* (Benjamin, New York, 1964); P. G. de Gennes, *Superconductivity of Metals and Alloys* (Addison-Wesley, Reading, MA, 1989).

[17] The solutions we find will have nonuniform charge density except at $\xi = 0$, and so we should, strictly speaking, include a corresponding mean-field term in the diagonal blocks of the BdG equations. An account of the role played by these terms will be presented elsewhere.

[18] Further details of our work will be provided in a subsequent publication. H. Akera, A. H. MacDonald, and M. R. Norman (unpublished).

[19] F. D. M. Haldane, in *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1990), 2nd ed., Chap. 8. In the lowest $(n=0)$ Landau level V_{2k}^{H} is the binding energy of a pair of electrons with relative angular momentum $2k$. States of definite relative angular momentum do not exist in higher Landau levels; however, an exact mapping exists which makes any Landau level equivalent to the lowest Landau level with an appropriate modification of the effective interaction, and hence of the Haldane pseudopotentials. In particular, δ-function interactions in higher Landau levels are equivalent to longer-ranged interactions in the $n=0$ Landau level for which pairing can occur in higher relative angular momentum channels.

[20] D. Yoshioka and H. Fukuyama, J. Phys. Soc. Jpn. **47**, 394 (1979); D. Yoshioka and P. A. Lee, Phys. Rev. B **27**, 4986 (1983); A. H. MacDonald, Phys. Rev. B **30**, 4392 (1984).

[21] F. D. M. Haldane and E. H. Rezayi, Phys. Rev. Lett. **60**, 956 (1988).

[22] S. M. Girvin and A. H. MacDonald, Phys. Rev. Lett. **58**, 1252 (1987).