Quantum Corrections Deflate Deep Bags

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(Received 26 July 1991)

We examine the formation of nontopological solitons in a Yukawa theory with N fermions and a single scalar field. We solve the theory to leading order in $1/N$, for any value of the Yukawa coupling g. We find that the full quantum theory supports nontopological soliton solutions, and that the quantum solitons differ significantly from those in the classical theory. For large g, the energy scales with g, while the radius decreases as $1/g$. Thus, quantum corrections invalidate the classical picture of tightly bound fermions inside deep bags.

PACS numbers: 11.10.Lm, 11.15.Pg, 12.40.Aa

The ever-increasing lower bounds on the top-quark mass have generated renewed interest in field theories with large fermion masses. In the standard model, the top mass is given by the product of a Yukawa coupling g and a vacuum expectation value v,

$$m = gv\,. \tag{1}$$

The value of v is fixed to be about 246 GeV, so large m implies large g.

Theories with large Yukawa couplings have a rich yet subtle phenomenology. A characteristic feature of such theories is that they support nontopological soliton solutions, called bags [1–7]. In particle physics, the SLAC bag played an early and important role in describing the confinement of quarks inside hadrons [3]. In nuclear physics, nontopological bags have been successfully used to model the binding of nucleons within nuclei [6,7]. More recently, bags have also been discussed in conjunction with the phenomenology of the heavy-top-quark–Higgs-boson system [8].

Nontopological solitons are coherent states in which the expectation value of a scalar field is reduced from its vacuum value by the presence of a fermion field. The solitons carry fermion number because the fermion is energetically bound to the bag. They are stable because they have lower energy than any other configuration with the same quantum numbers. The solitons form because the energy gained by decreasing the fermion mass is greater than the energy lost through the potential and gradient terms in the scalar-field Hamiltonian. At the classical level, as the Yukawa coupling gets large, the fermions become tightly bound inside deep bags, whose energy and radius are independent of g.

In the full quantum theory, however, quantum corrections can be very important [1–7], especially in the nonperturbative regime of large g. One must check to see whether bags still form. There are two types of quantum fluctuations to consider: those in the scalar field, which can destroy the coherent state, and those in the fermion field, which can collapse the bag and even destabilize the vacuum.

In this Letter we examine bag formation in a consistent quantum field theory. We consider a theory with N Dirac fermions ψ^i coupled to a real scalar field ϕ. We solve the quantum theory to leading order in the large-N expansion for any value of the Yukawa coupling g. We find that the full quantum theory supports nontopological bags. The bags correspond to bound states of N fermions, with a binding energy of less than about 5%. The quantum bags differ significantly from those in the classical theory, where the binding energy approaches 100% for large g. The quantum corrections invalidate the classical picture of tightly bound fermions inside deep bags.

We shall first present our model. The Lagrangian density is

$$\mathcal{L}_0 = \tfrac{1}{2}(\partial_\mu\phi_0)^2 - \frac{\lambda_0}{8N}(\phi_0^2 - u_0^2 N)^2 + \sum_{i=1}^{N} \bar\psi_0^i \left(i\partial\!\!\!/ - \frac{g_0}{\sqrt{N}}\phi_0 \right)\psi_0^i\,, \tag{2}$$

where the subscripts signify bare quantities, and all N dependence is explicitly shown. The Lagrangian is characterized by three parameters, u_0, λ_0, and g_0. The N dependence is chosen so that fermion-loop contributions are of the same order as the tree-level couplings. The boson-loop contributions, however, are suppressed by at least one factor of N. This implies that the bosonic fluctuations can be ignored, and the scalar field can be treated as classical for any value of the Yukawa coupling [9].

From Eq. (2) we see that the field ϕ_0 develops a vacuum expectation value $\langle\phi_0\rangle \neq 0$. To compensate for this, we shift $\phi_0 = \sqrt{N} v_0 + \sigma_0$, where v_0 is chosen so that $\langle\sigma_0\rangle = 0$. At tree level, v_0 is just u_0. Then the mass of the field σ_0 is $\mu_0 = \lambda_0^{1/2} v_0$, while the fermion mass is $m_0 = g_0 v_0$.

To solve the model to leading order in $1/N$, we compute all diagrams with a single fermion loop [10]. As usual, we must specify a renormalization condition for each bare parameter. We define the wave-function renormalizations in the standard way,

$$\left.\frac{d\Gamma^{(2)}_{\sigma\sigma}}{dp^2}\right|_{p=0} = 1, \quad \left.\frac{d\Gamma^{(2)}_{\psi\bar\psi}}{dp\!\!\!/}\right|_{p=0} = 1\,, \tag{3}$$

where $\Gamma^{(2)}_{\sigma\sigma}$ and $\Gamma^{(2)}_{\psi\bar\psi}$ are the renormalized one-particle irreducible two-point functions for the renormalized fields

σ and ψ. We then fix the bare parameters u_0, λ_0, g_0, and v_0 through the following renormalization conditions:

$$\Gamma_\sigma^{(1)}=0,\quad \Gamma_{\sigma\sigma}^{(2)}|_{p=0}=-\mu^2,$$
$$\Gamma_{\psi\bar\psi}^{(2)}|_{p=0}=-m,\quad \Gamma_{\sigma\psi\bar\psi}^{(3)}|_{p_i=0}=-g/\sqrt{N}\,. \tag{4}$$

The vanishing of the one-point function $\Gamma_\sigma^{(1)}$ ensures that we are expanding about the minimum of the effective potential. The other three conditions define the renormalized masses of σ and ψ, as well as the renormalized Yukawa coupling g.

The Lagrangian (2) together with the renormalization conditions (3) and (4) define the full quantum theory. The quantum effective Lagrangian can be written as the sum of two terms,

$$\mathcal{L}=\mathcal{L}_{\text{fermion}}+\mathcal{L}_{\text{scalar}}\,. \tag{5}$$

The first term receives no quantum corrections,

$$\mathcal{L}_{\text{fermion}}=\sum_{i=1}^{N}\bar\psi^i\left(i\not\partial-\frac{g}{\sqrt{N}}\phi\right)\psi^i\,, \tag{6}$$

where $\phi=\sqrt{N}v+\sigma$ and $v=m/g$. The second term is modified by the fermion loops. It can be written in a derivative expansion,

$$\mathcal{L}_{\text{scalar}}=\mathcal{L}^{(0)}+\mathcal{L}^{(2)}+\mathcal{L}^{(4)}+\cdots\,, \tag{7}$$

where $\mathcal{L}^{(0)}$ is minus the effective potential,

$$\mathcal{L}^{(0)}=-\mathcal{V}=-\left(\frac{\lambda}{8N}+\frac{g^4}{32\pi^2N}\right)(\phi^2-v^2N)^2-\frac{g^4}{16\pi^2N}\phi^2(\phi^2-v^2N)+\frac{g^4}{16\pi^2N}\phi^4\ln\left(\frac{\phi^2}{Nv^2}\right),$$

with $\lambda=\mu^2/v^2$, and

$$\mathcal{L}^{(2)}=\frac{1}{2}\left[1-\frac{g^2}{8\pi^2}\ln\left(\frac{\phi^2}{Nv^2}\right)\right](\partial_\mu\phi)^2\,,$$
$$\mathcal{L}^{(4)}=\frac{N}{160\pi^2}\frac{(\partial^2\phi)^2}{\phi^2}-\frac{11N}{1440\pi^2}\frac{(\partial^2\phi)(\partial_\mu\phi)(\partial^\mu\phi)}{\phi^3}+\frac{11N}{2880\pi^2}\frac{(\partial_\mu\phi)^4}{\phi^4}\,. \tag{8}$$

As usual for Yukawa theories, the full quantum theory is afflicted by many problems [10], including a Landau pole, a tachyon, and vacuum instability. They result from the fact that the theory is not asymptotically free, and indicate that (5) must be viewed as the solution to an *effective* theory, valid for energies and momenta below some scale Λ. The scale of Λ depends on the Yukawa coupling g. For small g, Λ is exponentially large and can be safely ignored. For large g, however, Λ plays an important role. The size of Λ can be found by expanding the scalar field propagator in powers of p^2/m^2,

$$\Gamma_{\sigma\sigma}^{(2)}=-\mu^2+p^2+\frac{g^2}{80\pi^2}\frac{p^4}{m^2}+\cdots\,. \tag{9}$$

For $\mu\ll m$, we find a tachyon pole at $-p^2\approx 80\pi^2v^2\equiv\Lambda^2$. Requiring $m\lesssim\Lambda$ puts a limit of $g\lesssim 30$ on the Yukawa coupling. Subject to this condition, (5) describes the complete solution to the quantum theory [11], to order p^4 in the derivative expansion, and to leading order in $1/N$.

We will now demonstrate that the quantum theory supports bag solutions. In the large-N limit, however, bags do not form about a single fermion, but only when many fermions are present. We shall see that the bag lowers the energy of the N-fermion state, indicating the formation of a bound state, similar to a baryon in the large-N expansion of QCD [12].

We expect the lowest-energy state of given fermion number to have a static scalar-field configuration. The energy of the state is

$$E=E_{\text{scalar}}+E_{\text{fermion}}\,. \tag{10}$$

For static configurations, the scalar energy is given by

$$E_{\text{scalar}}=-\int d^3x\,\mathcal{L}_{\text{scalar}}\,. \tag{11}$$

The fermion energy is found from the positive-energy solutions to the Dirac equation in the presence of the scalar field ϕ. If ζ_a denotes the spinor solution with (positive) energy ϵ_a,

$$\left(-i\boldsymbol{\alpha}\cdot\nabla+\frac{g}{\sqrt{N}}\beta\phi\right)\zeta_a=\epsilon_a\zeta_a\,, \tag{12}$$

normalized so that $\int d^3x\,\zeta_a^\dagger\zeta_a=1$, the total fermion energy is just

$$E_{\text{fermion}}=\sum_a n_a\epsilon_a\,, \tag{13}$$

where the n_a are the occupancy numbers of the energy eigenstates, and $\sum_a n_a=N$ is the total fermion number of the state.

In the lowest-energy state with fermion number 1, the scalar field has a constant expectation value, $\langle\phi\rangle=\sqrt{N}v$, and ζ is a free Dirac spinor of mass $m=gv$. Thus, there is a solution to the quantum theory with fermion number N and energy $E=mN$, consisting of N free Dirac spinors in a constant scalar-field background. For sufficiently large Yukawa coupling, however, there are also soliton solutions, with $\langle\phi\rangle<\sqrt{N}v$ in the presence of N fermions. The soliton is stable if this state has energy $E<mN$.

For large g, there are two effects to consider: the scalar field becomes strongly coupled to the fermion, and the quantum corrections become important. Therefore we proceed in two steps. We first examine the *classical* theory, which corresponds to dropping the terms $\mathcal{L}^{(n)}$ with $n>2$, as well as dropping the terms in $\mathcal{L}^{(0)}$ and $\mathcal{L}^{(2)}$ that depend on g. In this case it is well known that the theory supports finite-energy baglike solutions, with $\langle\phi\rangle=\phi(r)$, as shown in Fig. 1(a). (The solution is plotted for the values $g=25$ and $\lambda=1$. Our numerical work was done with the aid of the program COLSYS [13].) The solution has fermion number N when the lowest orbital of

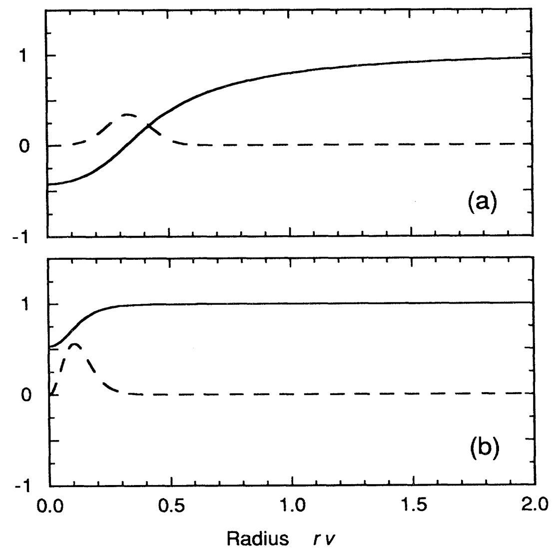

FIG. 1. Profiles of the bag solutions in the (a) classical and (b) quantum regime, for $g=25$ and $\lambda=1$. The solid line marks the bag profile $\phi(r)/\sqrt{N}v$, while the dashed line denotes the fermion number density.

each fermion is occupied. The energy per fermion is $E/N=6.3v \ll 25v$, so the N fermions are tightly bound. Such configurations are known as "deep bags" because $\phi(r)$ deviates significantly from its vacuum value v.

We now consider the *quantum* theory. For large g, the quantum corrections significantly modify the potential and the scalar gradient energy. They also induce higher-derivative terms in $\mathcal{L}_{\text{scalar}}$. It is straightforward to solve the quantum equations of motion [10]. We drop the terms $\mathcal{L}^{(n)}$ for $n>2$. (We have checked that this changes the soliton energy by less than 1% for $g \lesssim 30$ and $\lambda=1$.) As in the classical case, we find finite-energy soliton solutions, as shown in Fig. 1(b). (The solution is plotted for $g=25$ and $\lambda=1$.) We see that the quantum corrections dramatically alter the size and shape of the bag. As above, the solution has fermion number N. Now, however, the energy per fermion is $E/N=23.8v$, so the fermions are only weakly bound to the bag.

In Fig. 2 we see how the soliton energy scales with g for $\lambda=1$. For small g, the classical and quantum bags are similar, with $E<mN$ for $g \gtrsim 4$. For larger g, the bags begin to differ. In the classical case, the energy is independent of g for large g. In contrast, the energy of the quantum bag scales as $E/N \approx 0.95gv$, while the radius goes as $R \sim 1/gv$. The quantum corrections imply that the fermions are weakly bound to a small and shallow bag, with a binding energy approaching about 5% for large g. In fact, a simple scaling argument [10] shows that this asymptotic 5% binding energy is independent of λ.

In this model, the quantum corrections to the energy have a simple physical origin which can be understood in

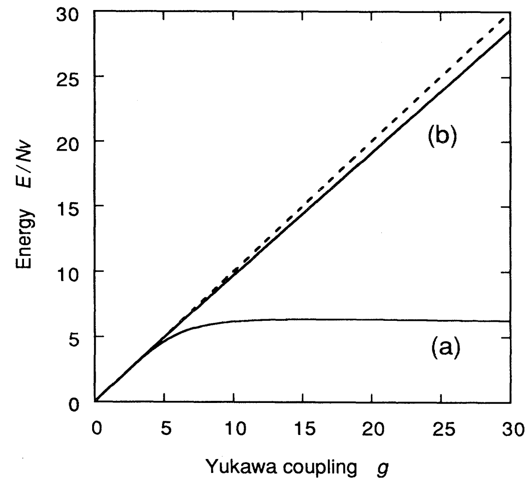

FIG. 2. The (a) classical and (b) quantum bag energies E/Nv, as a function of Yukawa coupling g, for $\lambda=1$.

terms of the Dirac equation. The presence of the bag changes the energy eigenstates and eigenvalues. It shifts the valence orbitals *and* the Dirac sea levels. Equation (13) explicitly accounts for the change in the valence orbitals. The shift in the Dirac sea is included implicitly, through the quantum corrections to $\mathcal{L}_{\text{scalar}}$. These corrections automatically sum the shift in the Dirac sea. To leading order in $1/N$, the two effects give the entire change in the energy [2].

We would like to stress, however, that the simplicity of the picture presented here is a feature of the large-N limit of the Yukawa theory. For finite N, the bosonic fluctuations modify the field equations for the nontopological soliton solution. This changes the details of our picture, but it does not alter our main point: that quantum solitons can differ dramatically from their classical counterparts.

In this Letter we have used the large-N expansion to find nontopological soliton solutions to a quantum Yukawa theory. For large couplings, the energy of the quantum bag scales with g. This implies that bags can indeed be used to model nuclei with their relatively small binding energies. In fact, bag formation provides a powerful, nonperturbative technique for finding bound-state solutions. We have also found that quantum effects deflate deep-bag solutions. This raises serious questions about using the SLAC bag as a realistic picture of quark confinement. If we trust the general features of our results all the way to $N=1$, we are also led to conclude that bag formation does not play a major role in top-quark physics.

We would like to thank M. Crescimanno, G. Feldman, C. Hill, and T. Truong for useful discussions. This work has been supported by the National Science Foundation, Grant No. PHY-90-96198, and by the Alfred P. Sloan Foundation.

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