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Acoustic Emission from Volcanic Rocks: An Example of Self-Organized Criticality

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Experimental evidence of ultrasonic emission from volcanic rocks has been produced for the first time by a survey of the Strombolian activity. The statistical analysis of the recorded acoustic wave bursts shows that the notion of self-organized criticality applies well to the mechanism responsible for the observed emission phenomenon.

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Recently, Bak, Tang, and Wiesenfeld [1] have proposed the notion of self-organized criticality (SOC) to describe the dynamics of systems which exhibit anomalously large fluctuations. A variety of open physical systems operate persistently at or close to states of *neutral equilibrium* (or critical) [2]. Such a stationary condition is achieved through ever-amplifying, self-adjusting activation processes with no length or time scales other than those set by the size of the system and the elementary activation mechanism. In other words, the behavior of such systems is *self-similar* in both their temporal and spatial fluctuations.

The notion of SOC has been extended quite naturally to interpret empirical observations on the occurrence and magnitude of earthquakes [3–5]. The earthquake statistics and the large-scale structure of the crust with its arrays of faults are both envisaged as closely related manifestations of the same stationary self-organization process. While the existence of steadily increasing stresses acting upon the crust is variously assumed, the predictions of all SOC models take the form of *scaling laws,* independent of the underlying stress mechanism.

In this Letter we suggest that the very same approach is well suited to account for recent observations of *high-frequency* acoustic emission (AE) in volcanic areas. Most notably, the amplitude and time distributions of the AE bursts are governed by scaling laws in good agreement with the relevant predictions based on SOC [3,5].

The earthquake models developed in Refs. [3] and [5] both start from a common ingredient, namely, a spatiotemporal dynamical system with stationary, locally fluctuating incoming and outgoing energy fluxes. The system is fragmented to form a self-similar composite structure. The loading mechanism [6] is governed by a "gap" dynamics: The energy input occurs locally and by a discrete quantity per time gap. The typical time duration of local energy releases of any size is taken to be instantaneous [1] (in the case of earthquakes, minutes compared to months or years [3]).

We can disregard the spatiotemporal complexity of the system and focus on the energy released at a time *t anywhere* within the system as a global variable $E(t)$. A wide variety of self-organized automata have been investigated numerically, which, in spite of rather small differences between the rules, give rise to rather different scaling exponents [7]. In particular, the SOC models of Refs. [3] and [5] lead to the following predictions: (i) Energy releases show up as bursts in the system response with random amplitude and time duration. The distribution function $N(\tau)$ of the time interval between two successive bursts obeys the scaling law

$$N(\tau) \sim \tau^{-\gamma}, \tag{1}$$

with critical exponents $\gamma \equiv \gamma_{SS} = 1$ (in mean-field approximation [3]) and $\gamma \equiv \gamma_{CBO} \simeq 1.3$ (from numerical simulation [7]). (ii) The energy released at any event, i.e., the burst amplitude E, is also distributed according to a scaling law, i.e.,

$$N(E) \sim E^{-\delta}. \tag{2}$$

The authors of Ref. [3] assume that the system components interact through *frictional* forces only, and, by means of a simple mean-field energy-conservation argument, obtain for the critical exponent $\delta \equiv \delta_{SS} = 2$. However, mean-field critical exponents fail to be correct for dimensions of the system below the relevant upper critical dimension d_{cr}. Obukhov [8] found that for the short-range SOC models $d_{cr}=4$, whence the above estimate for the critical exponent δ is too crude to be reliable. In the presence of *long-range* forces, instead, numerical evidence yields $d_{cr}=3$; the universality of such a class of models is still under investigation [2]. We assume, here, the more realistic view [5] that the crust is a dynamical system made of components of any size (with self-similar distribution) subject to both elastic compression (long-range forces) and static friction (short-range forces). A mean-field approach is, then, justified and leads to a smaller value [5] for the critical exponent in Eq. (2), i.e., $\delta \equiv \delta_{CBO}=1.5$.

We remark at this point that it is also possible to generate a random sequence of bursts with approximate scaling laws, like those predicted in Refs. [3] and [5], without having recourse to the notion of SOC. A typical example of some use later on for the discussion of our experimental results is presented in Ref. [9]. Let us consider the pulselike events triggered by a zero-mean-valued Gaussian noise $x(t)$ when it crosses a given level $x=b$. If σ_x^2 and τ_x denote the noise variance and correlation time, respectively, and $w_0(x)$ is its probability distribution, the density (i.e., the number per unit of time) of triggered pulses with time separation longer than a fixed positive number τ (but shorter than a certain cutoff value which depends on σ_x) is [9]

$$N(\tau)=w_0(b)\sigma_x(\tau_x/\pi\tau)^{1/2}. \qquad (3)$$

Analogously, one could assume that the amplitude A of the triggered pulses varies with the noise "excess area" S, i.e., the area enclosed below the curve $x(t)$ and above the line $x=b$, according to the law $A \sim S^\alpha$ with $\alpha>0$. The corresponding amplitude distribution of the triggered pulse sequence can be easily derived from Eq. (3.75) of Ref. [9] in the limit $b^2 \ll \sigma_x^2$,

$$N(A) \sim A^{-\beta}, \qquad (4)$$

with $\beta=1-2/3\alpha<1$. If, as for the operating conditions of acoustic detectors, $\alpha=1$, a simple triggered event dynamics might well explain scaling laws like those in Eqs. (1) and (2). The threshold mechanism in AE phenomena [10] would be provided by the static friction causing local slippages [4] or ruptures [5] at any length scale. However, the picture of the crust dynamics based on SOC is more comprehensive in that it relates the statistics of the local relaxation events to the spatial fragmentation of the crust itself. Now, since most of our observations are restricted to monitoring local seismic activity, a firm evidence in favor of a certain SOC model can only come from an *experimental* check on the scaling laws (1) and (2).

While historical records of seismic activity are rather incomplete, a particularly intense volcanic activity may provide a unique possibility to observe a large number of microseisms and eruptions in a relatively short period of time. With this in mind, we chose Stromboli (Aeolian Islands, Italy) as the most suitable site for revealing and monitoring possible underground ultrasonic emission. The expression "Strombolian activity" has been coined to denote a persistent release of crust energy in the form of volcanic tremors (with maximum surface displacement in the frequency range 0.5-10 Hz) and intermittent explosion quakes accompanied by the ejection of incandescent materials [6]. The related AE is thus expected to consist of a *random time sequence* of well-spaced bursts of acoustic waves with frequency up to 1 MHz (or higher) due to the microfractures, phase transformations, and, in general, the structural movements following the primary volcanic activity [6].

As in the first surveys of the AE in mines, our detectors are made of a piezoelectric sensor coupled to the free end of a steel rod tightly cemented into a rock-drill hole about 25 cm deep and 3 cm in cross section. The AE signals are generated by a transducer and amplified (gain factor 42 dB and dc output) with stable maximum sensitivity in a broad frequency range 0.2-1 MHz. Since we are interested in determining the statistics of the AE bursts with no regard to their spectral composition, the effective time constant of our data-acquisition system was set to the order of 0.2 s, i.e., much longer than the characteristic period of the burst acoustic waves, but mostly shorter than both the expected time duration of a single burst and the time interval between two successive bursts. Correspondingly, to avoid storage problems we adopted a relatively long sampling time (0.1 s) and acquired data for 11 days. Every single burst was thus integrated into a pulselike voltage signal with a rise time much shorter than 0.1 s and recorded as a function of time. In our first survey of the Strombolian activity the signal peak amplitudes have been sampled in the voltage range 0-1 V, only. Moreover, due to our choice of the data-acquisition time constant, in our statistical analysis we neglected every smaller amplitude signal within up to 0.2 s after each recorded burst. As a consequence, the effective background threshold for each detector depends on the intensity of the AE phenomenon as well. (Anyway, with our amplification setup, all detector backgrounds are estimated to be smaller than 0.1 V.)

All detectors operating at one time have been linked together to form a survey network. The first sensor (S_1) was placed at about 100 m above sea level, coupled to a steel rod drilled into a lava bed. Two more sensors were placed at 150 m (S_2) and 200 m (S_3) above sea level, the relevant steel rods being drilled into bulky rocks sticking 110 and 180 cm out of the mountain slope, respectively. Finally, a blank sensor (S_b) was connected to the survey network but carefully insulated from the ground. Signals detected in coincidence (i.e., within an interval of 0.1 s) by S_b and any other sensor of the network were recognized as due to electrical noise and, then, discarded. A few inches away from S_3 a glass detector rod with the same cross section as the steel rods was tested. It turned out to be as good an acoustic guide, thus leading to the conclusion that no appreciable emission was due to electrical current flows within the Earth's crust. Furthermore, we verified the following: (a) The mechanical properties of the *acoustic guides* have no noticeable impact on the outcome of our measurements. On varying the length and cross section of the steel rods coupled to the sensors, we only observed the expected variation of the overall detector efficiency [11], whereas the relevant statistics of the AE phenomenon described below turns out to be reproducible within the experimental error. The same result had been obtained when the sensors were placed directly in contact with the solid rock, i.e., in the absence of any acoustic guide. (b) In the operating conditions outlined above the *detector response* is linear. This property has been verified in laboratory tests where appropriate bursts of acoustic waves in the frequency range 0.1–1 MHz have been produced by means of a commercial analog generator and sent both to a single amplifier and to one of our detectors connected to an amplifier of the same type. The two outputs turn out to differ only by a multiplicative constant independent of the frequency. (c) *Thermal gradients* seem to play no role. Laboratory tests and the comparison between the recordings taken both at night and during the daytime allow us to conclude that the AE induced by the solar radiation is negligible. We checked that an abrupt heating of solid rocks, as caused by contact with flowing lava, may originate intense AE bursts indeed. However, the corresponding time and amplitude distributions are totally different from those observed during our survey. In conclusion, our *in situ* observations and further tests provide compelling evidence that the recorded AE signals have been generated by *underground sources.*

In the AE associated with the Strombolian activity two different emission regimes with random time duration have been observed (Fig. 1): (i) A *low-activity* regime characterized by mostly isolated and intense AE bursts and sparse volcanic explosions. An accurate statistical analysis [11,12] showed a close time correlation between the occurrence of a volcanic explosion and the AE bursts in this regime, which can be thus divided into precursor, coincident, and aftershock events. (ii) A *high-activity* regime, when a surprisingly intense AE is recorded with no correspondence with any anomalously increased explosive activity by the volcano. Tidal effects and local effects like volcanic microtremors, heat flows, and lava moments in side conduits have been advocated to explain such a spectacular AE phenomenon [12].

We discuss now the statistics of the AE bursts with respect to their amplitude and timing. Let A denote the amplitude (in volts) of a single AE burst. In order to eliminate possible spurious effects due to the detector noise background we introduced a variable threshold A_0 and counted the bursts with $A > A_0$ only. Moreover, it is well known that the energy release E, responsible for the generation of an AE burst, is proportional to its amplitude squared, i.e., $E \sim A^2$. In Fig. 2 we display the amplitude distribution of the AE bursts detected by S_2 and S_3 [10]. The scaling law (4) with $\beta = 2.0 \pm 0.1$ fits our experimental data very closely. Furthermore, on changing variables, $A \rightarrow E \sim A^2$, we conclude that this corresponds to the distribution law

$$N(E) \sim E^{-(1+\beta)/2}, \qquad (5)$$

in good agreement with the mean-field prediction of Ref. [5]—from Eq. (2) $\beta = 2\delta_{CBO} - 1 = 2$. The statistics of our data is high enough to rule out for the case under study the mean-field estimate $\delta_{SS} = 2$ of Ref. [3]. It is also clear that the triggered-event model for the AE bursts does not apply here, no matter what the time correlation for the driving noise $x(t)$, the exponent β in Eq. (4) always being smaller than unity.

The distribution of the time interval between two successive AE bursts with amplitude larger than A_0 is displayed in the logarithmic plot of Fig. 3. Our experimental data lie apparently on straight lines with slope

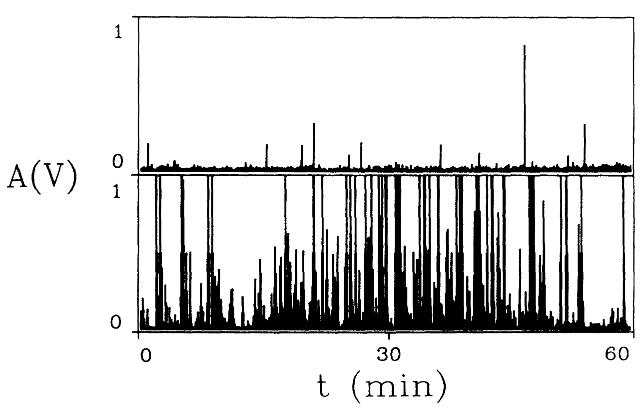

FIG. 1. Recorded samples of low- (above) and high-activity AE (below).

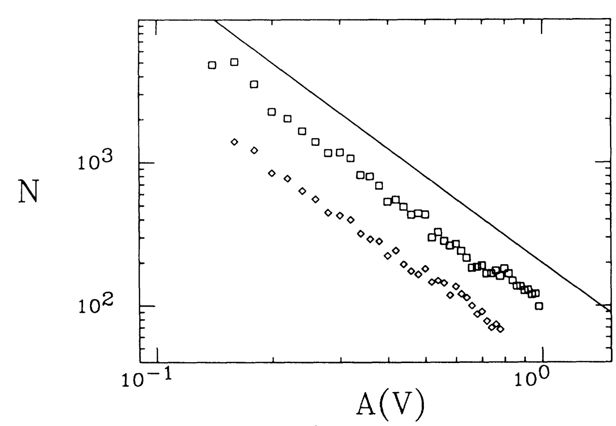

FIG. 2. Distribution of 10^3 AE bursts from detectors S_2 (lozenges) and S_3 (squares) with respect to their amplitude (in volts). The scaling law (4) with $\beta = 2$ is plotted for comparison.

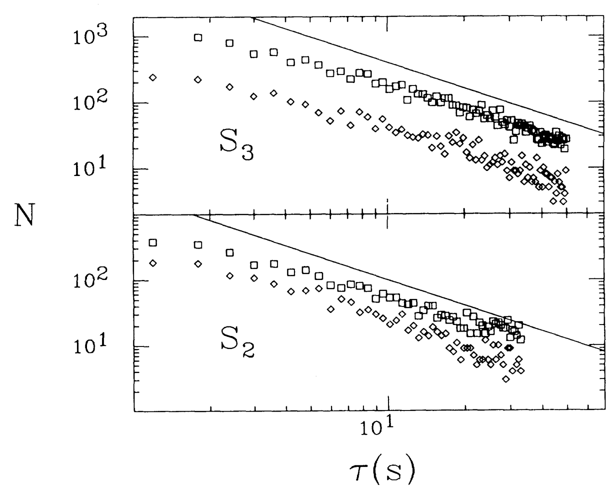

FIG. 3. Distributions of 10^3 AE bursts from detectors S_2 and S_3 with respect to their time separation. The threshold value A_0 is 0.15 V (squares) and 0.5 V (lozenges). The scaling law (1) with $\gamma = \gamma_{CBO}$ is plotted for comparison.

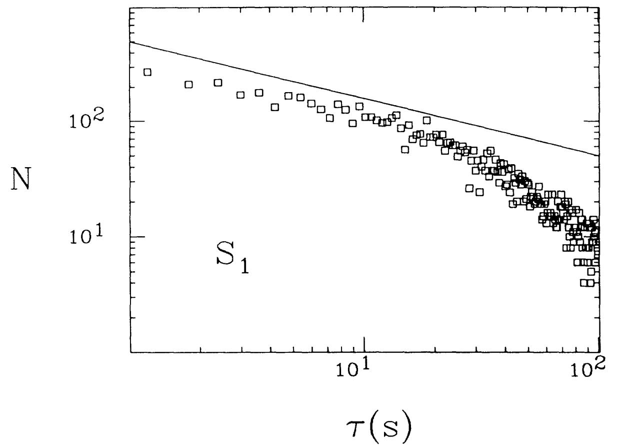

FIG. 4. Time distribution of AE bursts from detector S_1 with threshold value $A_0 = 0.1$ V. The scaling law (3) is plotted for comparison.

$\gamma = 1.2 \pm 0.1$. This corresponds to the scaling law (1) with γ very close to γ_{CBO}. In this case, as well, a triggered-event model predicts too small a critical exponent, i.e., $\gamma \lesssim 0.5$ [9], for such a class of models to be seriously considered in the analysis of the AE from volcanic activity.

An exceptional situation is represented in Fig. 4 where the time distribution of the AE bursts detected by S_1 is displayed for a relatively low value of the threshold A_0: Indeed, $N(\tau)$ approaches the triggered-event law (3), i.e., $\gamma \simeq 0.5$. Such an observation does not contradict our conclusions about the validity of the SOC models, because the data in Fig. 4 should be regarded as due to an "edge" effect [1]. As a matter of fact, a lava bed behaves like a diffusive interstitial medium, which damps the microtremors of the volcanic structure owing to its brittle consistency. It is no surprise that a triggered-event model may work in this case for a low value of A_0 (corresponding to the condition $b^2 \ll \sigma_x^2$ assumed in Ref. [9]).

Finally, we verified that SOC provides a robust model for the relaxational dynamics underlying the AE phenomena. We carried out separate statistical analyses for both the high- and low-activity phases of the AE recorded at Stromboli. Irrespective of the possibly different loading mechanisms feeding the energy (thermal and mechanical) released by AE, the time and amplitude distributions of the AE bursts obey very much the same scaling laws with indistinguishable critical exponents β and γ.

In conclusion, while providing a beautiful example of SOC, the universal properties of the volcanic activity reported above call for a new rationale in the surveys of microseismic activity in general. Because of the lack of characteristic time and length scales, the local crust dynamics is not very reminiscent of the loading mechanisms under scrutiny (plate indentation, lava flows, etc.), contrary to what is implied, for instance, by the linear-response theory. Significant surveys of volcanic activity are thus expected to cover frequencies and wavelengths well outside the applicability range of the SOC scaling laws.

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