Phase Transition in Small Metallic Junctions with Quasiparticle Dissipation

Vincenzo Scalia, Giuseppe Falci,[a] Rosario Fazio, and Gaetano Giaquinta
Istituto di Fisica, Facoltá di Ingegneria, Universitá di Catania, viale A. Doria 6, 95125, Catania, Italy
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We report Monte Carlo simulations for the $T=0$ phase diagram of a mesoscopic metallic junction. We present some evidence of a $T=0$ phase transition induced by the dissipation strength α. In the strong-coupling region the predictions of the spin-wave theory turn out to be correct and the algebraic decay of correlations implies quasi-long-range order, corresponding to the absence of a Coulomb-blockade voltage threshold. The critical junction resistance is estimated to be $R_t \approx 0.6\ \mathrm{k}\Omega$.

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The study of the quantum-mechanical behavior of macroscopic systems has attracted much attention from both the theoretical and the experimental points of view [1,2]. In their seminal paper, Caldeira and Leggett showed that in these systems it is crucial to consider the role of dissipation [2]. Starting from the Caldeira-Leggett model, Schmid has shown that a Josephson junction with Ohmic dissipation undergoes a ($T=0$) phase transition from a resistive to a superconducting state depending on the dissipation strength [3] α. The critical value of the dissipation is universal, i.e., it does not depend on the other coupling constants (the Josephson and the charging energies) of the system.

Guinea and Schön [4] investigated the dissipation arising from quasiparticles; they showed that also in this case the system has a phase transition. In this case, however, the critical value of α depends on the Josephson energy E_J and the charging energy E_C. In the limit in which E_J vanishes (normal-metal junction) the model is isomorphic to an XY model in one dimension with an interaction decaying as n^{-2} (n being the distance between the lattice sites). This last model was studied by Kosterlitz [5] who showed that it should not have any phase transition. This should imply that a normal mesoscopic junction at $T=0$ always exhibits a Coulomb-blockade region in the I-V characteristics [4] if $V \leq V_C$ (V_C is, eventually, exponentially small for large dissipation strength).

By contrast, in a recent work, Brown and Simanek [6] advanced the possibility of a phase transition in the one-dimensional XY model with $1/n^2$ interaction, which resembles the Kosterlitz-Thouless-Berezinskii (KTB) transition, by means of a Monte Carlo simulation. As is well known, a one-dimensional system can undergo a phase transition when the interaction is sufficiently long ranged (as in the 1D Ising model with $1/n^2$ interaction [7]). For the one-dimensional XY model the n^{-2} interaction is a marginal case and the situation is not clear. Using the Bogoliubov inequality it is possible to prove the absence of a spontaneous magnetization at finite temperatures. The analysis of the model in the spin-wave approximation showed that the power-law decay of the correlation functions implies an infinite susceptibility above a critical value of the coupling strength [8]. In the same approximation Odintsov [9] calculated the I-V curves, showing that they go like $I \approx V^a$ (a is larger than 1 and depends on the dissipation strength), with a vanishing threshold voltage.

The purpose of this Letter is to present the results of a Monte Carlo simulation of the system under consideration. The main result of this Letter is the calculation of the correlation functions in the strong-coupling region. In this regime the predictions of the spin-wave analysis turn out to be correct. Indeed, the correlator $g(n)$ (see below for the definition) is well described by the spin-wave result $n^{-\eta}$; the exponent, however, is renormalized with respect to that calculated in Ref. [8], and only far in the strong-coupling region is it inversely proportional to the coupling constant. The calculated values of η imply that this region is characterized by an infinite susceptibility. Therefore, in the large-dissipation limit, our simulations confirm the I-V characteristics of Ref. [9]. In the weak-coupling region the correlations will decay asymptotically like the interaction ($\eta=2$), the I-V are those of Ref. [4], and the susceptibility is finite. This implies that a phase transition takes place in the system: For a finite value of the dissipation strength the threshold voltage for the Coulomb blockade vanishes.

Ben-Jacob, Mottola, and Schön [10] showed that the partition function of a mesoscopic normal-metal junction can be expressed as a path integral over a phase field $\varphi(\tau)$. The Euclidean effective action reads (in units $\hbar=1$)

$$S[\varphi]=\frac{1}{4E_C}\int_0^\beta d\tau\left[\frac{d\varphi_i}{d\tau}\right]^2-\int_0^\beta d\tau\, d\tau'\,\alpha(\tau-\tau')\cos[\varphi(\tau)-\varphi(\tau')]\,, \tag{1}$$

where $E_C=e^2/2C$ is the charging energy (C being the geometric capacitance of the junction), and the phase $\varphi(\tau)$ is related to the voltage difference across the junction by the relation $d\varphi/d\tau=eV$. The dissipative kernel is defined as

$$\alpha(\tau)=\alpha\frac{(\pi/\beta)^2}{\sin^2(\pi\tau/\beta)}\,, \tag{2}$$

the nonlinear cosine form of the dissipative term being related to the discrete charge transfer between the electrodes. The dissipation strength is the dimensionless tunneling conductance α defined as

$$\alpha=\frac{1}{4\pi^2}\frac{R_0}{R_t}\approx\frac{0.66\,\mathrm{k\Omega}}{R_t},\quad R_0=\frac{h}{e^2}\approx 26\,\mathrm{k\Omega}\,. \tag{3}$$

We will refer to the infinite-susceptibility phase as the φ-ordered one. Since φ is canonically conjugate to the charge on the electrodes, order in φ means large quantum fluctuations of the charge, i.e., a disordered charge state. In the discretized-time version of the action the charging energy introduces a nearest-neighbor interaction in time so it is irrelevant as far as the critical properties are concerned (we will neglect it from now on; see Refs. [11,12]). The quasiparticle damping introduces a long-range correlation in time and therefore we are left with the analysis of the following effective action:

$$S[\varphi]=\sum_{n,n'}\alpha(n-n')\cos[\varphi_n-\varphi_{n'}]\,, \tag{4}$$

where the discretized-time version of the kernel is $\alpha(n)=\alpha(\pi/N)^2/\sin^2(\pi n/N)$; N is the number of lattice points in the time direction. The action at $T=0$ is then equivalent to the Hamiltonian of the one-dimensional classical XY model with $1/n^2$ interaction.

The Monte Carlo simulation was performed following the standard Metropolis algorithm. The ensemble averages were calculated over 100000 passes after 5000 thermalization steps in the strong-coupling region and 250000 steps in the critical region. Many independent runs, both heating and cooling, were performed and no hysteresis was detected. Starting from the strong-dissipation limit, the initial configuration with aligned phases was chosen. In our simulations we studied the phase-phase correlation function, the energy, and the specific heat; the last two quantities, although not very important for the metallic junction in itself, can help in the understanding of the dissipative phase transition. The largest lattice we considered was $N=100$ [13].

We start from the study of the phase-phase correlator defined as

$$g(n-n')=\langle\cos[\varphi_n-\varphi_{n'}]\rangle\,. \tag{5}$$

In the spin-wave approximation it is straightforward to evaluate the correlator, with the result

$$g(n)=\exp\left[-\frac{2}{\pi\alpha}V(n)\right]\approx n^{-\eta_{\mathrm{SW}}}, \tag{6}$$

noting that the function $V(n)$, defined as

$$V(n)=\frac{1}{N}\sum_{k_m>0}\frac{\sin^2(k_m n/2)}{k_m}\,,$$

is $\approx(1/4\pi)\ln(n)$ for large separation (yielding $\eta_{\mathrm{SW}}=1/2\pi^2\alpha$).

In Figs. 1 and 2 we present our results for the correlation function; $g(n)$ is plotted against the function $V(n)$ in order to minimize the effects of the boundary conditions [14,15] (in the figures the error bars are of the same order as the symbol sizes). In the strong-coupling region (Fig. 1) the straight lines point out that the system correlations decay following the behavior of Eq. (6). The value of the dissipation at which the system deviates from the spin-wave behavior is at $\alpha^{-1}\approx 0.71$; we can therefore bracket the critical resistance from below and obtain a value of $R_t\approx 0.47$ kΩ. In this region the I-V curves based on this algebraic law of the correlators do not exhibit the Coulomb blockade.

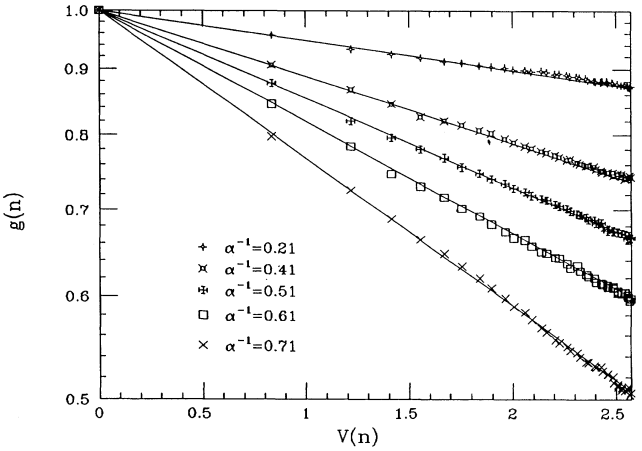

FIG. 1. The phase-phase correlation function plotted against the function $V(n)$ (which is essentially logarithmic but takes into account the periodic boundary conditions used in the simulations) in the strong-coupling region. The straight lines confirm that the correlations decay algebraically and therefore the system has quasi-long-range order.

In Fig. 2 we show the phase-phase correlation in the

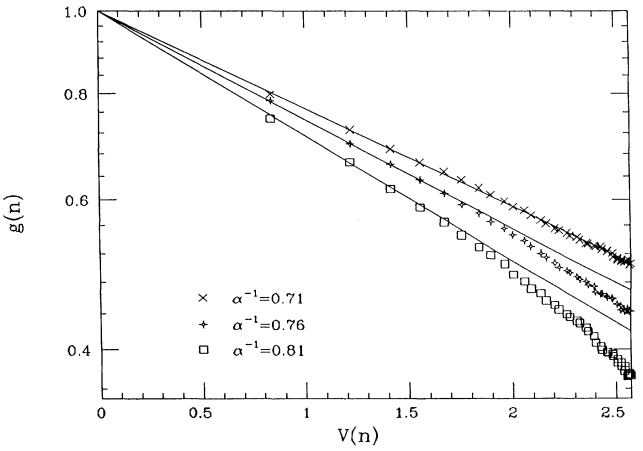

FIG. 2. The same as in Fig. 1 but in the critical region. The correlator calculated at $\alpha^{-1}=0.81$ starts bending downwards, showing a clear deviation from the spin-wave behavior.

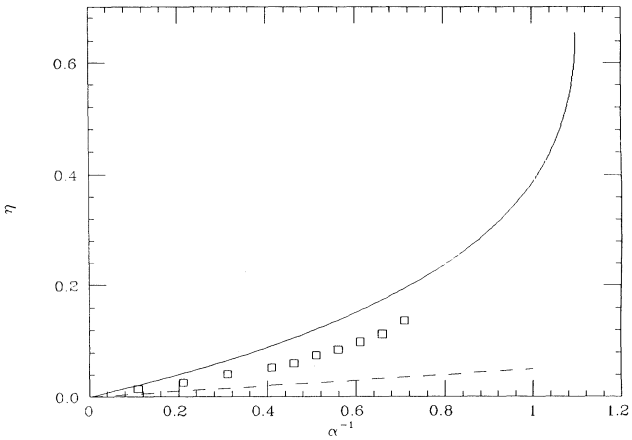

FIG. 3. The exponent η plotted against the dissipation strength; in the strong-coupling region the behavior is linear, as predicted by the spin-wave analysis (dashed line). We compare the results of the numerical simulation with the calculation of the exponent in the SCHA (continuous line), obtaining a good agreement. Near the critical region η deviates from the linear trend as in the analogous case of the 2D XY model with short-range interaction.

transition region. The $g(n)$ calculated at $1/\alpha=0.81$ deviates from the law of Eq. (6). In this region we also checked larger lattices ($N=250$), with the same outcomes. From the study of the specific heat and the energy per site the upper bound for the critical value of the resistance can be inferred. The energy per site versus the coupling constant has an inflection point which (roughly) separates the weak- and the strong-coupling regions. The specific heat, calculated from the fluctuation-dissipation theorem, is peaked at a value of $\alpha\approx 0.91$ (the curves being independent of the lattice dimensions) [16]. Our estimate for the critical resistance is therefore $R_t\approx 0.53 \pm 0.06\ \mathrm{k}\Omega$.

It is interesting to analyze also the behavior of the exponent η as a function of the dissipation (see Fig. 3). When the correlation functions follow the spin-wave–like functional behavior (i.e., for $\alpha^{-1}<0.71$) the experimental η is proportional to the slope of the curves in Fig. 1. In Fig. 3 we also report η_{SW} and the exponent calculated in the self-consistent harmonic approximation (SCHA) [16]. The latter also shows a good quantitative agreement with the Monte Carlo data (in the SCHA the critical threshold is $\alpha\approx 1$). The linear behavior of η vs α^{-1} typical of the bare spin-wave analysis is recovered up to $\alpha^{-1}\approx 0.4$. The topological excitations present in the system, although irrelevant in a renormalization-group procedure, lead to the renormalization of the spin-wave stiffness. When approaching the critical region the exponent substantially deviates from the linear behavior. This last result is very similar to what happens in the KTB transition. The exponent η, according to the scaling relations for the 2D Coulomb gas, is related to the inverse of the helicity modulus of the system [17]. However, in this case, in absence of a theory, we cannot employ the same relation.

Finally, we want to discuss a possible connection with the experimental results. In the $T=0$ I-V characteristics the phase transition is related to the vanishing of the threshold voltage for the Coulomb blockade at a finite value of the dissipation strength (according to our calculations α_{crit} is of the order of 0.8). On the other hand, some conclusions can also be drawn for the conductance $\sigma(T)$. Brown and Simanek [18] found that the junction changes its behavior from Ohmic to activated depending on the nominal tunneling resistance; the experiments carried on by the Delft group [19] showed a reasonably good agreement with the theory. From the experimental results it seems that the resistance tends to saturate at low temperatures for strong dissipation; in the opposite limit the resistance diverges strongly when T goes to zero. From Ref. [18] we know that $\sigma(T)$ is an integral of the phase correlator studied in this work; therefore a different temperature dependence is expected in the various regimes.

After the submission of this paper we became aware of a preprint by W. Zwerger where the same problem was studied.

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[a]Also at Department of Applied Physics, Technische Universitat-Delft, Lorentzweg 1, 2628 CJ Delft, The Netherlands.

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